THE EFFECT OF NON-UNIFORM IONOSPHERIC CONDUCTIVITY ON STANDING MAGNETOSPHERIC ALFVEN WAVES

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Abstract—We look at time-dependent normal mode solutions to the Alfven wave equation in a uniform magnetic field, between planar ionospheres. In particular, the effect of sharp gradients in ionospheric conductivity on the spatial and temporal structure of the waves is considered. We show that the electric field of the wave must always be perpendicular to any conductivity discontinuities present, and that this is achieved by the generation of circularly polarized Alfven waves at the discontinuity. The results are applied to an ionospheric strip of high conductivity; this being relevant to Pi2s.

INTRODUCTION
In the closed field line regions of the terrestrial magnetosphere, standing transverse Alfven waves may be excited on closed geomagnetic field lines (Southwood, 1974; Radoski, 1974; Chen and Hasegawa, 1974) and are responsible for the ultra low frequency geomagnetic pulsations observed on the ground. The existence of the standing structure depends on reasonable levels of reflection from the ionospheres at the feet of the flux tubes. Indeed, the structure and development of the waves are likely to be modified significantly by boundary conditions at the ionospheres.

The simplest situation to consider is one in which the conductivities in both ionospheres are equal and uniform. Alfven wave damping occurs as a result of Joule dissipation in the ionospheres. This paper considers the more complicated situation in which there are discontinuities in the ionospheric conductivity.

The rate of the damping depends critically on the relative magnitude of the ionospheric height-integrated Pedersen conductivity and the flux tube conductivity; the nearer they are matched, the more effective is the damping (Newton et al., 1978; Allan and Knox, 1979). That the form of damping proposed is very effective has recently been shown by Greenwald and Walker (1980), Crowley et al. (1985) and Lathuillere et al. (1986).

Their work is extended here to consider standing Alfven waves and their time development.

We will consider a single discontinuity in both Pedersen and Hall conductivities, and will apply the result to the case of a high conductivity strip in the ionosphere, as might be present in auroral regions during a substorm. The work is therefore particularly relevant to Pi2s, which are transient geomagnetic pulsations associated with substorm onset.

HYDROMAGNETIC WAVES AND THE IONOSPHERE
In a cold plasma there are two hydromagnetic modes, the fast and the transverse or Alfven mode. In this paper we shall be concerned with only one of these, the Alfven mode. It has long been recognized as the wave that is important for magnetospheric-ionospheric coupling. There are various ways of understanding why it is the Alfven mode that is important. The most popular is to point out that the Alfven mode is the mode that carries field-aligned current and thus is the only wave that can adjust the electrical coupling along individual flux tubes threading the two regions (see for example, Southwood and Stuart, 1980).

Mathematically, the reason for the dominance of the Alfven mode in studies of the ionosphere lies in the difference in the dispersion relation of the two modes in the magnetospheric plasma. As, for instance, shown by Hughes (1974) (see also Hughes and Southwood, 1974, 1976), a source of fast mode energy in the ionosphere attenuates vertically on a scale comparable to its horizontal wave number. The more localized a source is, the more rapidly it is attenuated.
vertically. The dispersion relation of the Alfven mode, in contrast, is independent of the variation perpendicular to the field (Dungey, 1967).

In circumstances where the ionosphere is horizontally uniform there is a very simple solution for the structure of the electric and magnetic fields of a magnetospheric Alfven wave. Hughes (1974) solved the full wave problem for a vertically stratified ionosphere and then pointed out that a simple sheet approximation for the ionosphere could reproduce his results. In the approximate solution, the electric field of the magnetospheric Alfven wave drives Pedersen currents in the ionosphere which shield the magnetic component of the Alfven wave from penetrating below the ionosphere. The electric field also drives Hall currents. These provide the ground magnetic signature and above the ionosphere give rise to a fast mode signal that attenuates vertically as mentioned above. In these circumstances, the boundary condition at the ionosphere is the familiar relation given later as equation (3) in this paper.

Similar considerations concerning the relative importance of Alfven and fast mode fields apply when the ionosphere is allowed to be horizontally non-uniform, as the vertical attenuation of fast mode signals from a localized source does not depend on the uniformity of the local ionosphere. Ellis and Southwood (1983) examined the reflection of an Alfven wave pulse from an ionosphere in which there was a relatively sharp horizontal step in conductivity such as might occur at the dawn or dusk terminator or at the equatorward edge of the auroral zone. Recently, Glassmeier (1983, 1984) has done some similar work allowing for a finite conductivity gradient. The results show that the major change is the need to allow for additional field-aligned currents flowing out of the ionospheric gradient region effectively as a polarization phenomenon. Excess current from the high conductivity region is carried away by field-aligned currents. In certain cases, much as we see later in this paper, subsidiary Alfven waves are created that carry the additional field-aligned current. Otherwise differential damping between the two regions gives rise to the current distribution naturally. All such treatments rely on the suppression of the long range propagation of fields associated with the fast mode. As mentioned above, this seems a reasonable assumption. In particular, in the very short gradient limit used by Ellis and Southwood (1983) and in this paper, it is clear that the discontinuity itself generates no electric field in the direction of inhomogeneity; Faraday's law requires that the electric field align with the discontinuity and thus the tangential field be continuous.

**DAMPING OF STATIONARY ALFVEN WAVES BETWEEN UNIFORMLY CONDUCTING IONOSPHERES**

It is useful to start by considering the problem with uniform ionospheres, since its solution forms the basis for solutions in more complicated situations.

A uniform magnetic field, $B_0$, is considered. At $z = \pm a$ there are uniformly conducting ionospheres of height-integrated Pedersen conductivity $\Sigma_p$. The symmetry of the system and the fact that it is absorptive imply that the Alfven wave magnetic and electric fields should take the following forms

$$b = b_0 e^{-\gamma t} e^{i\omega t} (e^{-ikz} \pm e^{ikz})$$

and

$$E = V_A b_0 e^{-\gamma t} e^{i\omega t} (e^{ikz} \mp e^{-ikz})$$

where $V_A$ is the Alfven speed.

The dispersion relation of the Alfven mode in a uniform plasma only involves the parallel wave number; thus no spatial variation across the field is imposed.

In equations (1) and (2), $K$ is complex, $\gamma$ and $\omega$ are real, and all three are determined by the ionospheric boundary condition for transverse Alfven waves,

$$b \times \mathbf{z} = \mu_0 \Sigma_p E$$

(Hughes and Southwood, 1976).

The result is:

$$\gamma = \frac{V_A}{2a} \ln(|R|^{-1}), \omega = \frac{mn}{2a} V_A, m = \text{integer}$$

$$K_i = \frac{1}{2a} \ln(|R|^{-1}), K_r = \frac{mn}{2a}$$

(Ellis and Southwood, 1983).

When $m$ is even (odd), upper (lower) signs are chosen in equations (1) and (2), corresponding to an electric (magnetic) node at the equator. The parameters $R$ and $\Sigma_A$ are the reflection coefficient and Alfven conductance given by

$$R = \frac{\Sigma_p - \Sigma_A}{\Sigma_p + \Sigma_A}, \quad \Sigma_A = \frac{1}{\mu_0 V_A}.$$

Thus the normal mode solution for Alfven waves between uniformly conducting ionospheres is of decaying standing wave form. The rate of decay of the waves and their structure are seen to depend upon the height-integrated Pedersen conductivity of the ionospheres.

Equations (1) and (2) describe the variation of the wave electric and magnetic fields with $z$ and $t$. However, since the electric field has a component only in $x$ and the Alfven wave is characterized by
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non-zero electric field divergence, the wave fields must vary in the \( x \) direction. We choose a harmonic form for this variation, viz. \( e^{i k_x x} \). Any more complex form could be constructed by Fourier synthesis from this.

The fields and ionospheric current closure of an Alfvén wave with such a variation are illustrated schematically in Fig. 1. The wave electric and magnetic fields and the ambient magnetic field are mutually perpendicular. Within the magnetospheric Alfvén wave structure, the current density consists of two parts, determined by \( \mu x j = \text{curl} \; b \); a field aligned component, \( j_{||} \), and a component \( j_{\perp} \), which is parallel to the electric field. In the ionosphere the parallel currents are closed by a dissipative Pedersen current, \( J_p = \Sigma_p E \).

Nowhere does the magnetospheric wave structure connect to Hall current flow in the ionosphere. The ionospheric Hall current is divergence free and flows normal to the plane illustrated.

We will now turn our attention to systems with field variations thus described, but which include ionospheric conductivity discontinuities. One can note that far away from any discontinuity we may expect the solution to take the form of (1) and (2).

**THE EFFECT OF CONDUCTIVITY DISCONTINUITIES**

In the following sections, we shall look at what effects horizontal discontinuities in ionospheric conductivity may have on Alfvén waves whose electric fields are linearly polarized in the \( \hat{x} \) direction and which vary as \( e^{i k_x x} \).

Two cases will be considered. In the first, the electric field in the unperturbed wave is perpendicular to the conductivity discontinuity and, in the second, parallel to it.

It is worth emphasizing at this point the differences between the treatment of Ellis and Southwood (1983) and the present treatment. The former approach does not assume any particular time variation, nor any variation in \( z \). It describes the reflection of a single Alfvénic pulse from one ionosphere. The present approach accommodates factors of \( e^{(i \omega + i \gamma) t} \) and \( (e^{-i K_z} + e^{i K_z}) \), which arise in the normal mode solution of the Alfvén wave equation restricted by the presence of two ionospheres. Despite these differences, the essential physics of the problem is common to both and the procedure that we adopt is the same as that used by Ellis and Southwood (1983).

(a) Wave electric field perpendicular to the conductivity discontinuity

The situation is outlined in Fig. 2. The conductivity discontinuity lies along \( x = 0 \) and the linearly polarized Alfvén wave electric fields are in the \( \hat{x} \) direction. \( \Sigma_{\text{H}} \) and \( \Sigma_{\text{p}} \) refer to the height-integrated Hall and Pedersen conductivities in regions (1) and (2) and the \( J_s \) refer to the corresponding currents. The ionosphere is at \( z = \pm a \).

One might expect that the values of \( \gamma \) and \( K \) differ on either side of the boundary because \( \Sigma \) differs. Let us investigate a trial solution explicitly allowing for spatially-dependent damping rates and complex propagation vector, \( K \).

\[
\begin{align*}
x < 0 & \quad b_1 = b_0 e^{i \omega t} e^{i k_x x} e^{-\gamma t} (e^{i K_z z} + e^{-i K_z z}) \hat{y} \\
& \quad E_1 = V_0 b_0 e^{i \omega t} e^{i k_x x} e^{-\gamma t} (e^{-K_z z} - e^{K_z z}) \hat{x};
\end{align*}
\](5)
FIG. 2. FIELDS AND CURRENTS OF A STANDING ALFVÉN WAVE, WITH ELECTRIC FIELD PERPENDICULAR TO AN IONOSPHERIC CONDUCTIVITY DISCONTINUITY. The ionosphere is in the x, y plane, and the discontinuity is along $x = 0$.

$E_2 = V_k b_0 e^{i k x} e^{-\gamma t} (e^{i k z} + e^{-i k z}) \hat{y}$

$E_2 = V_k b_0 e^{i k x} e^{-\gamma t} (e^{-i k z} - e^{i k z}) \hat{y}$

(6)

Note that we have chosen the sign of the solution which corresponds to an electric node at the equator. In the above let us assume further that $\omega$ and $\gamma_1, \gamma_2$ are determined by expressions of the same form as (4). Thus the assumed frequency, $\omega$, is quantized by the boundary conditions in $z$. The damping rates are inversely proportional to the Pedersen conductivity in each region.

The above expressions satisfy the Alfvén wave equation and the ionospheric boundary condition, (3), away from the discontinuity. It remains to show that current continuity is satisfied at $z = a, x = 0$, and that the electric field tangential to the discontinuity is continuous.

The latter requirement is trivially satisfied, since the electric field is perpendicular to the discontinuity everywhere. It follows from the field orientation chosen that the only discontinuity in $\hat{y}$-directed current in the ionosphere is that of the Pedersen current. In the uniform solution, ionospheric Pedersen current closes in the Alfvén waves in the magnetosphere. Note also that the Pedersen currents are the root of the dissipative process that causes the damping. Now the form of the solution we have chosen implies that the rate of loss of energy on each side of the discontinuity matches the rate of dissipation in the ionosphere on that side. The consequent differential rate of decay across the boundary will lead to a discontinuity in wave amplitude across the discontinuity. There is thus a net excess of Pedersen current, but, because of the polarization, not Hall current. Because of the natural configuration of the Alfvén wave wherein ionospheric Pedersen current is closed by magnetospheric current flow in the wave, no problem is created by the Pedersen current excess. Just as is described in the similar case discussed by Ellis and Southwood (1983), the excess ionospheric current is carried away by a sheet field-aligned current above the discontinuity. This current also just serves to provide the appropriate jump in tangential magnetic field in the wave across the field lines mapping to the discontinuity.

The situation is equivalent to two uniformly conducting ionospheres placed side by side. The currents on each side of the discontinuity close independently of those on the other side, the Alfvén waves therefore being of different magnitude on either side of the boundary for all time. This is clarified in Fig. 3. The sheet of enhanced field-aligned current per unit length of discontinuity at $x = 0$ is, from Ampère's law,

$$J_{\parallel} = E_n e^{i k x} [\Sigma_{\rho_1}(e^{-i k z} - e^{i k z})e^{-\gamma t} - \Sigma_{\rho_2}(e^{-i k z} - e^{i k z})e^{-\gamma t}].$$

(7)

Note how the field-aligned current grows in amplitude initially as the wave decays in the region with the larger damping decrement. Subsequently, the current decays as the more weakly damped wave also decays.

The simplicity of the solution (5) and (6) derives from the fact that the wave electric field is always perpendicular to the discontinuity, and so the constraint on the electric field parallel to the discontinuity is always satisfied. The solution is very similar in form to the corresponding instance discussed by Ellis and
Southwood (1983). In the next subsection we look for solutions in which the electric field in the unperturbed (or incident) signal is everywhere parallel to the discontinuity.

(b) Distant wave electric field parallel to the discontinuity

The situation is illustrated in Fig. 4. The electric fields $E_1$ and $E_2$ are the linearly polarized components that constitute the entire solution far from the discontinuity and are once again assumed to be in the $\hat{x}$ direction. However, the discontinuity is now located at $y = 0$ and thus aligned with the distant wave electric field. Near the discontinuity, the polarization must differ from the distant fields in this case. Firstly, as in the previous case, we expect the fields on either side to differ as time progresses, but in this instance the imposed field has a tangential component and thus tangential electric field continuity cannot be maintained without additional fields. Also, an excess of Hall current is created at the discontinuity and similarly requires additional fields to maintain charge balance as the Hall current in itself cannot be simply closed by a purely Alfvén wave structure in the magnetosphere.

Ellis and Southwood (1983) resolved the closure problem for a single Alfvén pulse (rather than the normal mode solution we seek here) by introducing circularly polarized subsidiary waves centred on the discontinuity and which decay in the direction. These waves carry parallel currents in a sheet mapping to the discontinuity and thus can be introduced to balance current flow in the vicinity of the discontinuity. Ellis and Southwood (1983) showed that the net effect of these subsidiary waves is to reduce the

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**Fig. 3.** Current structure for the wave configuration shown in Fig. 2. $\lambda$ Again represents one perpendicular wavelength of the signal. Over a uniform ionosphere, the net current out balances that into the ionosphere (Fig. 1). At a discontinuity in conductivity, e.g. at A, any excess Pedersen current is carried off in a sheet.

**Fig. 4.** Electric fields of a standing Alfvén wave, with distant electric field parallel to an ionospheric conductivity discontinuity (at $y = 0$). $E_1$ and $E_2$ represent the distant electric field on either side of the discontinuity. $E_1$, $E_2$ are the magnitudes of the subsidiary surface Alfvén waves, localized at the discontinuity.
component of electric field parallel to the discontinuity and so they partly suppress the flow of Hall current into the discontinuity. We take a similar approach here, modifying the form of the subsidiary waves to include their time dependence and their standing wave nature. We try therefore a solution made up of a superposition of the linearly polarized component:

\[ y < 0 \quad b_1 = b_0 \exp(i k_x x - \gamma t (e^{-i k_x x} + e^{i k_x x})) \]

\[ E_1 = V_A b_0 \exp(i k_x x - \gamma t (e^{i k_x x} + e^{-i k_x x})) \]

\[ y > 0 \quad b_2 = b_0 \exp(i k_x x - \gamma t (e^{i k_x x} + e^{-i k_x x})) \]

\[ E_2 = V_A b_0 \exp(i k_x x - \gamma t (e^{i k_x x} - e^{-i k_x x})) \]

and the circularly polarized subsidiary wave component:

\[ y < 0 \quad \tilde{b}_1 = (-i \tilde{b}_1, -\tilde{b}_1, 0) \]

\[ \tilde{E}_1 = (E_1, -i E_1, 0) \]

where

\[ \tilde{b}_1 = a_1 b_0 \exp(i k_x x - \gamma t (e^{i k_x x} + e^{-i k_x x})) \]

\[ \tilde{E}_1 = a_1 V_A b_0 \exp(i k_x x - \gamma t (e^{i k_x x} - e^{-i k_x x})) \]

\[ y > 0 \quad \tilde{b}_2 = (i \tilde{b}_2, -\tilde{b}_2, 0) \]

\[ \tilde{E}_2 = (\tilde{E}_2, i \tilde{E}_2, 0) \]

where

\[ \tilde{b}_2 = a_2 b_0 \exp(i k_x x - \gamma t (e^{i k_x x} + e^{-i k_x x})) \]

\[ \tilde{E}_2 = a_2 V_A b_0 \exp(i k_x x - \gamma t (e^{i k_x x} - e^{-i k_x x})) \]

\[ a_1 \text{ and } a_2 \text{ are constants.} \]

To find the values of \( a_1 \) and \( a_2 \), the two boundary conditions at the discontinuity are applied. Continuity of electric field parallel to the discontinuity at \( y = 0 \) implies:

\[ (e^{-i k_x z} - e^{i k_x z}) e^{-\gamma t} (1 - a_1) = (e^{-i k_x z} - e^{i k_x z}) e^{-\gamma t} (1 - a_2). \]

This must hold for all \( t \) and \( z \), hence

\[ (1 - a_1) = (1 - a_2) \]

\[ a_1 = a_2 \]

for \( \Sigma_{p1} = \Sigma_{p2} \).  \( (12) \)

\[ (1 - a_1) = (1 - a_2) = 0 \]

\[ a_1 = a_2 = 1 \]

for \( \Sigma_{p1} \neq \Sigma_{p2} \).  \( (13) \)

Immediately we see that the parallel component of the electric field must always be zero at a discontinuity of Pedersen conductivity. It is also necessary for the boundary condition of current continuity to be satisfied at the discontinuity. The field-aligned current per unit length of discontinuity at \( y = 0 \) (by Ampere's law) is

\[ J_{||} = \frac{i}{\mu_0} (\tilde{b}_1 + \tilde{b}_2) \]

The current in \( \tilde{y} \) in region (1) is

\[ J_{y1} = \Sigma_{H1} E_{x1} + \Sigma_{H1} \tilde{E}_1 - i \Sigma_{p1} \tilde{E}_1 \]

and in region (2) is

\[ J_{y2} = \Sigma_{H2} E_{x2} + \Sigma_{H2} \tilde{E}_2 + i \Sigma_{p2} \tilde{E}_2. \]

Continuity of current at \( y = 0, z = a \) implies:

\[ \Sigma_{H2} E_{x2} + \Sigma_{H1} \tilde{E}_2 + i \Sigma_{p1} \tilde{E}_2 + \frac{i}{\mu_0} (\tilde{b}_1 + \tilde{b}_2) = \Sigma_{H1} E_{x1} + \Sigma_{H1} \tilde{E}_1 - i \Sigma_{p1} \tilde{E}_1. \]

Now if \( a_1 = a_2 \) as required previously, and substituting in for \( \tilde{E}_1, \tilde{E}_2, \tilde{b}_2 \) we obtain

\[ \Sigma_{H1} (1 - a_1) (e^{-i k_x a} - e^{i k_x a}) e^{-\gamma t} + \frac{i}{\mu_0} a_1 (e^{i k_x a} - e^{-i k_x a}) e^{-\gamma t} + \frac{i}{\mu_0} a_1 (e^{i k_x a} + e^{i k_x a}) e^{-\gamma t} = \Sigma_{H1} (1 - a_1) (e^{-i k_x a} - e^{i k_x a}) e^{-\gamma t} - i \Sigma_{p1} a_1 (e^{i k_x a} - e^{-i k_x a}) e^{-\gamma t}. \]

Again this must hold for all time. So, for \( \Sigma_{p1} \neq \Sigma_{p2} \), equating coefficients of \( e^{-\gamma t} \) and those of \( e^{-\gamma t} \) gives:

\[ \mu_0 V_A \Sigma_{p2} = \frac{e^{-i k_x a} + e^{i k_x a}}{e^{i k_x a} - e^{-i k_x a}} \]

\[ \mu_0 V_A \Sigma_{p1} = \frac{e^{-i k_x a} + e^{i k_x a}}{e^{i k_x a} - e^{-i k_x a}} \]

which are the required conditions on \( K_x, K_z \) already demanded by the ionospheric boundary condition (3) on either side of \( y = 0 \).

For \( \Sigma_{p1} = \Sigma_{p2} \), the solution of (14) gives the same result, with \( a_1 = a_2 = 1 \).

The full solution is therefore

\[ y < 0 \quad b_1 = b_0 \exp(i k_x x - \gamma t (e^{-i k_x x} + e^{i k_x x})) \]

\[ E_1 = V_A b_0 \exp(i k_x x - \gamma t (e^{i k_x x} + e^{-i k_x x})) \]

\[ y > 0 \quad b_2 = b_0 \exp(i k_x x - \gamma t (e^{i k_x x} + e^{-i k_x x})) \]

\[ E_2 = V_A b_0 \exp(i k_x x - \gamma t (e^{i k_x x} - e^{-i k_x x})) \]
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We have thus derived a solution for a mode standing in the $z$ direction in which the fields decay in time at different rates on either side of the boundary marked by the discontinuity in conductivities, much as we had in the previous case. However, we have had to introduce subsidiary waves which carry current in sheets along the discontinuity and whose amplitudes decay in space away from the boundary. Ellis and Southwood (1983) similarly introduced such signals into their solution.

There is an apparently significant difference between the fields derived by Ellis and Southwood (1983) and those given here. In the case here, the tangential electric field is entirely suppressed across the boundary at $y = 0$. This feature of our solution is independent of the magnitudes of the conductivities and is due to the distinct time variation for the fields on each side of the discontinuity that we explicitly introduced ab initio. On the other hand, in the impulse reflection problem, no time variation is assumed (Ellis and Southwood, 1983; Sun and Kan, 1985), and the induced surface wave fields only act on each reflection to reduce the tangential electric field and Hall current flow into the discontinuity. The reduction in tangential field after reflection depends on the relative magnitude of the ionospheric conductivities on each side. We discuss this matter further later.

AN APPLICATION

The previous analysis will now be applied to a high conductivity strip in the ionosphere. Such conductivity enhancements are present in auroral regions during substorms, and so the work may indicate the development of stationary Alfvén waves associated with the substorm process. The most notable manifestation of these waves is the Pi2.

The geometry we consider is illustrated in Fig. 5. We again consider a linear polarized component of electric field in $\hat{x}$ and the necessary subsidiary wave component.

This time there are two discontinuities, at $y = \pm y_0$.

The linear wave contribution is

$$\mathbf{b}_{1,2} = b_0 e^{ik_z y} e^{i\phi_0} e^{-\gamma_1 t} (e^{-ik_z z} + e^{iK_z z}) \mathbf{\hat{x}}$$

$$E_{1,2} = V_A b_0 e^{ik_z y} e^{i\phi_0} e^{-\gamma_1 t} (e^{-ik_z z} - e^{iK_z z}) \mathbf{\hat{x}}$$

$$\times \left[ \mathbf{\hat{y}} \right]$$

We discuss this matter further later.

The surface wave contribution is as follows:

For the discontinuity $y = -y_0$,

$$\mathbf{b}_1 = (-i \mathbf{b}_1, - \mathbf{b}_1, 0)$$

where

$$\mathbf{b}_1 = c_1 b_0 e^{i\phi_0} e^{ik_z y} e^{-\gamma_1 t} (e^{-ik_z z} + e^{iK_z z}) \mathbf{\hat{x}}$$

For the discontinuity $y = y_0$,

$$\mathbf{b}_2 = (-i \mathbf{b}_2, - \mathbf{b}_2, 0)$$

where

$$\mathbf{b}_2 = c_2 b_0 e^{i\phi_0} e^{ik_z y} e^{-\gamma_1 t} (e^{-ik_z z} + e^{iK_z z}) \mathbf{\hat{x}}$$

The corresponding electric fields are given by

$$E_{\hat{z}} + E_1 = E_{\hat{z}} + E_2 (i) + E_2 (ii)$$

which implies

$$c_1 = 1, \quad c_2 + c_3 e^{-2|K_z|y_0} = 1$$

FIG. 5. GEOMETRY OF HIGH CONDUCTIVITY STRIP IN THE IONOSPHERE.
and at \( y = y_0 \) gives

\[
E_{x2} + \tilde{E}_{x2}^x(i) + \tilde{E}_{x2}^y = E_{x3} + \tilde{E}_{x3}
\]

which implies

\[
c_4 = 1, \quad c_2 e^{-2|k_z|y_0} + c_3 = 1.
\]

Therefore we have

\[
c_1 - c_4 = 1\]

\[
c_2 = c_3 = \frac{1}{1 + e^{-2|k_z|y_0}}.
\]

As before, it is necessary that current continuity is satisfied at the discontinuities.

Integrating Ampere's law across each boundary reduces to the conditions on \( K_2, K_1 \) (15) and (16).

So the full solution is:

\[
y < y_0 \quad b_1 = b_0 e^{i|k_z|y} e^{-ik_1y} + e^{ik_1y} [x (1 - e^{ik_2(y + y_0)} - i e^{ik_1(y + y_0)} x] \quad \text{(19a)}
\]

\[
E_1 = V_A b_0 e^{i|k_z|y} e^{-ik_1y} (e^{-ik_2y} - e^{ik_2y})
\]

\[
-b_2 = b_0 e^{i|k_z|y} e^{-ik_1y} e^{-ik_2y} [e^{-ik_1y} + e^{ik_1y}]
\]

\[
\times \left[ i \bar{x} e^{-ik_1(y + y_0)} + e^{ik_1(y + y_0)} x - e^{-ik_2(y + y_0)} + e^{ik_2(y + y_0)} x \right]
\]

\[
E_2 = V_A b_0 e^{i|k_z|y} e^{-ik_1y} e^{-ik_2y} [e^{-ik_1y} + e^{ik_1y}]
\]

\[
\times \left[ i \bar{x} e^{-ik_1(y + y_0)} + e^{ik_1(y + y_0)} x - e^{-ik_2(y + y_0)} + e^{ik_2(y + y_0)} x \right]
\]

\[
y > y_0 \quad b_3 = b_0 e^{i|k_z|y} e^{-ik_1y} e^{-ik_2y} + e^{ik_1y} \left[ i \bar{x} e^{-ik_1(y + y_0)} + e^{ik_1(y + y_0)} x \right] \quad \text{(20a)}
\]

\[
E_3 = V_A b_0 e^{i|k_z|y} e^{-ik_1y} e^{-ik_2y} (e^{-ik_1y} - e^{-ik_1y})
\]

\[
- e^{ik_1y} \left[ x (1 + e^{-ik_1(y + y_0)} + i \bar{x} e^{-ik_1(y + y_0)} + e^{ik_1(y + y_0)} x \right]
\]

\[
\text{FIELD LINE RESONANCE EFFECTS}
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A further important feature of the real magnetosphere that we have not treated here is the effect of the spatial dependence of field line resonance frequencies. The problem is complicated and has been dealt with using various simple models (originally by Southwood, 1974; Chen and Hasegawa, 1974) and is still the subject of intensive study (Allan et al., 1986; Kivelson and Southwood, 1986). For pulsations excited by an impulsive source one expects a spatially dependent spectrum of hydromagnetic waves to be excited (Radoski, 1974). Intuitively, one might expect that the signal would be dominated by the resonance
frequencies of the field lines on which the source field-aligned currents flow. As noted above, we do not feel that our model treats the distribution of field-aligned current well. What we do feel secure about in our model and which should appear in any modified form of it, are the pulsating field-aligned currents in the surface waves on the edges of the high conductivity regions. We expect that the frequency response of the effect on the ground of damped oscillation fields in the ionosphere with a spatially varying frequency has recently been done by Poulter and Allan (1985). Their work suggests that field line resonance effects may be smeared out in many situations.

**SUMMARY**

We have examined simple models of Alfvén waves trapped between partially reflecting ionospheres. The signals have a damped form in time, the damping rate being a function of the local ionospheric Pedersen conductivity. Using the extreme example of sharp jumps in conductivity, we have investigated the modification in wave structure and polarization introduced by horizontal nonuniformity in ionospheric conductivities. In all cases that we have considered, the electric field at any discontinuity is constrained to align perpendicular to the discontinuity. Unless the applied field by the magnetospheric source is polarized to match such a condition, the condition is achieved by the generation of circularly polarized subsidary Alfvén waves centred at the boundary corresponding to the field lines mapping to the discontinuity. Evidently these localized signals can rotate the polarization by as much as a right-angle in the vicinity of the discontinuity and one expects that the discontinuity would have a very significant influence on signal structure and morphology even when recorded on the ground.

**REFERENCES**


