A new formulation for the ionospheric cross polar cap potential including saturation effects

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Manuscript submitted to
Annales Geophysics
Manuscript version from February 28, 2005
Abstract

It is known that the ionospheric cross polar cap potential (CPCP) saturates when the interplanetary magnetic field (IMF) $B_z$ becomes very large. Few studies have offered physical explanations as to why the polar cap potential saturates. We present 13 events in which the reconnection electric field (REF) goes above $12 mV/m$ at some time. When these events are examined as typically done in previous studies, all of them show some signs of saturation (i.e., over-prediction of the CPCP based on a linear relationship between the IMF and the CPCP). We show that by taking into account the size of the magnetosphere and the fact that the post-shock magnetic field strength is strongly dependent upon the solar wind Mach number, we can better specify the ionospheric CPCP. The CPCP ($\Phi$) can be expressed as

$$\Phi = (10^{-4}v^2 + 11.7B(1 - e^{-Ma/3})\sin^3(\theta/2))r_{ms}/11$$

where $v$ is the solar wind velocity, $B$ is the combined $Y$ and $Z$ components of the interplanetary magnetic field, $Ma$ is the solar wind Mach number, $\theta = \cos(B_z/B)$, and $r_{ms}$ is the stand-off distance to the magnetopause, assuming pressure-balance between the solar wind and the magnetosphere. This is a simple modification of the original Boyle et al. (1997) formulation.

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1 Introduction

It has been shown in a number of studies that many ionospheric electrodynamic properties can be described as being linearly related to the interplanetary magnetic field (IMF) and solar wind velocity. For example, Papitashvili et al. (1994) and Friis-Christensen et al. (1985) show that ground magnetic perturbations can be linearly related to the IMF $B_z$ and $B_y$ components. These relationships can be combined with an ionospheric conductance pattern to determine a linear relationship between the IMF and the ionospheric potential pattern. Ridley et al. (2000) use the assimilative mapping of ionospheric electrodynamics (AMIE) technique (Richmond and Kamide, 1988) to calculate a large number of ionospheric electric potential maps from ground-based magnetometers and show that the potentials are linearly related to the IMF $B_z$ and $B_y$. Papitashvili and Rich (2002) show that electric potential derived from in-situ measurements of the ionospheric plasma flow also shows a linear relationship to the IMF. Most of the above analysis was completed for small magnitude IMF time periods.

Only a small number of studies have attempted to examine the saturation that may occur under strong driving of the solar wind and IMF. Reiff et al. (1981) compares in-situ measurements of ionospheric plasma flow (or electric fields and the resulting potential) to different magnetospheric coupling functions (such as the Kan and Lee, 1979, function). They find that using an amplified magnetic field (due to the bow shock compression) works best, but that the amplified field has to be limited to get the best fits. The best amplification factor is 7-8, with a limiting value of $\sim 60$ (corresponding to a maximum IMF of $\sim 8nT$).

Weimer et al. (1990) investigates the saturation of the auroral electrojet (AE) index to both the IMF $B_z$ and the solar wind velocity ($V$) multiplied by $B_z$. They show that the maximum AE reaches a saturation value at $B_z = -15nT$, or $VB_z = -8mV/m$. They further point out that the AE index has been related to the cross polar cap potential (CPCP) (Ahn et al., 1984), so this indicates that the CPCP most likely saturates at similar values.

Russell et al. (2000) examines the high-latitude ionospheric electric potential and Joule heating saturation during the September 24-25, 2000 storm. They attempt to show that the high-latitude features saturate while the ring current injection rate does not. They further argue that the saturation takes place when the solar wind velocity times $B_z$ (i.e., the $Y$ component of the interplanetary electric field, or IEF) reaches a level of $3mV/m$. This is an equivalent magnetic field $B_z$ of $7.5nT$ with a solar wind speed of $400km/s$ (similar to
the Reiff et al., 1981, value). Russell et al. (2001) shows five time periods which arguably show signs of saturation in the potential and Joule heating. They continue to state that the saturation occurs near an IEF of 3 mV/m. Liemohn and Ridley (2002) take issue with the claims of saturation stated by Russell et al. (2001). They argue that the presented events can be fit with a linear function with similar error, and that the saturation occurs closer to 10 mV/m.

Nagatsuma (2002) shows that, on a statistical basis, the saturation occurs around 5 mV/m (or a \(B_z\) of 12.5 nT). Nagatsuma (2002) include all available data from 1995-1999, which was during solar minimum and the rise to solar maximum. From Figure 3 in their study, there is an indication that the relationship may be more complicated than simple saturation - there is a huge scatter in the points above 5 mV/m. The Nagatsuma (2002) uses the polar cap index (PCI) as a proxy for the CPCP. Troshichev et al. (1996) show that the PCI can be related to the ionospheric potential by:

\[
\Phi = 19.35 PCI + 8.78,
\]

where \(\Phi\) is the ionospheric CPCP in kV. Ridley and Kihn (2004) also show that the PCI is linearly related to the CPCP.

Shepherd et al. (2002) show that the SuperDARN radars clearly show that the ionospheric potential saturates somewhere around an IEF of approximately 15-20 mV/m. These studies use 1638 10-minute time periods in which there was very steady IMF and solar wind to show that the potential rarely reaches above 100 kV.

Siscoe et al. (2002) is one of the only papers that attempts to explain why the potential saturates. They argue that the saturation of the cross polar cap potential is an internal process - the region 1 currents flowing into the ionosphere tend to reduce the magnetic field near the subsolar magnetosphere, inhibiting reconnection. Equation 13 of the Siscoe et al. (2002) study relates the ionospheric cross polar cap potential \(\Phi_{pc}\) (i.e. \(\Phi\) above) to the electric field \(E_{SW}\) and pressure \(p_{SW}\) in the solar wind, the IMF clock angle \(F(\theta)\), the dipole strength \(D\), a geometrical factor \(\xi\), and the ionospheric conductance \(\Sigma_0\):

\[
\Phi_{pc} = \frac{57.6 E_r p_{sw}^{1/3} D^{4/3} F(\theta)}{p_{sw}^{1/2} D + 0.0125 \xi \Sigma_0 E_r F(\theta)}
\]

where \(\Phi_{pc}\) is in kV. This formulation takes into account the compression of the magnetosphere due to the solar wind dynamic pressure, the reconnection efficiency, and those terms included directly above.
Shepherd et al. (2003) show that in order to get the SuperDARN measurements of the saturation to match the Siscoe et al. (2002) formulation, the ionospheric conductance has to be 23 mhos, which is rather high. They further show that the SuperDARN data has a tendency to decrease in potential as a function of solar wind ram pressure, rather than increasing, like the Siscoe et al. (2002) predicts.

This study suggests that the saturation of the ionospheric potential may actually be an external process. We present ionospheric cross polar cap potentials derived from the PCI, similar to the Nagatsuma (2002) study. The polar cap index is derived from a single station per hemisphere. There are therefore two independent polar cap indices - one derived from the Thule magnetometer (Northern) and one from the Vostok magnetometer (Southern) (see, for example, Lukianova et al., 2002). All calculations within this study are conducted using the Northern PCI computed by the Danish Meteorological Institute (DMI). Because the PCI index is calculated on a 15 minute time cadence, the IMF data is also averaged to 15 minutes. This means that some of the peaks in the electric fields may be reduced.

One of the problems with examining how the saturation of the cross polar cap potential occurs is that there are only a few time periods in which the IMF is extremely large compared to the number of time periods in which the IMF is small. In order to overcome this difficulty, this study only examines time periods surrounding and including events in which the reconnection electric field exceeded $12 \text{ mV/m}$. The REF is defined as by Sonnerup (1974) and Kan and Lee (1979):

$$E_r = V B_{yz} \sin^2(\theta/2),$$

where $E_r$ is the solar wind (or interplanetary) electric field, $B_{yz} = \sqrt{B_z^2 + B_y^2}$, and $\theta = \cos^{-1}(B_z/B_{yz})$, and $V$ is the solar wind speed.

The $12 \text{ mV/m}$ limit allows an examination of the individual events and possibly may allow a general cause for the saturation of the potential to be illuminated. In addition, $12 \text{ mV/m}$ is four times the value suggested by Russell et al. (2001) and Reiff et al. (1981), less than twice the value suggested by Weimer et al. (1990), and just over the value suggested by Liemohn and Ridley (2002). This value is a good compromise between using more events because of a value too low and using few events because of a very high value; both of which would tend to skew the results.

This paper first suggests that using the Boyle et al. (1997) formulation as the expected potential is not physically accurate, since the length of the reconnection line is not considered.
Therefore to account for this, the definition of the relationship between the potential and the solar wind and IMF is altered. We then show that the expected potential still overpredicts the PCI inferred potential in a large number of events. This implies that these events are saturated. It is then suggested that the solar wind Alfven Mach number may influence the potential, so the Boyle et al. (1997) formulation is further altered to include the solar wind Alfven Mach number. When this is done, the modeled and measured potential agree much better than without the modifications. It further suggests that the mechanim for the saturation of the ionospheric potential may be external to the magnetosphere, and not an internal mechanism, as suggested by the Siscoe et al. (2002) study.

2 Results

Figures 1 and 2 show 13 periods in which the reconnection electric field (REF) becomes larger than 12 mV/m for some interval of time. For each event, we show the \( B_z \) and \( B_y \) components of the IMF, the cross polar cap potential estimated by the PCI index and the CPCP estimated by an analytical function which assumes a linear relationship between the IMF and the ionospheric potential. This linear analytical function is by Boyle et al. (1997):

\[
\Phi = 10^{-4} v^2 + 11.7 B \sin^3(\theta/2),
\]

(4)

where we took \( B = B_{yz} \), \( \theta \) as defined above, and \( v = v_x \). Each cluster of plots also includes two scatter plots that demonstrate various ways in which the ionospheric cross polar cap potential can be shown to have saturation. The left plot shows the CPCP estimated by Boyle et al. (1997) and the CPCP inferred from the PCI, both as a function of the reconnection electric field (as defined by Equation 3). The right plot shows the PCI inferred CPCP versus the Boyle et al. (1997) estimated CPCP, with a line showing where they would be equal. In addition, there is an indication of the average difference between the two estimates.

These clusters of plots show three different ways in which saturation of the ionospheric CPCP has typically been shown: (1) the time series plots show that the Boyle et al. (1997) estimation of the CPCP typically becomes larger than the PCI inferred CPCP when the IMF \( B_z \) component becomes large and negative; (2) the left scatter plot shows that the Boyle et al. (1997) estimated CPCP is linear for large REFs, while PCI inferred CPCP is significantly lower than this linear estimate; and (3) when the Boyle et al. (1997) estimated and PCI inferred CPCPs are plotted against each other, the Boyle et al. (1997) estimated potentials are much larger than the PCI inferred values. While the data are shown differently,
each of these three plots show exactly the same thing: a linear relationship between the
reconnection electric field and the ionospheric cross polar cap potential overestimates the
potential when the REF becomes large. All of the events show this to be true. The exact
value of the REF at which this overestimation starts to occur can be debated, but it is clear
that it does occur in all of the events.

In most previous studies of the saturation of the cross polar cap potential (e.g. Russell
et al., 2001; Merkine et al., 2003; Nagatsuma, 2002), they show plots such as Figures 1
and 2, implying a relationship between an electric field and a potential. However there is
something missing in this relationship – a length. An electric potential is the integral of an
electric field along a path of some length. The above plots do not indicate any length scale
at all.

This can be problematic, because when the reconnection electric field becomes large,
often the solar wind density and velocity also become large. This compresses the magneto-
sphere, reducing the length-scale for the integration of the electric field. This means that
the cross magnetospheric potential could possibly decrease.

By modifying the Boyle et al. (1997) formulation to include a length scale, we can bet-
ter relate the solar wind and IMF to the ionospheric CPCP. While the Boyle et al. (1997)
formulation technically does not contain an electric field (since the first term only has a \(v^2\)
term, and the second term does not contain a \(v\) term), it should still be dependent upon the
size of the magnetosphere. Multiplying by a ratio of the size of the magnetosphere to a
nominal size, this size dependence is achieved:

\[
\Phi = (10^{-4}v^2 + 11.7B\sin^3(\theta/2)) \frac{r_{ms}}{11}. 
\]  

(5)

The radius of the magnetosphere \((r_{ms})\) can be approximated through a pressure balance
between the solar wind pressure and the magnetospheric magnetic field pressure (in \(R_e\)):

\[
r_{ms} = \left( \frac{(2B_s)^2}{2\mu_0 P_{sw}} \right)^{1/6}. 
\]  

(6)

\(B_s\) is the surface magnetic field, and \(P_{sw}\) is the ram and magnetic pressure of the solar wind:

\[
P_{sw} = \frac{B^2}{2\mu_0} + nM_p v^2. 
\]  

(7)

\(N\) is the solar wind number density, \(B\) is the magnitude of the IMF, and \(M_p\) is the mass of
a proton. Typically, the solar wind ram pressure \((nM_p v^2)\) is almost an order of magnitude
larger than the magnetic pressure. In these extreme cases, though, the magnetic pressure
can become comparable to the ram pressure, so it needs to be included. It is interesting to note that the radius of the magnetosphere along the Earth-Sun line should have a seasonal dependence and a local time dependence because of the seasonal tilt and the offset of Earth’s dipole from the center of the planet. The effect on the radius of the magnetosphere should be less than about five percent, though. Furthermore, during periods in which the magnetic field in the solar wind becomes large, the shape of the magnetopause can be distorted (e.g. Raeder et al., 2001; Siscoe et al., 2002). Although this distortion should be taken into account, it is most likely highly dependent on the direction of the IMF (i.e., parallel versus perpendicular shocks), and therefore one would need a global magnetospheric model to do this. Because we are not using a large-scale model of the magnetosphere in this research, it is beyond the scope of the current study.

Figures 3 and 4 show the pressure balanced radius of the magnetosphere, the PCI inferred CPCP and the estimated potential using Equation 5. In addition, the average differences between the PCI inferred and Equation 5 estimated potentials are displayed on the plots. The average difference between the potentials decreases by 29% over the Boyle et al. (1997) formulation when the magnetospheric size is considered.

Figures 3 and 4 compared to Figures 1 and 2 show a decrease in the amount of over-prediction of the ionospheric potential, especially on May 23, 2000; September 17, 2000; April 11, 2001; and October 21, 2001. These four events show very little saturation at all, implying that considering the radius of the magnetosphere should be important when examining large IMF and solar wind events. The other nine events still show significant over-predictions of the CPCP, although they are reduced in many cases. It is evident that a modified formulation must be determined that takes into account the saturation of the potential.

Let us consider a single event chosen at random, March 30, 2001. Figure 5 shows all of the relevant quantities for this time period, such as the reconnection electric field and the radius modification factor (\(r_{ms}/11\), as discussed above). The fourth plot shows the PCI inferred CPCP as well as the CPCP estimated from Equation 5. The middle plot shows the solar wind Alfven Mach number, which is defined as:

\[ M_a = \frac{V_{sw}}{C_a}, \]  
(8)

where

\[ C_a = \frac{B}{\sqrt{\mu_0 n M_p}}. \]  
(9)
It is interesting to note that when the potential is saturated (i.e., the predicted potential is significantly larger than the PCI inferred value), the Alfven Mach number is less than four. Indeed, this is true of all other events, and could in fact be the cause of the saturation.

Taking the Alfven Mach number into consideration, we can express the ionospheric cross polar cap potential as:

\[
\Phi = (10^{-4}v^2 + 11.7B(1 - e^{-M_a/3}) \sin^3(\theta/2)) \frac{T_{ms}}{T}. \tag{10}
\]

The term \((1 - e^{-M_a/3})\), which multiplies the magnetic field, will be justified below, when the physics of the bow shock are discussed. When the Alfven Mach number is in its typical range (approximately eight), the last term is approximately 0.93, so it does not modify the potential very much at all. When the Mach number decreases significantly (when the magnetic field becomes very large, or when the solar wind density decreases significantly – both of which occur in magnetic clouds) the last term decreases significantly. At a Mach number of three, the term is 0.63, and at a Mach number of one, it is 0.28.

Figure 6 shows examples of how this term modifies the ionospheric potential as a function of solar wind density and solar wind density. The plots on the left have an input solar wind velocity of 400 km/s, while those on the right have an input velocity of 800 km/s. From top to bottom, the input solar wind densities are 1, 5, and 20 \(cm^{-3}\). These plots show that as the solar wind density and velocity increases, the saturation of the potential occurs at a much higher REF. The upper left plot almost never occurs, because magnetic clouds with the lowest densities typically have high speeds, so they are more likely to be the top right plot.

When Equation 10 is applied to all of the events, Figures 7 and 8 result. These plots show the Alfven Mach number as well as the estimated potential using Equation 10. The saturation is reproduced in almost all cases, and strongly suggests that the Alfven Mach number may be tied to the saturation. The average difference is decreased by 41% over using simply the Boyle et al. (1997) formulation and 24% over using Equation 5. Equation 10 is a much better estimation of the cross polar cap potential than the Boyle et al. (1997) formulation during strong driving conditions.

3 Discussion

The study by Reiff et al. (1981) discusses the fact that when the IMF is advected through the bow shock, it increases in strength. For a typical solar wind density and flow speed, the IMF...
can increase by a factor of four from the solar wind to the magnetosheath. In addition, as the magnetic field in the solar wind becomes larger, the ratio between the shocked magnetic field and upstream field should decrease. This is because the Alfvén Mach number decreases with increasing magnetic field. Reiff et al. (1981) show that they can increase the correlation between the IMF and the cross polar cap potential by “shocking” the IMF up to a certain level and having values above that be constant.

Recently, Lopez et al. (2004) discuss the role of the solar wind number density in controlling the strength of the cross polar cap potential and ionospheric Joule heating. They show that the solar wind density and the magnetic field strength control the compression ratio of the bow shock. During nominal solar wind and IMF conditions, the magnetic field is always increased by a constant factor (independent of the solar wind density) as it goes through the shock. As the magnetic field decreases, the solar wind density gains more control over the shock compression.

This can be easily quantified if one only considers magnetic fields that are tangential to the bow shock (i.e., only \( B_y \) and \( B_z \) components of the IMF). The following set of equations can be used to determine the increase in field strength and density across the bow shock (Roberge and Draine, 1993):

\[
p_u = n_u k T_u \\
C_{su} = \sqrt{\frac{\gamma p_u}{\rho_u}} \\
C_{Au} = \frac{B_u}{\sqrt{\mu_0 \rho_u}} \\
M_{su} = \frac{V_u}{C_{su}} \\
M_{Au} = \frac{V_u}{C_{Au}} \\
C = \gamma - 1 + 2M_{su}^{-2} + \gamma M_{Au}^{-2} \\
\frac{\rho_d}{\rho_u} = \frac{B_d}{B_u} = \frac{2(\gamma + 1)}{C + \sqrt{C^2 + 4(\gamma + 1)(2 - \gamma) M_{Au}^{-2}}}
\]

where symbols with a subscript “\( u \)” are upstream of the bow shock, and values with a subscript “\( d \)” at downstream of the bow shock. \( T \) is the temperature of the solar wind, \( k \) is Boltzmann’s constant, \( n_u \) is the solar wind number density, \( \rho_u \) is the solar wind mass density, \( p_u \) is the kinetic pressure of the solar wind, \( C_{su} \) is the solar wind sound speed, \( C_{Au} \) is the solar wind Alfvén speed, \( V_u \) is the solar wind speed, and \( M_{Au} \) and \( M_{su} \) are the Alfvén and sound Mach numbers.
An important consideration in this formulation is that it is for a shock in which $B$ is perpendicular to the shock normal. This is only true (at the subsolar point) when $B_x = 0$. When $B_x$ is non-zero, this formulation will be incorrect for the subsolar point. The relative strength of $B_x$ to $\sqrt{B_y^2 + B_z^2}$ will determine how relevant it is. During large events in which the main components of the IMF are in $B_z$ and $B_y$, this formulation holds true.

Typically, the sound speed in the solar wind is on the order of $50\text{km/s}$, depending on the solar wind temperature. This means that the sound Mach number is on the order of 7 to 20 (given solar wind speeds of $350\text{km/s} - 1000\text{km/s}$). The Alfven Mach number is typically in the range of eight for nominal solar wind and IMF conditions. These large values of $M_{Au}$ and $M_{sw}$ imply that:

$$B_d \simeq \frac{\gamma + 1}{\gamma - 1} B_u = \frac{5/3 + 1}{5/3 - 1} B_u = 4B_u. \quad (18)$$

The magnetosheath magnetic field is approximately four times the IMF for tangential fields and nominal solar wind conditions.

Figure 9 shows the relationship between the IMF and the solar wind Alfven Mach number for a number of different solar wind number densities. The grey shaded region is considered nominal values (i.e., $2.5cm^{-3} < n_u < 10cm^{-3}$ and $1nT < B_u < 10nT$). In this regime, the Alfven Mach number is always above three. It is clear that, as the number density of the solar wind decreases, the Mach number also decreases, meaning that the solar wind can become sub-Alfvenic at lower magnetic field values. For example, with a number density of $1cm^{-3}$, an 18 $nT$ magnetic field means a sub-Alfvenic solar wind (if the solar wind speed is $400 \text{ km/s}$). When the solar wind number density is $25cm^{-3}$, the solar wind becomes sub-Alfvenic only when the IMF is larger than $90nT$, which is a very rare occurrence. When one considers that the cores of magnetic clouds are regions of high magnetic field strength, low temperature, and low density, they are in the exact region that can easily become sub-Alfvenic. These are also the times in which saturation of the CPCP occurs.

Figure 10 offers a possible explanation for the saturation of the cross polar cap potential. This plot shows the shocked (i.e., magnetosheath) magnetic field strength as a function of the upstream magnetic field strength for a number of different solar wind number densities. If one of the lines is followed, there is a sharp, linear rise of the magnetic field when the Alfven Mach number is very large (i.e., $B_u$ is small). This line is simply $B_d = 4B_u$. As the Mach number decreases below three, the sheath field saturates at around $B_d = 2B_u$, and actually starts to decrease. When the Mach number passes below one, there is no longer a
shock, so Equation 17 is no longer valid.

Equation 10 multiplies the magnetic field of the Boyle et al. (1997) equation by a factor of \((1 - e^{-M_a/3})\), which has a very similar dependence on the Alfven Mach number as Equation 17. Figure 11 shows the ratio of the downstream and upstream magnetic fields (Equation 17) as a function of upstream magnetic field. In addition, \(4(1 - e^{-M_a/3})\) is over-plotted to show that the lines almost overlay each other. This means that by taking into account the shocking of the solar wind, either with Equation 17 (divided by four), or with a simple exponential dependence, the saturation of the cross polar cap potential can be accurately modeled.

It should be noted that the solar wind velocity decreases in speed by the same ratio as the magnetic field through the shock, meaning that the electric field remains the same through the shock. At the subsolar point though, the velocity decreases to zero as it approaches the magnetopause (independent of the shock strength), while the magnetic field increases to some value that is most likely controlled by the shocked magnetic field strength. The original Boyle et al. (1997) formulation does not contain the velocity in the primary coupling term, so the decrease in the velocity through the shock need not be compensated for in this term. The viscous interaction term, on the other hand, does have a \(v^2\) term. It is not reduced in the formulation above because the viscous interaction takes place on the sides of the magnetosphere, after the solar wind has accelerated back up to some significant fraction of the original velocity.

Recently, Borovsky et al. (2005) showed that during time periods of low Alfven Mach numbers, the magnetosphere can exhibit global sawtooth oscillations. It could be possible that these two phenomena both occur during similar driving conditions, and may both be ramifications of the different coupling that may occur between the IMF and the magnetosphere during low Alfven Mach number conditions.

4 Conclusions

In this study we present 13 events in which the reconnection electric field becomes larger than 12 mV/m for some time period. At some point during all of these 13 events, the ionospheric cross polar cap potential calculated from the Boyle et al. (1997) formulation over-predicts the inferred CPCP from the PCI index. We show that when the size of the magnetosphere is considered, the modified Boyle et al. (1997) formulation better matches...
all of the events (by 29%), and in four events, the over-prediction is almost completely eliminated.

We further show that during all of the time periods in which the over-prediction of the CPCP occurs, the solar wind Alfven Mach number is reduced beneath its nominal value. When we take this into account, almost all of the over-prediction of the CPCP is accounted for. We therefore conclude that the solar wind Alfven Mach number may play a significant role in the magnetosphere solar wind coupling. This is quite similar to sawteeth oscillations, which occur under similar driving conditions.

If the magnetosheath magnetic field is considered to be more important than the actual solar wind magnetic field in the coupling between the solar wind and the magnetosphere, then the compression of the magnetic field across the bow shock must be considered. We show that for nominal conditions, the magnetic field is increased by almost a factor of four across the bow shock. Under stronger magnetic field conditions (i.e., smaller Alfven Mach numbers), the compression is reduced below four, reaching two when the Mach number goes below three. When this is taken into account, the CPCP can be expressed as \( \Phi = \left(10^{-4} v^2 + 11.7 B (1 - e^{-Ma/3}) \sin^3(\theta/2)\right) \frac{r_{ms}}{r_0}. \) This is a simple modification of the original Boyle et al. (1997) formulation, and explains the saturation of the cross polar cap potential during these intervals.

Arguing that the solar wind Alfven mach number can control the saturation of the ionospheric cross polar cap potential suggests that the saturation is caused by a process external to the magnetosphere, while Siscoe et al. (2002) argues that the saturation is caused by processes internal to the magnetosphere. Further studies are needed to determine whether the saturation is caused by an internal or external mechanism.

Acknowledgements. I wish to thank G. Toth and K.C. Hansen in their helpful discussions on shock compressions. The ACE solar wind and IMF data was downloaded from the NASA CDAWEB FTP site. The Northern hemisphere polar cap index data was downloaded from the World Data Center's FTP site at the Danish Meteorological Institute. This research was supposed by NSF through grant ATM-0077555 and the DoD MURI program grant F4960-01-1-0359.
References


Figure Captions

**Fig. 1.** Six events in which the reconnection electric field went above 12 mV/m. The top plot in each cluster shows the IMF $B_z$ (solid) and IMF $B_y$ (dashed). The middle plot shows the ionospheric cross polar cap potential (CPCP) as inferred from the PCI index (solid) and estimated from the Boyle et al. (1997) formulation (dashed). The bottom left plot shows the ionospheric CPCP versus the reconnection electric field for the Boyle et al. (1997) formulation (stars) and inferred CPCP from the PCI index (diamonds). The bottom right plot shows the PCI inferred potential versus the Boyle et al. (1997) formulation potential.

**Fig. 2.** Seven more events in the same format as Figure 1.

**Fig. 3.** The same six events in Figure 1, plotted in the same way, except Equation 5 was used rather than estimating the CPCP with Boyle et al. (1997). The top plot is the radius of the magnetosphere, as estimated by Equation 6.

**Fig. 4.** The same seven events in Figure 2, plotted in the same way, except Equation 5 was used rather than estimating the CPCP with Boyle et al. (1997). The top plot is the radius of the magnetosphere, as estimated by Equation 6.

**Fig. 5.** From top to bottom: the reconnection electric field, using Equation 3; the magnetospheric radius divided by 11; the solar wind Alfvén Mach number; the PCI inferred cross polar cap potential (CPCP, solid) and the estimated CPCP using Equation 5; and the PCI inferred CPCP (solid) and the estimated CPCP using Equation 10. The time period is March 30-31, 2001.

**Fig. 6.** Examples of CPCP curves as a function of REF using Equation 10 (solid) and Equation 5 (dashed). The left curves use an input solar wind velocity of 400 km/s, while the right curves use 800 km/s. From top to bottom, the input solar wind density is changed from $1 cm^{-3}$ to $5 cm^{-3}$ to $20 cm^{-3}$. The vertical line indicates when the solar wind Mach number is four. Points to the right are less than four. It should be noted that the vertical scales are the same on all plots, but the left plots stop at REF = $20mV/m$, while the right plots stop at REF = $40mV/m$.

**Fig. 7.** The same six events in Figure 1, plotted in the same way, except Equation 10 was used rather than estimating the CPCP with Boyle et al. (1997). The top plot is the Alfvén Mach number.
**Fig. 8.** The same seven events in Figure 2, plotted in the same way, except Equation 10 was used rather than estimating the CPCP with Boyle et al. (1997). The top plot is the Alfven Mach number.

**Fig. 9.** The solar wind Alfven Mach number as a function of the IMF strength for a number of solar wind number densities. The solar wind speed is 400 km/s in this plot. The shaded region represents typical values of the solar wind number density and IMF strength.

**Fig. 10.** The magnetosheath magnetic field strength as a function of the upstream IMF strength for a number of solar wind number densities. The bottom right area represents a region in which the Alfven Mach number is less than one, so it is not considered. The shaded region represents typical values of the solar wind number density and IMF strength. The solar wind speed is 400 km/s in this plot.

**Fig. 11.** The ratio of the downstream and upstream magnetic fields as a function of upstream magnetic field strength (i.e., Equation 17) are plotted as solid lines for five densities. The formula $4(1 - e^{-M_a/3})$ is over-plotted as dotted lines for the five different densities.
Fig. 1. Six events in which the reconnection electric field went above 12 mV/m. The top plot in each cluster shows the IMF $B_z$ (solid) and IMF $B_y$ (dashed). The middle plot shows the ionospheric cross polar cap potential (CPCP) as inferred from the PCI index (solid) and estimated from the Boyle et al. (1997) formulation (dashed). The bottom left plot shows the ionospheric CPCP versus the reconnection electric field for the Boyle et al. (1997) formulation (stars) and inferred CPCP from the PCI index (diamonds). The bottom right plot shows the PCI inferred potential versus the Boyle et al. (1997) formulation potential.
Fig. 2. Seven more events in the same format as Figure 1.
Fig. 3. The same six events in Figure 1, plotted in the same way, except Equation 5 was used rather than estimating the CPCP with Boyle et al. (1997). The top plot is the radius of the magnetosphere, as estimated by Equation 6.
Fig. 4. The same seven events in Figure 2, plotted in the same way, except Equation 5 was used rather than estimating the CPCP with Boyle et al. (1997). The top plot is the radius of the magnetosphere, as estimated by Equation 6.
Fig. 5. From top to bottom: the reconnection electric field, using Equation 3; the magnetospheric radius divided by 11; the solar wind Alfven Mach number; the PCI inferred cross polar cap potential (CPCP, solid) and the estimated CPCP using Equation 5; and the PCI inferred CPCP (solid) and the estimated CPCP using Equation 10. The time period is March 30-31, 2001.
Fig. 6. Examples of CPCP curves as a function of REF using Equation 10 (solid) and Equation 5 (dashed). The left curves use an input solar wind velocity of 400 km/s, while the right curves use 800 km/s. From top to bottom, the input solar wind density is changed from $1 \text{ cm}^{-3}$ to $5 \text{ cm}^{-3}$ to $20 \text{ cm}^{-3}$. The vertical line indicates when the solar wind Mach number is four. Points to the right are less than four. It should be noted that the vertical scales are the same on all plots, but the left plots stop at $\text{REF} = 20 \text{ mV/m}$, while the right plots stop at $\text{REF} = 40 \text{ mV/m}$. 
Fig. 7. The same six events in Figure 1, plotted in the same way, except Equation 10 was used rather than estimating the CPCP with Boyle et al. (1997). The top plot is the Alfvén Mach number.
Fig. 8. The same seven events in Figure 2, plotted in the same way, except Equation 10 was used rather than estimating the CPCP with Boyle et al. (1997). The top plot is the Alfven Mach number.
Fig. 9. The solar wind Alfven Mach number as a function of the IMF strength for a number of solar wind number densities. The solar wind speed is 400 km/s in this plot. The shaded region represents typical values of the solar wind number density and IMF strength.

Fig. 10. The magnetosheath magnetic field strength as a function of the upstream IMF strength for a number of solar wind number densities. The bottom right area represents a region in which the Alfven Mach number is less than one, so it is not considered. The shaded region represents typical values of the solar wind number density and IMF strength. The solar wind speed is 400 km/s in this plot.
Fig. 11. The ratio of the downstream and upstream magnetic fields as a function of upstream magnetic field strength (i.e., Equation 17) are plotted as solid lines for five densities. The formula $4(1 - e^{-M_a/3})$ is over-plotted as dotted lines for the five different densities.