Penetration of Low-Energy Protons Deep into the Magnetosphere

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By using a simple model with constant convection electric field and a dipole magnetic field, plasma flow patterns are calculated with the convection fields as scale factors. Unlike other particle trajectories, the flow patterns for protons with certain relative magnetic moments show double forbidden regions; one is composed of orbits that circle the earth; the other is composed of orbits that do not circle the earth. These protons can penetrate very close to the earth through the space between the two forbidden regions. The calculations based on the model of constant electric field with charge exchange as a loss mechanism indicate that protons of a few hundred electron volts convected in from the tail to \( L = 3-4 \) could be responsible for the storm-time ring currents.

INTRODUCTION

The existence of convection electric fields in the magnetosphere has been known for many years (see, e.g., the review by Axford [1969]). Many authors have discussed trajectories of particles drifting through the magnetosphere under the influence of simple assumed electric fields [see e.g., Alfvén, 1939, 1950; Axford and Hines, 1961; Karlson, 1962, 1963, 1969; Taylor and Hones, 1965; Block, 1967; Brice, 1967; Kavanagh et al., 1968]. The motion of protons with low energy (but non-zero energy) in the superimposed convection and corotation electric fields has never been discussed in detail. The trajectories of these low-energy protons are topologically quite different from those of other classes of particles. This paper describes the differences. The topologies of the particle trajectories are illustrated in terms of a simple model for the electric and magnetic fields, a model for which all the expressions regarding the properties of particle trajectories are analytically represented and can be described in terms of no more than fourth-order polynomials.

This paper follows an analytical approach similar to that taken by Schield [1969], and our results are in agreement with his; however, the present paper extends the range of calculations.

FORMULATION OF PROBLEM

To obtain analytic expressions for the trajectories, we make the following assumptions:

1. The geomagnetic field is that of a dipole.
2. The electric field consists of two components. One is the constant convection electric field \( E_0 \) with the direction perpendicular to the sun-earth line. The other is \( -(K/R)\delta \delta \), due to the rotation of the earth. \( K \) is equal to 14.5 \( \text{mv/m} \), when \( R \) is in earth radii.
3. The pitch angle of the particles is 90°, so that they drift in the equatorial plane only.
4. All drift motion is adiabatic.

Under these assumptions, the drift of a particle is a combination of \( E_0 \times B \), gradient drift, and corotation, but the motion is the same as pure \( E \times B \) drift in an equivalent electric field: \( E_{eq} = -\nabla \Phi \), where

\[
\Phi = -\frac{K}{R} - E_0 R \sin \varphi + \frac{\mu M_B}{q R^2} \tag{1}
\]

The term proportional to \( K \) (= 14.5 \( \text{mv/m} \)) is due to the earth's rotation; \( R \) is radial distance in earth radii; \( E_0 \) is the magnitude of the convection electric field; \( \varphi \) is azimuthal angle measured counter-clockwise from the solar direction; \( M_B \), geomagnetic dipole moment, equals \( 8.09 \times 10^{25} \) gauss cm³; \( \mu \) and \( q \) are the particle's magnetic moment and charge, respectively. The following definitions are used:
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\[ R_m = \frac{K^{1/2}}{E_0^{1/2}} = 3.81 E_0^{-1/2} R_R \]  

(2a)

\[ \mu^* = \frac{qK^2}{12 M e E_0} = \frac{q}{|q|} \frac{3.60 \text{ ev/} \gamma}{E_0} \]  

(2b)

\[ R' = R/R_m \]  

(2c)

\[ \mu' = \mu/\mu^* \]  

(2d)

where \( E_0 \) is expressed in millivolts per meter. Equations 1 can then be reduced to the dimensionless expression

\[ \Phi' = -\frac{1}{R'} - R' \sin \varphi + \frac{\mu'}{12 R'^3} \]  

(3)

where \( \Phi' \) differs \( \Phi \) by a constant factor only. The drift velocity is given by

\[ \mathbf{v}_d = \frac{\mathbf{B} \times \nabla \Phi}{B^2} \]

\[ = \left( \frac{K}{E_0} \right)^{1/2} \omega \left[ \left( R' - R'^2 \sin \varphi - \frac{\mu'}{4R'} \right) \beta_\varphi \right. \]

\[ \left. + R'^3 \cos \varphi \beta_R \right] \]  

(4)

where \( \omega \) is the angular velocity of the earth. Aside from the scale factor, the drift velocity does not depend on the convection electric field explicitly, but rather depends on \( R' \) and \( \mu' \). These two variables in turn depend on the convection electric field through equations 2a–2d.

**Shapes of Trajectories**

**Stagnation points.** Equation 4 implies that any point at which the drift velocity is zero must lie in the dawn-dusk meridian plane (cos \( \varphi \) = 0). There are, in fact, generally three stagnation points, two on the dusk side and one on the dawn side. They are labeled \( a, b, \) and \( c \) in the figures. For \( \varphi = 90^\circ \) the radial distances of the stagnation points \( a \) and \( b \) are given by the following expressions:

\[ R'_a = \left[ 1 + \frac{1}{2} \left( \mu - \mu' \right)^{1/2} \right]^{1/2} \]  

(5a)

\[ R'_b = \left[ 1 - \frac{1}{2} \left( \mu - \mu' \right)^{1/2} \right]^{1/2} \]  

(5b)

The closed equipotential contour with the equipotential given at point \( a \) is the forbidden-

![Fig. 1. Drift paths in the equatorial plane for electrons and protons with \( \mu' = 0 \). The full circle in the center represents the size of the earth with convection electric field of 0.36 mv/m. The positions of the magnetopause and the forbidden region are indicated by solid lines. This flow pattern is universal in the sense that, for a different convection electric field, the radius of the circle representing the earth is multiplied by a factor of \( (E_0/0.36 \text{ mv/m})^{1/2} \).](image)
region boundary. The equation describing this contour is

$$-R' \sin \varphi - \Phi_m' R'^3 - R'^2 + \frac{\mu'}{12} = 0$$  \hspace{1cm} (6)

where

$$\Phi_m'(R_a') = -\frac{1}{R_a'} - R_a' + \frac{\mu'}{12} \frac{1}{R_a'^3}$$

Point $b$ corresponds to a minimum in the equipotential $\Phi'$. In other words, the particle at point $b$ can never drift; the particles in its neighborhood drift about point $b$.

The radial distance of stagnation point $c$ (which is at $\varphi = -90^\circ$) is given by

$$R_c' = \left[\frac{-1 + (1 + \mu')^{1/2}}{2}\right]^{1/3}$$  \hspace{1cm} (7)

The equation describing the equipotential contour $c$ is

$$-R' \sin \varphi - \Phi_m'' R'^3 - R'^2 + \frac{\mu'}{12} = 0$$  \hspace{1cm} (8)

where

$$\Phi_m''(R_c') = -\frac{1}{R_c'} + R_c' + \frac{\mu'}{12} \frac{1}{R_c'^3}$$

Zero-energy particles. The flow pattern for $\mu' = 0$ is shown in Figure 1. The $x$ axis is pointed in the solar direction, and the $y$ axis is directed from the dawn to the dusk meridian. Particles from infinity can reach any point outside the closed solid line (the boundary of the forbidden region) but can never cross the solid line. As the electric field due to corotation is inversely proportional to $R^2$, the drift motion is dominated by corotation within the forbidden region; outside of this region, the convective...
electric field becomes more important than the corotation electric field. The flow pattern is universal in the sense that the magnitude of the convection field does not affect the shape of the diagram; it only determines the scale. The circle shown in the center of the figure represents the size of the earth for a convection electric field of 0.36 mv/m. To use this diagram for another value $E_o$ of the electric field, simply multiply the radius of the circle representing the earth by a factor of $(E_o/0.36 \text{ mv/m})^{1/2}$. For example, if the electric field is reduced by a factor of 4, the radius of the circle representing the size of the earth will be reduced by a factor of 2. To avoid confusion, $E_o$ is fixed at 0.36 mv/m for the succeeding figures if not otherwise specified.

**Protons with non-zero energy.** For $\mu' \neq 0$ the flow patterns become more complicated. Figure 2a shows the trajectories for $\mu' = 0.2$. The size of the forbidden region is a little smaller than that for $\mu' = 0$. As in Figure 1, the full circle at the center corresponds to the earth for a convection electric field of 0.36 mv/m. The magnetic moment of the proton is then $2 \text{ ev}/\gamma$.

The dashed curve connecting the two stagnation points $b$ and $c$ is the contour on which the azimuthal drift velocity is zero. Note that this contour is not an equipotential. The drift motion within this contour is dominated by gradient drift, whereas in the space between this contour and the outer boundary of the forbidden region the corotation electric field is more important. The convection field dominates again outside the forbidden region. The dash-dotted contour represents the equipotential with respect to the point $c$. Particles within this contour are trapped and drift about point $b$. They do not circle the earth. Figure 2b is for $\mu' = 0.3$. General features are about the same except that the sizes of the dash-dotted and dashed curves increase more. If the relative magnetic moment $\mu'$ is steadily increased, the stagnation point $c$ eventually reaches point $d$ on the boundary of forbidden region and the outer por-

Fig. 2b. For $\mu' = 0.3$. 
Figure 3 is the same as Figure 1 for different values of \( \mu' \). For \( \mu' = 0.346 \), the closed flow line passing point c becomes open and coincides with the boundary of the forbidden region. In other words, for \( \mu' > 0.346 \) the particles can penetrate much closer to the earth on the dusk side.

The flow pattern for this critical value is shown in Figure 3a. Now, the particles from infinity can penetrate much closer to the earth. The original forbidden region is separated into two parts so that the common boundary is accessible to the particles at infinity with the distance \( y' = -\Phi_{m'}(R_0) \) from the \( x \) axis. When the relative magnetic moment is increased further, the two contours passing points a and c separate from each other again. As shown in Figure 3b for \( \mu' = 0.4 \), the dash-dotted curve moves beyond the full line. The particles at infinity with \( y' \) between \( y'_a \) and \( y'' = -\Phi_{m''}(R_0) \) can drift up to the dusk side nearest the earth.

Figure 4a shows the flow pattern for \( \mu' = 0.6 \). It can be seen that the trapped region on the dusk side of the earth shrinks further, and the inner trapped region on the right-hand side expands slightly.

Finally, the trapped region on the left-hand side shrinks to a point when \( \mu' = 1.0 \), as shown in Figure 4b. For even higher \( \mu' \), the particle
Electron trajectories. We should mention that trajectories for electrons with $\mu' < 0.346$ are generally similar to the trajectories of the particles with zero magnetic moment, except that the size of the forbidden region is larger for electrons. There is no dramatic change in the electron trajectories at $\mu' = 0.346$. Because the gradient drift and corotation are in the same direction for electrons, the complicated double

Fig. 3b. For $\mu' = 0.4$.

Fig. 4a. For $\mu' = 0.6$.

Fig. 4b. For $\mu' = 1.0$. 
Figure 4 is the same as Figure 1 but for different values of \( \mu' \). The space between two flow lines passing \( a \) and \( \frac{1}{2} \) becomes larger to allow more particles drifting close to the earth for \( \mu' = 0.6 \) than for \( \mu' = 0.4 \). When \( \mu' = 1.0 \), the kidney-shaped forbidden region shrinks to a point, and \( a \) and \( b \) coincide with each other. For \( \mu' > 1 \), only one stagnation point exists, point \( c \) on the dawn side. The flow patterns are similar to those given by Alfvén [1950].

Forbidden regions never occur. The single forbidden region just grows large as \( \mu' \) increases.

Regions Accessible to Particles from the Tail

It is useful at this point to summarize our results on the accessibility of various \( L \) shells to particles of various energies. Figure 5 summarizes particle trajectories crossing the midnight meridian plane for different values of \( \mu' \). Each straight line shows the variation of particle energy with \( L \) shell for a given \( \mu' \) and convection electric field. In other words, the same magnetic moment \( \mu \) will correspond to different straight lines for different convection electric fields.

Protons. The dots at the ends of the straight lines represent the deepest penetrations for protons of various \( \mu' \). The dotted lines signify the 'forbidden regions' for \( \mu' \) between 0.346 and 0.465 due to the intersection of kidney-shaped forbidden regions with the \( x \) axis. To explain the diagram more clearly, let us first take \( E_0 = 0.09 \) mv/m, i.e., choose the innermost scales. For \( \mu' = 1 \), the corresponding magnetic moment \( \mu \) is 40 ev/\( \gamma \). These particles can drift to 3 \( R_s \) attaining an energy of 16 kev. To use the same diagram for an arbitrary electric field \( E_0 \) simply locate the origin of \( R-E \) coordinates at the specific \( E_0 \) along the \( E_0 \) axis. For instance, the \( R-E \) coordinates will correspond to the middle scales for \( E_0 = 0.36 \) mv/m. For \( \mu = 40 \) ev/\( \gamma \), the corresponding \( \mu' \) becomes 4. These particles, then, can drift up to 3.7 \( R_s \), attaining an energy of 26 kev. That is, the particles with the same magnetic moment \( \mu \) can drift closer to the earth and be further energized under the influence of a higher convection electric field.

Another important point that should be emphasized concerns the critical relative magnetic moment \( \mu' = 0.346 \). For \( E_0 = 0.09 \) mv/m, the particles with \( \mu = 4 \) ev/\( \gamma \) can drift up to 6.1 \( R_s \) and reach an energy of 0.4 kev. For \( E_0 = 0.36 \) mv/m, however, these same particles can drift up to 1.5 \( R_s \) and attain an energy of 26 kev. These dramatic changes are due to the fact that \( \mu' \) is less than 0.346 in the former case and slightly greater than 0.346 in the latter case.

Electrons. The cross on each straight line in Figure 5 represents the electrons' deepest penetration for a given \( \mu' \). The radial distance increases monotonically with \( \mu' \), in contrast to the complex behavior for protons. Furthermore,
Fig. 5. Summary of particle trajectories crossing the midnight meridian plane. The numbers on the straight lines are the values of $\mu'$; the lines differ by equal increments between the sequential pairs of the specified numbers 0.1, 1, 10, and 100 unless otherwise specified. The three-dimensional coordinates are $R$, the radial distance in earth radii; $E$, the particle energy in kilo electron volts; and $E_0$, the convection electric field in units of 0.09 mv/m. The scale for the first two is logarithmic and the scale for $E_0$ is square-rooted. The origin of the coordinate system is located along the $E_0$ axis for a given convection electric field. Each straight line represents particle energy versus $L$ shell for constant $\mu'$ with convection electric field as a parameter. The dots represent the deepest proton penetration. The cross on each line stands for the electrons' deepest penetration with the same magnetic moment as the protons.

the radial distance for the electrons is larger than the radial distance for protons with the same $\mu'$. The differences become smaller for higher $\mu'$ particles and are almost zero if the gradient drift is much larger than corotation.

Penetration of Low-Energy Protons near Earth

Penetration to 3 $R_E$. We have found that the trajectories of low-energy protons can reach deep into the magnetosphere on the dusk side, particularly if the cross-tail convection electric field is high. Large numbers of protons of relatively low energy have, of course, been observed during storm time at $L = 3-5$ on the dusk side [Frank, 1967a, b]. These protons are generally believed to be the source of the asymmetric ring current [Kavanagh et al., 1968]. It is tempting to interpret, at least semiquantitatively, the observed low-energy protons in terms of the computed trajectories.

Some caution must of course be exercised when comparing experimental data with calculations based on the assumption of a uniform electric field across the magnetosphere, an assumption that must represent a drastic oversimplification. At present, however, there is no generally accepted interpretation of how the convection field deviates from uniformity under different conditions of geomagnetic activity (see, e.g., Axford [1969], Taylor and Hones [1965], Moser and Berling [1969], Wolf [1969]). Without
a reliable complex theory for the electric field distribution, it seems reasonable to see how consistent our simplified uniform-field model picture is with Frank's observations of low-energy protons.

Of course, if we are to have any hope of a meaningful comparison with the data, we must include the effects of particle loss.

There is some evidence that the dominant loss mechanism for kev protons in the inner magnetosphere is charge exchange with neutral hydrogen atoms [Frank, 1967a; Swisher and Frank, 1968]. Assuming that charge exchange is the only loss mechanism and using standard lifetime calculations [Liemohn, 1961; Swisher and Frank, 1968], we have computed the percentages of surviving particles when they are convected to a point 3 $R_e$ from the earth, in the midnight meridian plane, from a distance of 10 $R_e$ out in the tail. The equation governing the calculation is

$$p.c. = \exp \left\{ - \int_{0}^{3 R_e} \frac{dR}{1 + \frac{v_R}{v_e}}, \frac{1}{T} \right\}$$  \hspace{1cm} (10)$$

where $p.c.$ is the percentage of surviving particles, $v_R$ is the radial component of drift velocity, and $T$ is the charge-exchange lifetime, which depends on the $L$ shell and the particle energy.

For each of several magnetic moments $\mu$, Table 1 shows the convection electric field required to drive the particles in to 3 $R_e$, the corresponding particle energies, and the percentages of surviving particles at different radial distances.

Another possible loss mechanism is Coulomb scattering. Similar calculations have been made with the lifetime determined by Liemohn [1961]. However, the results show no significant loss due to Coulomb scattering for the energy range considered in Table 1.

**RING CURRENT PARTICLES**

During the moderate magnetic storm of early July 1966, directional intensities of protons (31 $\leq E \leq 49$ kev) become essentially zero at 5.5 and 2.7 $R_e$ for pre-storm and main phase, respectively [Frank, 1967b]. There are substantial fluxes observed beyond those radial distances. During the main phase, the proton (200 ev $\leq E \leq 50$ kev) number density deduced by Frank at $L = 4$ ($\lambda_m = 27^\circ$) is about 8 particles/cm$^2$. If one assumes that these particles were convected in from the tail, conserving the first two adiabatic invariants, with no loss by charge exchange, we find that the observed ring current particles at $L = 4$ correspond to 0.17 particle/cm$^2$ with energies between 12.8 ev and 3.2 kev at the inner edge of the tail ($L \approx 10$).

The 'bulge' portion of Figure 5 is redrawn as Figure 6 for increased clarity. The scales for $R$ and $E$ are now linear (different from Figure 5), but the scale for $E_0$ is the same as in Figure 5. The curve represents the maximum particle energy at a given radial distance for a specific convection electric field. With some manipulation, we find that the convection electric fields for pre-storm and main phase are 0.29 and 0.44 mv/m, respectively.

In the pre-storm period, protons with $\mu = 174$ ev/$\gamma$ can drift up to 5.5 $R_e$ with energy of 31 kev. Particles with $\mu < 174$ ev/$\gamma$ can penetrate farther; however, the particle ener-

**TABLE 1. Percentages of Surviving Protons with Different Magnetic Moments along the Radial Distances at Midnight Meridian Plane**

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$E_0$, mv/m</th>
<th>$\mu_e$, ev/$\gamma$</th>
<th>$E$, kev</th>
<th>$R_e$, $R_E$</th>
<th>p.c., %</th>
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<tr>
<td>0.4</td>
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* $R$ refers to the radial distance (in earth radii) in the midnight meridian plane.
Fig. 6. The close-up of the 'bulge' portion in Figure 5. The curve represents the maximum particle energy at a given radial distance for a specific convection electric field. The scale for \( R \) and \( E \) is linear, and the scale for \( E_0 \) is square-rooted.

energies are below 31 kev, so that they cannot be detected at the channel of \( 31 < E < 49 \) kev.

In main phase period, on the other hand, protons with \( \mu = 30 \) ev/\( \gamma \) can be brought in at 2.7 \( R_E \). Particles with \( \mu < 20 \) ev/\( \gamma \) can penetrate farther; however, they can hardly survive because of the slower drift velocities and the shorter lifetimes to charge exchange from 2.7 \( R_E \) inward. Furthermore, pitch angle scattering tends to prevent particles from coming close to the earth [Kennel, 1969].

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