An evaluation of the statistical significance of the association between northward turnings of the interplanetary magnetic field and substorm expansion onsets

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[1] An outstanding problem in magnetospheric physics is deciding whether substorms are always triggered by external changes in the interplanetary magnetic field (IMF) or solar wind plasma, or whether they sometimes occur spontaneously. Over the past decade, arguments have been made on both sides of this issue. In fact, there is considerable evidence that some substorms are triggered. However, equally persuasive examples of substorms with no obvious trigger have been found. Because of conflicting views on this subject, further work is required to determine whether there is a physical relation between IMF triggers and substorm onset. In the work reported here a list of substorm onsets was created using two independent substorm signatures: sudden changes in the slope of the $AL$ index and the start of a Pi 2 pulsation burst. Possible IMF triggers were determined from ISEE-2 observations. With the ISEE spacecraft near local noon immediately upstream of the bow shock, there can be little question about propagation delay to the magnetopause or whether a particular IMF feature hits the subsolar magnetopause. Thus it eliminates the objections that the calculated arrival time is subject to a large error or that the solar wind monitor missed a potential trigger incident at the subsolar point. Using a less familiar technique, statistics of point process, we find that the time delay between substorm onsets and the propagated arrival time of IMF triggers are clustered around zero. We estimate for independent processes that the probability of this clustering by chance alone is about $10^{-11}$. If we take into account the requirement that the IMF must have been southward prior to the onset, then the probability of clustering is higher, $\sim 10^{-5}$, but still extremely unlikely. Thus it is not possible to ascribe the apparent relation between IMF northward turnings and substorm onset to coincidence.

INDEX TERMS: 2788 Magnetospheric Physics: Storms and substorms; 2784 Magnetospheric Physics: Solar wind/magnetosphere interactions; 2744 Magnetospheric Physics: Magnetotail; KEYWORDS: point process, interplanetary magnetic field, magnetospheric substorm, triggering, statistical analysis


1. Introduction

[2] Twenty-five years ago, Caan et al. [1975] used superposed epoch analysis to demonstrate that a maximum in the tail lobe field is associated with the onset of the expansion phase of a magnetospheric substorm as measured by midlatitude positive bays. Quite by accident they also found that this maximum, and hence the substorm onset, is also associated with an apparent northward turning of the interplanetary magnetic field (IMF). Caan et al. [1975] pointed out that a similar statistical association could be seen in an earlier study of isolated substorms [Foster et al., 1971]. In a subsequent study Caan et al. [1977] demonstrated that this same association was present in time series of IMF $B_z$ and traces of auroral zone magnetometers. Sudden decreases in the horizontal component of the magnetic field occurring at the time of a brightening of the aurora appeared to be associated with northward turnings of the IMF. They concluded that the change in the orientation of the IMF was somehow “triggering” the onset of the magnetospheric substorm.

[3] These results did not receive much attention until Rostoker et al. [1983] published several particularly clear examples of apparent triggering. Dmitrieva and Sergeev [1983] and Sergeev et al. [1986] also suggested that the explosive phase of isolated substorms could begin under the influence of a change in the IMF $B_z$ component. Shortly thereafter Samson and Yeung [1986] used the technique of generalized superposed epoch analysis to again demonstrate a statistical association between substorm onset as defined
by ground magnetograms from former USSR stations and northward turnings of the z component of the IMF. Most researchers at this time dismissed this apparent relation as simply coincidence. The B_z component of the IMF is constantly changing from positive to negative. Substorm expansions occur frequently when the IMF has been southward for about an hour. This is close to what is thought to be the typical time that the IMF remains southward so it is inevitable that some onsets will appear to occur about the time of a northward turning. To counter this view McPherron et al. [1986] performed a statistical examination of all substorms occurring in a 6-month interval. They found that nearly half of all substorms could be associated with northward turnings of the IMF.

[4] These results were again ignored until Lyons [1995] proposed a substorm model ultimately requiring that all substorm expansions be triggered by northward turnings of the IMF or changes in IMF B_z. This view was so extreme that it immediately drew criticism from numerous researchers. Horwitz [1985] and McPherron et al. [1986] had published several examples of substorm onsets that occurred when there was no apparent change in IMF B_z. Lyons [1996] dismissed these examples as either not being substorms or as having been triggered by changes in the IMF B_z component. However, Henderson et al. [1996] immediately published examples that were unambiguously substorms (as determined by synchronous particle injection), yet there were no changes in any component of the IMF. The conclusion appeared to be that triggering of substorm expansion by the IMF is not necessary. Many researchers continued to believe that those events that appeared to be triggered were simply a matter of coincidence.

[5] A preliminary attempt to assess the statistical significance of the association between possible IMF triggers and substorm onsets was conducted by Lyons et al. [1997]. The authors found that the average number of associations between IMF triggers and substorm onsets was much smaller than the observed values if the triggers are randomly distributed in time. From a small set of substorms they estimated that the probability of chance coincidences between IMF triggers and substorm onsets was about 1 in 10^6 –10^9. They also noted that the increase in time association between IMF triggers and substorm onsets was much smaller than the observed values if the triggers are randomly distributed. Another problem with the coincidence. Unfortunately, this conclusion was based on only 20 highly selected substorm events. The risk in using such a small sample is that the results can sometimes be extremely biased. The best way to avoid this problem is to increase the size of sample. Another problem with the procedure used by Lyons et al. [1997] was that they could not easily specify the exact time delay between IMF B_z triggers and substorm onsets. They were able to estimate this time delay for only seven substorm onsets for which the solar wind monitor was close to the Earth-Sun line. This smaller set of well-timed events made the results even less convincing.

[6] Obviously, a more comprehensive investigation is necessary to decide whether IMF B_z triggers are truly associated with substorm onsets. This is the purpose of this paper. To examine the association we have created a much larger substorm database [Hsu and McPherron, 1996, 1998]. With this database we utilize a less familiar statistical technique to investigate whether the association between IMF triggers and substorm onsets is statistically significant. This technique is called the statistics of point processes. In the following sections we briefly explain this technique and use it to examine the time association between substorm onsets and IMF triggers. We will show that even though only half of all substorms are associated with apparent IMF triggers, the probability that this association is due to chance is less than 1 in 10^11 confirming the estimate of Lyons et al. [1997].

2. Point Processes and Their Related Parameters

[7] A point process is a phenomenon in which isolated events occur randomly in time and is represented by discrete points (often called arrivals) on the time axis. The variety of data that arise in practice and have point process character is staggering. Examples include neuronal electrical activity, heartbeats, radioactivity, seismology, accidents or failure processes. The statistics of point processes have been used to identify the causes of such phenomena as machinery malfunctions or nerve cell discharges [e.g., Cox, 1955; Cox and Lewis, 1968; Griffith and Horn, 1963]. These statistics were used in geophysics to determine whether there is an association between earthquakes and volcanic eruptions [Mulargia, 1992].

[8] In ordinary time series, the first and second-order moment functions are useful quantities that quantify statistical properties of the series, e.g., the mean and auto (or cross) correlation functions. In the analysis of point processes two analogous parameters are defined to measure the average rate of occurrence of points and degree of association between different points of the sequence. These are respectively the first-order product density (mean intensity or mean rate) and second-order product density function. The definition of these functions will be introduced in the next few paragraphs.

[9] A realization or a sample of the point process labeled A may be represented by a counting measure defined as

\[ N_A(t) = \#\{a_j, 0 < a_j < t\}, \] (1)

where the notation \#\{\cdot\} means the count of all members of the set defined by the condition within the curly brackets. In this case a_j are the times at which events of type A occur on the t axis and the condition that selects members of the set is that the event times must occur between the times \( t' = 0 \) and \( t' = t \). This count depends on the value of \( t \) hence the functional notation on the left side. \( N_A(t) \) can be visualized as a stair step function starting at zero and increasing by one each time an event is encountered. The first difference of this function is a sequence of spikes of unit magnitude located at the time of each event.

[10] Usually, it is assumed (1) that the points of the process A do not occur simultaneously, i.e., the events are isolated, separated by finite distances, (2) the parameters characterizing the process do not change with time, i.e., the process is stationary. In geophysics, the first assumption is usually satisfied. However, the second assumption is usually not satisfied. In this case, it is still possible to break the series into quasi-stationary sections and study each of them separately.

[11] Important parameters of a stationary point process A include the mean intensity, \( \lambda \), and the second-order product
density, $P_{AA}(u)$. The mean intensity of a point process $A$ can be defined as

$$P_A = \lim_{h \to 0} \frac{\text{prob}\{\text{type } A \text{ event in } (t-h, t+h)\}}{2h}.$$  \(2\)

While the second-order product density is defined as

$$P_{AA}(u) = \lim_{h \to 0} \frac{\text{prob}\{\text{type } A \text{ event in } (t+h, t+2h) \text{ and type } A \text{ event in } (t-h', t+h')\}}{4hh'}.$$  \(3\)

$$P_{AB}(u) = \lim_{h \to 0} \frac{\text{prob}\{\text{type } A \text{ event in } (t+h, t+2h) \text{ and type } B \text{ event in } (t-h', t+h')\}}{4hh'}.$$  \(4\)

where $h$ and $h'$ are small and nonnegative. $P_{AA}$ and $P_{AB}$ are respectively the second-order auto and cross product density functions.

[12] The mean intensity determines whether there are many events or just a few in a given time interval. When $P_A$ is large there tend to be many points in the neighborhood of $t$ and vice versa. The second-order product density measures the relative positioning of pairs of events in a single train while the cross product intensity associates events in one train with those in another.

[13] In statistics, it is sometimes convenient to use the conditional probability to examine whether two types of events are dependent or independent. The analogous parameter for point processes is called conditional mean intensity.

[14] We can define a conditional mean density $m_{AB}$ at lag $u$ as

$$m_{AB}(u) = \lim_{h \to 0} \frac{\text{prob}\{\text{type } A \text{ event in } (t+h, t+2h) \text{ given a type } B \text{ event at } t\}}{2h} = \frac{P_{AB}(u)}{P_B}.$$  \(5\)

[15] If the type $A$ points are distributed independently of the type $B$ points we have $m_{AB} = P_A$. Thus

$$P_{AB}(u) = P_A P_B.$$  \(6\)

[16] Since this relation is derived from the assumption of independence between type $A$ and $B$ processes, a comparison of $P_A$, $P_B$, and $P_{AB}$ might provide some useful information to estimate the degree of association between type $A$ and $B$ processes.

### 3. Estimation of the Parameters in a Point Process

[17] In an elementary situation the probability that a single type of event will occur out of $N$ possible events can be estimated from $n/u/N$. In this notation, $n$ is the number of events that occurred out of $N$ possible events. Similarly, we can use this idea to estimate the mean intensities $P_A$, $P_B$ and the second-order cross product density function $P_{AB}$. Suppose that type $A$ and $B$ events occur uniformly throughout the interval $(0, T)$. Also assume that the events are ordered, i.e., $a_1 < a_2 \ldots < a_M$ and $b_1 < b_2 < \ldots < b_N$, and $M$, $N$ are the total number of events for the type $A$ and $B$ processes respectively. Then,

$$\hat{P}_A 2h = \lim_{h \to 0} \frac{\text{prob}\{A \text{ event in } (t-h, t+h)\}}{2h} \sim \frac{2h}{T} M.$$  \(7\)

[18] Thus we can estimate $\hat{P}_A$ as $M/T$. Likewise, we can estimate $\hat{P}_B$ as $N/T$.

[19] Now we estimate the cross product probability density, $\hat{P}_{AB}$. To do this we must define the association number between two point processes. Assume that we have two point processes $A$ and $B$ with events occurring at times $a_i$ and $b_j$ as illustrated in Figure 1. For every point on the $B$ axis construct an interval of width $2h$ centered at a time $b_j + u$. For each $b_j$ obtain the count of all $a_i$ that fall within this interval. This is the number of $A$ events associated with a particular $B$ event. Using the notation introduced in equation (1) we have

$$n(u, h, j) = \#\{a_i ; |a_i - b_j - u| < h\}. \label{eq:8}$$

i.e., the number of $a_i$ events that fall into the interval $[b_j + u - h, b_j + u + h]$. This count depends on which $B$ event is considered, the lag $u$, and the width of the interval $2h$. The association number is the sum of all such counts, i.e.,

$$n(u, h) = \sum_{j=1}^{N} n(u, h, j) = \sum_{j=1}^{N} \#\{a_i ; |a_i - b_j - u| < h\}. \label{eq:9}$$
Using the same idea as in our estimation of \( \hat{P}_B \) we can estimate the conditional probability \( m_{AB}(2h) \) by \( n(u,h)/N \). Therefore, by using (9), we have

\[
\hat{n}_{AB}(u)2h = \frac{n(u,h)}{N} = \hat{P}_{AB}(u)2h/\hat{P}_B.
\]

This relation can be written as

\[
\hat{P}_{AB}(u) = \frac{n(u,h)\hat{P}_B}{N2h} = \frac{n(u,h)}{2hT}.
\]

It should be pointed out that this measure for \( \hat{P}_{AB} \) is not exact, but it should be near the correct value, especially when \( T \) is large. This method of estimating \( n(u,h) \) was proposed by Griffith and Horn [1963]. The figure shows two types of events and some intervals of length 2h. The darker arrows represent type A events while lighter arrows represent type B events. The summation of all counts of the number of A events associated with each B event gives the estimation of \( n(u,h) \) at time lag \( u \).

4. Evaluation of the Chance Coincidence Between IMF Bz Triggers and Substorm onsets

Once we have estimated \( n(u,h) \) we can evaluate the significance of the degree of association between the two point processes using a method proposed by Mulargia [1992]. This procedure uses statistical hypothesis testing. The first step is to formulate a null hypothesis. Our null hypothesis is that the association between substorm onsets and IMF triggers is a chance occurrence. If at least one of the point processes (substorm or IMF triggers) is a Poisson process, i.e., events are randomly distributed along the time axis according to a Poisson distribution, then this hypothesis can be evaluated using the upper part of the cumulative Poisson probability with a mean value of \( \mu = N_{sub}/T \).

In the estimation of \( \mu \), \( N_{sub} \) and \( N_{IMF} \) represent the number of substorms and number of IMF triggers respectively, \( T \) is the total observation period, and \( \Delta t \) is the association time window, which is equal to 2h. Thus the significance level \( \alpha \), which is the probability of making a false rejection of a true hypothesis, can be calculated as

\[
\alpha = \sum_{k=n(u,h)}^{\infty} \frac{\mu^k}{k!} e^{-\mu},
\]

which is the equation (11b) from [Mulargia, 1992]. As \( \alpha \) gets smaller, the probability of falsely rejecting the true hypothesis declines. Therefore \( \alpha \) can be used to measure the degree of association between two point processes. For every specified \( \alpha \), we can find a corresponding \( n_{ah}(u,h) \), which represents the “threshold” for the confidence level. If the calculated \( n(u,h) \) is less than the threshold number \( n_{ah}(u,h) \) then the examined hypothesis cannot be rejected at the significance level of \( \alpha \). Usually, in statistical hypothesis testing \( \alpha \) equals either 1% or 5%.

5. Event Selection

5.1. Selection of Substorm Onsets

A procedure proposed by Hsu and McPherron [1996, 1998] was used to select substorm onsets. Six-hour plots of the AU and \( AL \) indices were scanned interactively on the computer. A cross hair was set at the beginning of sharp decreases in \( AL \) provided that the decrease exceeded 100 nT and persisted for more than 30 min. A list of all such onsets was written to a digital file. Then high time resolution magnetometer data (<3 s) from either the AFGL or IGS magnetometer chains were analyzed. The north components from the entire chain of stations were band pass filtered in the Pi 2 band (40 s < T < 150 s). The filtered traces from each station were then stacked plotted on the computer screen with resolution such that individual cycles of the Pi 2 waveform could be resolved. The plots were scanned only when some station in the network was within 3 hours of either side of midnight. A Pi 2 was identified as a short burst of wave activity of duration of order 10 min that grew rapidly in amplitude out of the background. At least three stations in the network must have recorded the event to be considered. A cross hair was then moved to the earliest time in the event where it was obvious that the Pi 2 waveform exceeded the background noise present in the data. The earliest time of all the stations observing the event was taken as the Pi 2 onset time and written to a digital file. In the final step the list of Pi 2 onsets associated with the list of sharp \( AL \) onsets. Any \( AL \) onset without a Pi 2 onset within plus or minus 20 min was dropped. Similarly any Pi 2 onset without a corresponding negative bay in \( AL \) was dropped. In the case of multiple Pi 2 onsets near a single \( AL \) onset, the closest Pi 2 onset was taken as the main substorm onset.

5.2. Selection of IMF Bz Triggers

Since it has been argued by Lyons et al. [1997] that the probability of detecting IMF triggers is a strong function of the distance of the monitor from the Earth-Sun line we tried to minimize this problem through selection of data close to the Earth-Sun line. Indeed, the scale size of IMF structure in a direction normal to the IMF may be less than the 40 Re width of the magnetosphere [e.g., Crooker et al., 1982; Russell et al., 1980]. Thus we used IMF observations taken by the ISEE spacecraft (apogee \( \approx 22 \) Re) when it was near noon just upstream of the bow shock [Formisano, 1982]. Because of seasonal precession of its highly elliptic orbit the ISEE spacecraft was in this region during the fall season. From the distribution of \( \rho \), the distance from solar wind monitor to the Earth-Sun line shown in Figure 3, it is clear that ISEE-2 IMF observations were made very close to the Earth-Sun line. The median distance to the Earth-Sun line is about 6–7 Re. Because of this orbital advantage, we selected October of 1978 and September–October of 1979 as our study period.

A procedure proposed by Lyons et al. [1997] was used to select IMF triggers. A computer program was written to choose IMF events satisfying a set of criteria optimized to maximize the association with substorm onsets. Obvious failures of this algorithm, mostly due to missing or bad data, were eliminated through a final visual inspection. An example of IMF triggers selected by this
procedure is presented in Figure 4. The upper panel shows the ISEE-2 IMF observations and the bottom panel shows the AL index. In this interval three possible IMF triggers were selected by the Lyons et al. [1997] procedure. However, an obvious IMF trigger around 1300 UT was not identified because of the preceding data gap. This particular IMF trigger seems to have had no effect on the corresponding AL index plotted in the panel below, i.e. there was no sudden sharp drop in AL.

Figure 2. An example of substorm onset timing. Two Pi 2 pulsation bursts occur between 0300 and 0600 UT. The first Pi 2 is clearly associated with the sudden change in slope of the AL index and is taken as the substorm onset. The vertical line indicates the onset of the first Pi 2 pulsation burst. The second Pi 2 burst is associated with a minor perturbation in the slope of the AL index during the recovery phase of the earlier substorm and thus was not used as a substorm onset time.

We have estimated the propagation time of each IMF trigger to Earth by calculating $T_{\text{prop}} = T_{\text{solar wind}} + T_{\text{magnetosheath}} + T_{\text{field line}}$ where $T_{\text{solar wind}}$ is the solar wind propagation time from ISEE-2 to Earth assuming solar wind discontinuities are aligned along the average spiral angle of the solar wind [Baker et al., 1983], $T_{\text{magnetosheath}}$ (4 min) is the propagation time from bow shock to magnetopause, and $T_{\text{field line}}$ (2 min) is the propagation time from magnetopause to earth [e.g., Lockwood et al., 1989].
After adding the propagation delay the estimated arrival times of these IMF triggers at Earth are plotted as vertical thin lines in the bottom panel of Figure 4. Three dotted lines represent the main Pi 2 onset times determined from AFGL network pulsation data for each AL onset [Hsu and McPherron, 1996, 1998]. It is apparent that one of the substorm onsets is associated with an IMF northward turning at 1427 UT, i.e. it appears to have been triggered by the IMF. Other substorm onsets are apparently not associated with IMF northward turnings. An interesting feature in this figure is that the triggered substorm seems to have a sharper AL onset than do other substorms. Whether this is a real phenomenon or coincidence remains to be seen. Nevertheless, in this example, three possible IMF triggers and three substorm onsets are identified. Among them, only one trigger and substorm onset meets our criteria for association. This corresponds to an association number \( n(u, h) = 1 \) at zero and adjacent lags.

6. Statistics of the IMF Triggers and Substorm Onsets

The total time span of IMF data available from ISEE-2 in the 3 months was 61,731 data points at 1-min resolution. During this time the automatic procedure identified 145 substorm onsets and 367 possible triggers. From this we can estimate the mean intensity for both series. We find that \( \hat{P}_{\text{substorm}} = 2.3 \times 10^{-3} \) and \( \hat{P}_{\text{triggers}} = 5.9 \times 10^{-3} \). It is apparent that there are nearly three times as many IMF \( B_z \) triggers as substorm onsets. The expected mean value for chance association can be estimated as \( m = \frac{(2h \times 145 \times 367)}{61731} \). Usually, because of the uncertainty of the arrival time of IMF triggers at Earth, \( h = \pm 10 \) min is used for the width of the association window [e.g., Lyons et al., 1997]. In this case \( m = 17.2 \) is the number of associations expected by chance.

7. Significance of Time Association Between IMF Triggers and Substorm Onsets

A graph of \( n(u, h) \) is presented in the upper half of Figure 5. It is obvious that there is a large peak near zero time delay. The meaning of this peak is that there is a very high probability of a propagated northward turning of the IMF arriving at the earth in a 20-min interval centered on the time of a substorm onset. In other words, substorms appear to be triggered.

A simple procedure for estimating the significance of the time association between substorm and IMF triggers makes use of the asymptotic value \( n(u, h) \) as the offset \( |u| \to \infty \). Intuitively, as \( |u| \to \infty, n(u, h) \) should approach a limiting value corresponding to chance occurrences of association between the two series of events (IMF \( B_z \) triggers and substorm onsets). If we assume that the IMF \( B_z \) triggers and substorm onsets are independent of each other for \( |u| < 6 \) hours (to \( |u| < 20 \) hours), we obtain an average \( n_{\infty}(u, h) \) of about 15 with a standard deviation of 4.5. The distribution of this "random" association number is presented in the bottom half of Figure 5. While the mean value \( n_{\infty}(u, h) \) from this sampling is consistent with the theoretical value \( \mu \) calculated above it will be noticed that a small difference does exist. This may be due to the limited

![Figure 3](image-url) **Figure 3.** A schematic of the ISEE-2 orbit in fall 1978 and 1979. The ISEE-2 orbit has a semiminor axis of approximately 7 Re and samples the solar wind upstream of the subsolar point of the bow shock. At the bottom of this figure there is plotted a histogram of the perpendicular distance of ISEE-2 from the Earth-Sun line during the interval used in the study. It is clear that ISEE-2 was within \( \sim 10 \) Re of the Earth-Sun line most of the time.

![Figure 4](image-url) **Figure 4.** (opposite) The upper panel shows the automatically selected IMF \( B_z \) triggers, which are presented as thick circles. The bottom panel shows the \( AL \) index and substorm onsets at 1048, 1423, and 2017 UT, which are presented as vertical lines. The estimated arrival of IMF \( B_z \) triggers is shown by vertical dashed lines. The estimated time delay of these possible IMF \( B_z \) triggers are shown on the top of this figure. It is clear that only one of the IMF \( B_z \) triggers is associated with substorm onset at 1423 UT. The other triggers do not cause clear change in the \( AL \) index.
Automatic Identified ISEE-2 IMF Triggers With AL Index 09/24/79

\[ B_z \text{ (nT)} \]

\[ T_{\text{prop}} = 7 \text{ mins.} \quad T_{\text{prop}} = 8 \text{ mins.} \quad T_{\text{prop}} = 7 \text{ mins.} \]
Time association number (similar to cross-correlation)

### ISEE-2

- $\alpha: 10^3$
- $\alpha: 10^2$
- $\alpha: 5 \times 10^2$

Asymptotic mean

Asymptotic $n(u, h)$ at $6 < |u| < 20$ hours

- Mean: 14.7
- Std: 4.5

Asymptotic Association Number; $n(u, h)$
sample of the “random” association number, or possibly bias from the data gaps in our IMF and pulsation data. Since the ISEE-2 IMF data could not be used inside the magnetopause it is possible that as we “cross-correlate” two point series, these data gaps cause reductions in the association number \( n_{\text{as}}(u, h) \) thus giving a smaller mean value. Similarly, we may miss accurate determinations of Pi 2 onsets as the stations rotate out of the midnight region.

13 The average \( n_{\text{as}}(u, h) \) is superimposed in the upper half of Figure 5 for comparison. It is obvious that the peak at zero lag exceeds the average value by more than 7 standard deviations. If we assume that the \( n_{\text{as}}(u, h) \) are normally distributed we find the peak value corresponds to a chance probability of \( 10^{-14} \). This suggests very strongly that the relation between IMF triggers and substorm onsets is not due to coincidence! However, in reality this is a binomial experiment because of the failure or success nature of this time association. Thus a binomial distribution is more appropriate to use in this examination. For an experiment with a large number of samples a binomial distribution can be replaced by a Poisson distribution [e.g., Walpole and Raymond, 1985].

34 In this study we found that \( 2hN_{\text{substorm}} \ll T \) and \( N_{\text{trigger}} \) and thus equation (7) derived from a Poisson distribution is a good approximation for estimating the chance probability. It should be noticed that in order to use this formula at least one of the point series (substorm or IMF triggers) must be a Poisson process. To examine this assumption, we fit these two series with a stationary Poisson process, i.e., exponential fit for time intervals between consecutive events. A Kolmogorov-Smirnov two-tail test yields a KS value of 0.13 for IMF triggers and 0.20 for the substorm onsets. These correspond to probabilities that these are stationary Poisson process equal to 0.98 and 0.65 respectively. Thus this result suggests that a stationary Poisson process is a good approximation for the IMF trigger series but is inappropriate for the substorm onset series. This is not a surprise because substorms may occur both periodically and randomly [e.g., Borovsky et al., 1993; Prichard et al., 1996]. Periodic substorms would not exhibit a Poisson distribution.

15 After validating the use of equation (12) we can estimate the chance probability between IMF triggers and substorms. In the upper half of Figure 5 three different horizontal dashed lines are plotted. They represent three different \( n_{\text{as}}(u, h) \) corresponding to chance “thresholds” at significance levels of \( \alpha = 0.05, 0.01 \) and 0.001 respectively. The peak value of \( n_{\text{as}}(u, h) \) exceeds all of these “thresholds” and in fact even passes the significance level of \( 10^{-11} \). This implies that there is less than 1 chance in \( 10^{-11} \) that this result could occur by chance alone. It is thus not possible that the association between the IMF triggers defined by the Lyons procedure and substorm onsets can be attributed to coincidence. Also note that no other significant peak is present in the graph. Our calculation supports with even higher confidence the result obtained by Lyons et al. [1997] that the chance probability is \( 10^{-6} - 10^{-5} \). Physically, this gives strong support to the contention that at least some substorms are triggered by IMF northward turnings.

8. Choosing the Length of the Time Association Window

36 Past studies have not examined how conclusions regarding substorm triggering depend on the width of the association window. In fact, different studies have used different widths. To determine the appropriate width we use the procedure described above to calculate the association number as a function of \( u \) for different values of \( h \). The result is shown in Figure 6. The upper panel shows some selected association number graphs calculated with different values of \( h \). For each \( h \) the curve of \( n(u, h) \) is different. The dashed lines through each curve represent the “threshold” value of \( n_0(u, h, \alpha = 10^{-3}) \) for each \( h \). It is clear that as \( h \) increases there are more possible associations and \( n(u, h) \) also increases. However, the value of \( n(u, h) \) above which we are confident that the association is significant increases even faster and eventually exceeds the curve. Similarly, as \( h \) decreases there are fewer possible associations. Again the level corresponding to a fixed confidence exceeds the curve. This suggests that there is an “optimal” choice of the width of the time association. Since the absolute difference between \( n(u, h) \) and the related “threshold,” \( n_0(u, h, \alpha = 10^{-3}) \), changes significantly with \( h \) it is not easy to determine the optimum width directly. A solution to this problem is to plot a map in the \((u, h)\) plane corresponding to each value of the graph of \( n(u, h) \). This map is presented in the bottom half of Figure 6. An optimal area is clear as the dark hole in this map. This extreme area corresponds to a probability of 1 in \( 10^{-11} \) that this value of \( n(u, h) \) could happen by chance. Thus the optimal time association window ranges from 16–24 min \((h = 8–12)\) at time delay about \(-3\) min with respect to substorm onset.

9 Discussion

9.1 Probability of Northward IMF Turning From Southward IMF

37 In our analysis of the association between IMF triggers and substorm onsets, we used two point processes: substorm onsets and IMF northward turnings. We found that the probability of their chance association appears to be extremely low. However, it might be suggested that the high probability of association is a consequence of the natural tendency of the IMF to turn northward after an interval of southward IMF. If it were the case that all intervals of southward IMF were exactly 1 hour long, and all substorm growth phases were as well, then every onset would be correlated with a northward turning. Such a circumstance is

Figure 5. (opposite) The upper panel shows the time association number \( n(u, h) \) at \( h = 10 \) min. A large peak near zero time delay is obvious. There are no other significant peaks throughout the time span. The lower panel shows the asymptotic association number at very large time delay \( u \). The peak value in the upper panel has a significance level at \( 10^{-14} \) if a normal distribution is assumed. More appropriately, if a Poisson distribution is used the significance level of the peak is \( 10^{-11} \). Three different thresholds of significance level at 0.05, 0.01 and 0.001 are added for comparison.
Figure 6. The upper panel shows traces of the total association number as a function of the offset \( u \) for four different values of \( h \), the half width of the time association window. It is clear that \( n(u, h) \) increases as \( h \) increases. Yet the “deviation” between \( n(u, h) \) peak value and the threshold value (\( \alpha = 10^{-3} \)) also changes with \( h \). The bottom panel presents the significance level of \( n(u, h) \) at different \( h \). An optimal choice of 16–24 min (\( h = 8 – 12 \)) with \( \alpha = 10^{-11} \) is clear in this figure.
 unlikely to occur because of the power law spectrum of the IMF, and because the duration of the growth phase of substorms varies over a range of 20–200 min, with 55 min most typical [e.g., McPherron, 1991]. Nonetheless we are led to consider the question: What is the probability for a northward turning of the IMF after some extended interval of southward IMF? If the probability of an IMF polarity change after some specified interval of southward IMF $B_z$ is extremely high, then the association between IMF $B_z$ northward turnings and substorm onsets may still be coincidental rather than physical. In our case our trigger criteria requires that the IMF have been southward for at least 20 of the 30 preceding minutes.

A similar question has been considered for earthquakes [Sornette and Knopoff, 1997]. Is the “big one” due? They have discussed this hypothesis, asking (can it be): The longer it has been since the last earthquake, the longer the expected time till the next? This question is very similar to what we are examining here. Therefore we use the method developed by these authors to examine the hypothesis, $Q$, of whether the longer there has been southward IMF, the shorter the expected time till a northward turning.

Let us assign $p(t)$ as the probability density that the time interval between southward and northward turnings of the IMF $B_z$ is $t$. The question we must answer is: what is the probability density function $g(t)$ that we must wait an additional time $t'$ until the northward turning IMF $B_z$, given the time $t$ since the southward turning. From elementary statistics, the conditional probability $P(A/B)$ can be written as

$$P(A/B) = \frac{P(A, B)}{P(B)},$$

(13)

given the knowledge of the joint probability $P(A, B)$, and the marginal probability $P(B)$.

Thus we have $P(A, B) = \int_{-\infty}^{+t'} p(u)du = \int_{-\infty}^{+t'} p(u)du$, which is the probability that the northward turning of IMF $B_z$ will occur at time $t'$ from $t$, and $P(B) = \int_{-\infty}^{+t} p(s)ds$, which is the probability that the IMF $B_z$ has been southward for a time $t$. Therefore, we have

$$g(t') = \frac{p(t + t')}{\int_{-\infty}^{+t} p(s)ds},$$

(14)

which is clearly normalized since $\int_{0}^{\infty} g(t')dt' = 1$.

Using this formula, we can calculate the expected time until the northward turning of IMF $B_z$, $\langle t' \rangle$ as a function of the time since the southward turning of IMF $B_z$. As suggested in [Sornette and Knopoff, 1997], the answer to our hypothetical question $Q$ is given by the sign of $\frac{d\langle t' \rangle}{dt}$, if $\langle t' \rangle$ exists.

From equation (14), the average expected time to the northward turning of IMF $B_z$ is

$$\langle t' \rangle = \frac{\int_{0}^{\infty} t' p(t + t')dt'}{\int_{-\infty}^{+t} p(u)du}.$$  

(15)

By a simple change of variable, we have

$$\langle t' \rangle = \frac{\int_{-\infty}^{+t} (u - t)p(u)du}{\int_{-\infty}^{+t} p(u)du}.$$  

(16)

We can integrate the numerator of (16) by parts and get

$$\int_{-\infty}^{+t} (u - t)p(u)du = (u - t) \int_{-\infty}^{+t} p(s)ds|_{0}^{+t} - \int_{-\infty}^{+t} du \int_{-\infty}^{+t} p(s)ds.$$  

(17)

The first part of (17) is zero at both ends of the integration because $\lim_{x \to -\infty} \int_{x}^{+t} p(s)ds = 0$ and thus we have

$$\langle t' \rangle = \frac{\int_{-\infty}^{+t} du \int_{-\infty}^{+t} p(s)ds}{\int_{-\infty}^{+t} p(u)du}. $$

(18)

In (18) the denominator is the first cumulative integral and the numerator is the second cumulative integral of the probability density. We can simply write the expected time $\langle t' \rangle$ as

$$\langle t' \rangle = f(t) \int_{-\infty}^{+t} f(t')dt',$$

(19)

where $f''(t) = p(t)$, i.e., $f(t)$ is the second cumulative integral of $p(u)$. This from of Sornett and Knopoff [1997, equation (6)]. Finally, we have

$$\frac{d\langle t' \rangle}{dt} = \frac{f(t)f''(t) - [f'(t)]^2}{f'(t)^2} > 0.$$  

(20)

Thus an examination of the probability density function $p(u)$ will answer the question of whether the northward turning of IMF $B_z$ will occur in the near future or whether we have to wait longer given a southward IMF $B_z$ duration $t$.

[40] To estimate the probability distribution between southward and northward turnings of IMF $B_z$ we used 6 continuous months of 5-min resolution ISSE-3 data (January to June 1979). The duration of southward IMF $B_z$ intervals was calculated and used to construct the probability distribution. In Figure 7, this complementary cumulative probability distribution is plotted. Using a least squares fit we find that this distribution, $p_s$, can be represented as $p_s(t) = 1.4e^{-t/13.1}$, in which $t$ is the time duration of the southward IMF $B_z$. It should be noticed that $P_s(t)$ is actually $f'(t)$, the cumulative integral of $p(u)$. In essence, this distribution is a Weibull distribution with an exponent less than 1 [see also Sornette, 2000, chapter 6; Sornette and Knopoff, 1997], a stretched exponential distribution. A particular characteristic of this distribution is that the longer we have waited since the last event, the longer the time to the next event. This is demonstrated in the next paragraph.

[41] Given that $f'(t) = ce^{-\sqrt{t}/\tau}$, we have $f(t) = -2e^{-\sqrt{t}/\tau}e^{-\sqrt{t}/\tau} = \frac{e^{-\sqrt{t}/\tau}}{\sqrt{t}/\tau} e^{-\sqrt{t}/\tau}$ in which we have set $c = 1.4$ and $\tau = 33.1$ for simplicity. After substituting these formulas into equation (20), we have

$$f(t)f''(t) - [f'(t)]^2 = \frac{e^{-2\sqrt{t}/\tau}}{\sqrt{t}/\tau}. $$

(21)

It is apparent from equation (21) that $\frac{d\langle t' \rangle}{dt} > 0$ for finite $t$. If $t \to \infty$, $\frac{d\langle t' \rangle}{dt} \to 0$. However, this would imply that IMF $B_z$ can remain southward forever, a nearly impossible situation for IMF $B_z$. Thus this result suggests that the
hypothesis Q, the longer it has been since the last southward turning of IMF, the shorter the time expected till the northward turning, is not correct. Instead, the opposite hypothesis that the longer it has been since the last southward turning of IMF $B_z$, the longer the time expected till the northward turning is true. This implies that the probability of a northward turning of IMF $B_z$ after our “preselected” southward IMF $B_z$ period of 20 min used in determining a trigger may not be “extremely” high as we imagined.

A study done by [Rostoker et al., 1988] examining the duration of intervals of IMF polarity obtained a similar result. Their study found that 65.7% of the IMF $B_z$ data do not change their polarity in a 1-hour time interval. This is consistent with our finding, i.e., the polarity change from southward IMF to northward IMF occurred in a time frame much longer the 20 min used in our trigger selection.

[50] We can estimate the probability that a northward turning will have occurred within some specified time after the southward turning by using the cumulative probability distribution in Figure 7. Now the problem is to find the northward turning probability $P_{north}(t)$ after a time $t$. In Figure 7 we have the cumulative probability that the duration of southward IMF $B_z$ exceeds a time $T$. Thus the probability that $B_z$ will have turned northward within an additional time $t$ can be estimated by the difference in cumulative probabilities at $t$ and $t + t$, which is $P_{north}(t) = P_{north, t + t} - P_{north, t}$. Thus

$$P_{north}(t) = \frac{P_{north}(t + t) - P_{north}(t)}{P_{north}(T)} = 1 - e^{\frac{-\sqrt{(T+t)/\pi}}{\sqrt{T/\pi}}},$$

(22)
which is normalized because $P_{\text{north}} = 0$ at $t' = 0$ and $P_{\text{north}} = 1$ as $t' \rightarrow \infty$.

[51] In our case, we can set $T = 20$ min because this is the growth phase requirement in the “IMF trigger” selection procedure [Lyons et al., 1997]. The result presented in Figure 8 shows that the probability of a northward turning within the first 10 min after 20 min of southward IMF is less than 15%. It is thus not correct to say that there is an extremely high probability of a northward turning within our ±10-min association window if the field has been southward for 20 min.

[52] Based on this result, it does not seem likely that the occurrence of northward IMF after 20 min of southward IMF is highly probable in the next 10–20 min. On the contrary, there is only a 50% chance that the polarity change from southward to northward has occurred 50 min beyond the end of our trigger selection window.

[53] The preceding argument can be extended to estimate the number of substorm onsets that will appear to be associated with a trigger as a result of our selection criterion that the IMF must have been southward for longer than 20 min before the northward turning. Suppose that all of the substorms in our list had a growth phase with duration 60 min, i.e. that the time of the substorm onset was actually 60 min after the southward turning. We then want to know the probability of a northward turning within a 20-min interval centered at 60 min. Let $T = T_0 - h$ and $t' = 2h$ in equation (22) so that we obtain

$$P_{\text{north}}(T_0 \pm h) = \frac{1}{e^\sqrt{(T_0+h)/T_0}} - \frac{1}{e^\sqrt{(T_0-h)/T_0}}.$$ (23)

[54] For the chosen values this reduces to $P_{\text{north}} = 0.2019$. If the typical growth phase were either 30 min or 90 min the corresponding probabilities would be 0.2756 and 0.1679. Note that the probability of a chance association decreases with increasing duration of the growth phase because of the likelihood that the IMF has already turned northward at an
earlier time. Since the most probable duration of substorms in our list was 55 min (data not shown) it is apparent that the probability of a chance association will be about 20%. If we multiply the 145 observed substorm onsets by this fraction we obtain 29 events that were possibly a result of chance. This number should be compared to our observation of 15 chance associations at times far from the expected arrival at Earth of the IMF trigger. The number of associations is clearly larger than what we would expect if the two events were independent, but is still much smaller than the 52 associations actually observed at zero lag. The difference divided by the standard deviation is (52−29)/4.5 or 5.11 standard deviations from the background level. For a normal distribution the probability of obtaining this difference by chance is about $6.8 \times 10^{-6}$. This number is considerably larger than our earlier estimate for independent processes, but still represents an exceedingly unlikely coincidence.

[55] A more accurate estimate of the associations resulting from our growth phase criterion would require knowledge of the actual duration of every substorm growth phase in our list of substorms. This list could then be used to weight the probabilities of chance association for different durations. If this distribution were skewed it could either increase or decrease our estimate somewhat. Unfortunately, this information was not retained in our survey of the data because our null hypothesis viewed the two processes as completely independent. However, since most substorms have about an hour-long growth phase we are confident that our estimate is reasonably correct, and that our primary conclusion remains unchanged.

9.2. Time Propagation Error

[56] In this paper, we have tried our best to estimate the time propagation error from the solar wind monitor (ISEE-2) to the Earth-Sun line. Unfortunately, the ISEE-2 plasma instruments failed in June 1978 so it was not possible to get the solar wind velocity from the ISEE-2 plasma instrument. Plasma data are available from the IMP 8 spacecraft, but only rarely were both spacecraft in the solar wind at the same time. An examination of the measured IMP-8 values of speed during the study interval gave 295, 390 and 485 km/s for the median and quartiles of speed. Thus we used a solar wind speed of 400 km/s to propagate the solar wind to the magnetopause. The difference in velocity between the upper and lower quartiles and the median causes about ±30 s difference in time delays. Since the time resolution of the data is 1 min, the errors caused by using the median solar wind speed are negligible.

[57] Another possible propagation error comes from our assumption of a Parker spiral IMF. When IMF $B_z$ is large, the varying orientation of the IMF may cause significant uncertainties. Fortunately, as shown in Figure 3, the medium value for $p$, from the ISEE-2 to the Earth-Sun line is only about 6–7 Re. Therefore, even if the IMF $B_z$ may be important in the propagation formula, for such small $y$ offsets at 22 Re or less they do not change the propagation delay by more than 2–3 min for typical values of $B_z$. This is small compared to the 20-min association window used in our paper.

9.3. Significance of Time Association

[58] It can be seen above that the width of the curves of association numbers $n(u, h)$ versus time delay increases approximately linearly with an increasing time window (h) (see Figure 6). The change in width of the peak in association number is the result of convolving a boxcar (the association window) with a delta function (the peak in association). As long as the peak lies within the moving window it will contribute exactly the same number of associations to the convolution. Thus the output will be a boxcar with width equal to the sum of widths of the boxcar and the delta function. Since the delta function is narrow the result is nearly the same width as the window.

[59] However, the window will detect an additional number of chance associations proportional to the width of the window so the total association number will also increase. This can be seen in the upper panel of Figure 6. The $n(u, h)$ at the positive $u$ is higher than $n(u, h)$ at the negative $u$. This is understandable because of the requirement of southward IMF for a certain amount of time [Lyons et al., 1997]. Eventually, the southward IMF will reverse its polarity. This means that we have discussed the probability for polarity change after southward turning. It is found that the probability is not very high at 20–30 min after the southward turning, and neither is the probability of substorm onset. The polarity change is more likely to occur as the duration of the southward interval increases. It should also be remembered that some of the substorms do not seem to be triggered by IMF northward turning. The IMF for these “nontriggered” substorms will eventually turn northward. For most of the substorms in this class of “nontriggered” substorms the polarity change will occur after substorm onset. This may explain why there appear to be more IMF triggers after substorm onset as the interval becomes wider.

[60] Another question to consider is the choice of the width of the time association window. This selection is a matter of compromise. It must be wider than zero to capture any events. It must be narrower than the time between triggers and between substorms so as not to capture too many chance events. The optimum width we have calculated utilizes these statistics for the two time series to give the best possible time resolution at the same time as it guarantees the highest level of significance. We could get either finer or coarser time resolution at less significance by narrowing or widening the window. Our Figure 6 clearly shows that there is very little change in the width of the association peak from 5 to 30 min. This suggests that over this range the width is determined by errors in timing and variability of the physical mechanism. Changing the width of the time association window by small amounts would not change our conclusions, but would reduce our confidence in them.

[61] Finally, the extremely high level of significance we have calculated makes us confident that we have a physical relationship. We could have chosen a significance level of only $10^{-7}$, which is reasonable high, but may not be high enough to justify the claim of a physical relationship. As a matter of fact, it has been suggested that a much lower significance level, as low as $10^{-2}$ should be selected to have confidence that a possible physical phenomenon has been identified [see Anderson, 1990].

10. Conclusion

[62] In this paper we examined the association between substorm onsets and northward turning of the IMF. As in
previous studies we find that some substorms appear to be triggered. But in contrast to previous studies we conclusively demonstrate that the association is statistically significant at a confidence level so high that there is little possibility that the association is a result of chance. Furthermore, we demonstrate that this high correlation is not due to the preselected “growth phase” condition for extended southward IMF period. The probability of northward IMF turning is not high enough to account for this significant correlation and it thus solidifies our conclusion that The IMF triggering of substorms is a real phenomenon. [63] The optimal choice of the width of the association window is about $b = 8$–12 min. With a window of this half width we find a peak in association with our propagated triggers arriving 3 min before substorm onset. Apparently, there is an additional 3-min delay between arrival of the pulse at Earth’s surface and substorm onset. This may represent the time it takes for the signal to propagate back into the tail, initiate the expansion onset, and then the Pi 2 pulsation burst. The 8–12 min half width of the optimum time association window is somewhat larger than we expected but turns out to be the frequent choice of substorm triggering studies [e.g., Lyons et al., 1997]. Without a more sophisticated formula to correct the arrival time of IMF discontinuities this $\pm 10$ min window may be our best choice in investigations of substorm triggering. Another surprising result is that that the substorm onsets are almost simultaneous with the arrival of IMF triggers at Earth. If IMF discontinuities are propagated to the tail location of substorm onset in the solar wind then $10$–$12$ Re seems the most likely source region for the onset. If, however, the auroral ionosphere can communicate with the more distant plasma sheet through Alfvén waves originating in the ionosphere then $30$–$40$ Re could be the source region. Without a thorough coverage of the tail with satellites it is nearly impossible to precisely locate the source region of the substorm expansion.

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