Generation of Vortex-Induced Tearing Mode Instability at the Magnetopause

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A two-dimensional magnetohydrodynamic simulation is performed to study the generation of the vortex-induced tearing mode instability at the magnetopause. When a sheared magnetic field exists along with the velocity shear, the tearing mode will couple with the Kelvin-Helmholtz (K-H) instability. The shear Alfvén Mach number \( M_a \) plays an essential role in determining the linear properties and nonlinear evolution of the coupled instability. When the magnetic field is fixed, if \( M_a < 0.4 \), the spontaneous tearing mode is dominated; when \( 0.4 < M_a < 1 \), the tearing mode is apparently modified by the K-H instability; as \( 1 < M_a \), the coupled instability, called the vortex-induced tearing mode (VITM) instability, appears to be intrinsically different from the conventional tearing mode instability. The long time asymptotic quasi-static state for the VITM instability is characterized by a large-scale fluid vortex together with a concentric magnetic island. The linear properties and nonlinear evolution of the VITM instability are not significantly changed with different Reynolds and magnetic Reynolds number used in the simulation.

1. INTRODUCTION

Plasma instabilities at the magnetospheric boundary have attracted space scientists' attention for quite a long time. In the past decade, considerable effort has been given to the tearing mode instability and the Kelvin-Helmholtz (K-H) instability at the magnetopause [Sonnerup and Ledley, 1979; Quest and Coroniti, 1981; Sonnerup, 1984; Paschmann et al., 1986; Galeev et al., 1986; Russell, 1984; Fu and Lee, 1985; Southwood, 1979; Lee et al., 1981; Walker, 1981; Pu and Kivelson, 1983a; Kivelson and Pu, 1984; Miura, 1984, 1987]. A great deal has been achieved. For example, the nonlinear evolution of the collisionless guide-field tearing mode instability has been investigated, and its efficacy and role in magnetopause reconnection have been estimated [Coroniti and Quest, 1984]. The multiple reconnection model based on the forced tearing mode instability has presented an interesting picture for flux transfer events at the dayside magnetopause [Lee and Fu, 1985]. The properties of unstable K-H waves in compressional plasmas and collisionless plasmas have been extensively studied [Pu and Kivelson, 1983a, b; Pu, 1989]. Nonlinear effects associated with the K-H instability have been found to contribute significantly to the magnetospheric boundary dynamics [Miura, 1984, 1987]. It is well known that the tearing mode instability occurs when (at least one component of) the magnetic field is reversed within a short distance, while the K-H instability appears in a sheared flow field. Almost all previous work treated these two instabilities separately. However, when the interplanetary magnetic field (IMF) has a southward component, a sheared flow and a sheared magnetic field exist simultaneously near the magnetospheric boundary. What will happen in this case if the tearing mode and K-H instability are coupled with each other? The problem is obviously of great importance to solar terrestrial physics.

Recently, Liu et al. and La Belle-Hamer et al. suggested that local magnetic field reconnection can be caused by vortices in the flow field [Liu and Hu, 1988; Hu et al., 1988; La Belle-Hamer et al., 1988]. They pointed out that if there is a strong velocity shear in the current sheet, the K-H instability is excited to produce large-scale fluid vortices; the magnetic field response to the vortices causes the magnetic field lines to twist and so to generate local reconnection. Apparently, in this case, coupling of the tearing mode with the K-H instability has occurred, leading to the change of features of instability in the transition region. How do the instability features vary with the flow speed? Under what conditions can the vortex-induced reconnection take place? And how is the vortex-induced reconnection different from conventional reconnection? In this paper a two-dimensional MHD simulation is performed to find answers to these questions. It is seen that the Alfvén Mach number \( M_a \) plays an essential role in
determining the properties of the coupled instability. In the case of $B_0 = 1.0$, $R_{in}^{-1} = 2.0$, and $R^{-1} = 0.1$, if $M_A < 0.4$, the spontaneous tearing mode is dominated in the system. When $0.4 \leq M_A \leq 1$, the tearing mode is apparently modified by the K-H instability. As $1 < M_A$, the coupled instability, called the vortex-induced tearing mode (VITM) instability, appears to be intrinsically different from the conventional tearing mode. The vortex-induced magnetic reconnection (VIMR) discussed by Hu et al. [1988] appears in this circumstance. The long time asymptotic quasi-static state (AQSS) for the VITM is characterized by a large-scale vortex together with a concentric magnetic island. The structures of magnetic islands and fluid vortices as $R_{in}^{-1} < 2.0$ and $R^{-1} < 0.1$ are quite similar to those for $R_{in}^{-1} = 2.0$ and $R^{-1} = 0.1$. The critical value of $M_A$ for generating the VITM is still found to be $\approx 1.0$. The linear and nonlinear instability properties of the VITM are not significantly changed with different $R_{in}^{-1}$ and $R^{-1}$.

2. SIMULATION MODEL

2.1. Basic Assumptions

For simplicity, we make the following assumptions: (1) In the initial state a one-dimensional velocity shear $V_0(z) = V_0 \tanh (z/\ell_0)$ exists, which varies with $z$ only. (2) The initial magnetic field $B_0(z)$ lies on the $(x, z)$ plane and only varies with $z$ as well. In addition, $B_{0x}(z)$, the $x$ component of $B_0(z)$, is also reversed within a short distant in the $z$ direction. (3) All perturbation quantities are invariant in the $y$ direction, i.e., $\partial / \partial y = 0$. (4) Plasmas can be regarded as incompressible and the number density is uniform everywhere.

2.2. Dimensionless Equations

In assumptions 1–4 we find that neither $B_x(x, z, t)$ nor $V_y(x, z, t)$ appears in the equations governing other quantities. The two-dimensional dimensionless MHD equations can then be written as

with $V = V_x \epsilon_x + V_y \epsilon_y$, $B = B_x \epsilon_x + B_y \epsilon_y$, $\nabla^2 \Phi = \Omega$, and $\nabla^2 A = -J$, where $A$, $\Omega$, and $J$ represent the $y$ component of the magnetic potential vector, fluid vorticity, and current density, respectively. $R = \ell_0 / \ell_0$ and $R_{in} = \ell_0^2 / \ell_0$ refer to the Reynolds and magnetic Reynolds number, $\ell_0 = \ell_0 / \nu_A$. $\ell_0$ indicates a unit length used to measure all distances in the system, $\nu_A = B_0(\mu_0 \rho_0)^{1/2}$, $B_0$ is a characteristic magnitude of $B_x$, $\rho_0 = n_0 m_p$ denotes the mass density and $\mu$ and $\mu_m$ are the fluid and magnetic viscosity, respectively. In the present work, we only need to deal with equations (1), (2), (5), and (6).

2.3. Initial and Boundary Conditions

The simulation region is taken to be a square of $-L/2 < x < L/2$, $-L/2 < z < L/2$ on the $(x, z)$ plane. The boundary conditions imposed at $x = \pm L/2$ are periodic in the $x$ direction, while at the upper and lower boundaries, $z = \pm L/2$, perturbations all tend to zero. The initial profiles of $V_0$ and $B_0$ are assumed to be

\begin{align*}
V_0 &= V_0 \tanh (z/\ell_0) \epsilon_x \quad (7) \\
B_0 &= -B_0 \tanh (z/\ell_0) \epsilon_x \quad (8)
\end{align*}

with $\ell_0$ indicating the scale length of the shear region.

2.4. Numerical Method

A mesh system of $(32 \times 32)$ grid points is used. The differential derivatives in space are replaced by the central difference formulas, while for time evolution we use the fourth-order Runge-Kutta scheme. The following parameters are chosen in the simulations: $B_0 = 1.0$, $\ell_0 = 6.4$, $R^{-1} = 0.1$, $R_{in}^{-1} = 2.0$, except for a few cases specially mentioned in the text.

3. SIMULATION RESULTS

1. Suppose that small perturbations appear in both $\Omega$ and $A$ at $T = 0$. Figure 1 shows how the maximum values of $|\Delta \Omega|$ and $|\Delta A|$ evolve with time when $B_0 = 1.0$ and $M_A = \nu_0 / \ell_0 = 0.3$. It can be seen clearly that $\Omega$ and $A$ grow simultaneously at the same growth rate in the linear region, while during the nonlinear stage they will gradually stop growing. Figure 2a plots how $\alpha = \gamma_{TM}$ varies with $M_A$ when $B_0 = 1.0$, where $\gamma$ is the linear growth rate of the coupled instability and $\gamma_{TM}$ denotes the linear growth rate for the spontaneous tearing mode instability ($M_A = 0$) under the same condition. If $M_A < 0.4$, $\alpha \to 1$, the tearing mode is dominated. When $0.4 \leq M_A \leq 1$, the growth rate gradually increases. This means that the tearing mode is enhanced and modified by the K-H instability. For $M_A >$
1, α rises very rapidly, implying that instability properties have been changed and the system is controlled by the K-H instability in this circumstance. Figure 2b shows how α varies with $M_A$ when $V_0 = 1.0$. It is found that when $M_A > 1$, α drops rapidly with decreasing $M_A$ (or increasing $B_0$), for the magnetic tension reduces the K-H instability, while for $0.4 < M_A < 1$, α increases as $M_A$ further decreases. For $M_A < 0.4$, α tends to 1.

2. Figure 3 plots $W$, the average perturbed magnetic energy defined as

$$W = \frac{(B_z^2 + (B_x - B_0)^2)}{B_0^2}$$

(9)

varying as a function of time for $B_0 = 1.0$ and $M_A = 0.3$. It is seen that when $T > T_s$, $W$ is saturated. The maximum value of $W$, denoted by $S$ in this paper, is usually called the saturation level of the instability. Hereafter we define $T_s$ as the saturation time of the system, which is also determined by $M_A$. Figure 4a shows how $\Delta T_s = (T_s)_{TM} - T_s$ varies with $M_A$, where $(T_s)_{TM}$ represents the saturation time for the spontaneous tearing mode instability. We see that if $M_A < 0.4$, $\Delta T_s$ is close to 0. When $0.4 < M_A < 1$, $\Delta T_s$ apparently increases. As $M_A > 1$, $\Delta T_s$ gradually tends to an asymptotic value. Figure 4b shows how $S$ varies with $M_A$ for $B_0 = 1.0$. It can be seen that there also exists a transition for $S(M_A)$ in the range $0.4 < M_A < 1$. If $M_A < 0.4$, $S$ is close to its minimum value $S_{min}$ at $M_A = 0$, while for $M_A > 1$, $S$ is much larger than $S_{min}$.

3. Simulation results indicate that for $M_A > 0.2$, the difference between the occurrence time for the magnetic island and the fluid vortex reflects, to some extent, the relative importance of the $B_0$ field shear over the flow shear in determining the instability development. It is found that if $M_A < 0.8$, magnetic islands appear earlier than vortices, while for $M_A > 0.8$, vortices are formed earlier than magnetic islands. Figures 5, 6, and 7 present the occurrence time for the large-scale magnetic island and vortex for $M_A = 1.0$, 0.7, and 0.8, respectively.

4. Time evolution of the instability and patterns of magnetic islands and vortices developed for distinct ranges of $M_A$ are more easily found to be intrinsically different from each other. If $M_A < 0.4$, plasmas in the shear region are driven to the centerline ($z = 0$) and then move to the flank edges, leading to the appearance of two pairs of
Fig. 3. Time evolution of $W$, the average perturbed magnetic energy, for the case of $B_0 = 1.0$ and $M_A = 2.0$.

Fig. 4. (a) $\Delta T_s = (T_s)_{ru} - T_s$ varying as a function of $M_A$, where $T_s$ denotes the saturation time, $(T_s)_{ru}$ refers to $T_s$ for the spontaneous tearing mode instability. The time unit in the figure equals $12.8 t_0$. (b) The saturation level $S$ varying as a function of $M_A$.

Fig. 5. The occurrence time of the magnetic island and fluid vortex for $B_0 = 1.0$ and $M_A = 1.0$.

vortices, with the scale length in the $z$ direction smaller than that in the $x$ direction. Two $x$ points occur at $z = 0$ where field lines with opposite directions merge. A magnetic island is thus formed with the O point in the center of the simulation square. Figure 8a plots the AQSS

Fig. 6. The occurrence time of the magnetic island and fluid vortex for $B_0 = 1.0$ and $M_A = 0.7$. 
Fig. 7. The occurrence time of the magnetic island and fluid vortex for $B_0 = 1.0$ and $M_A = 0.8$.

The system finally achieves for $M_A = 0.3$, which looks quite similar to that of the spontaneous tearing mode presented in Figure 8b. As $0.4 \leq M_A \leq 1$, the vortex and magnetic island develop independently at first. The vortex is of the K-H type with its center being $L/2$ apart from that of the magnetic island. However, as time goes on, a new vortex concentric with the island appears, which gradually grows and finally dominates the flow system. Figure 9 plots the field lines and flow patterns at $T = 1330$ as $M_A = 0.8$, showing the AQSS for this medium range of $M_A$. Furthermore, when $M_A > 1$, as we can see in Figure 10, which plots the stream lines and magnetic field lines for $M_A = 2.0$ at $T = 234, 251, 320, \text{ and } 338$, a large-scale fluid vortex occurs as soon as the K-H instability has developed. The vortical motion in this case is strong enough to twist significantly the magnetic field lines and to drive the field lines with opposite directions on each side of the vortex center close to each other. Magnetic reconnection then takes place with two $x$ points occurring near the left and right edges of the vortex. Two magnetic islands are thus formed, one being concentric with the vortex and the other being limited to the center region of the simulation square. The former grows and expands as reconnection of field lines continues while the latter becomes smaller and smaller and finally disappears. After saturation, the system gradually approaches an AQSS, which is composed of a large-scale fluid vortex together with a concentric magnetic island. This process is called the vortex-induced magnetic reconnection (VIMR), proposed by [Liu and Hu] [1988] and [Hu et al.] [1988].

Fig. 8. (a) The Asymptotic quasi-static state (AQSS) for the case of $B_0 = 1.0$ and $M_A = 0.3$. (b) The AQSS of the spontaneous tearing mode ($M_A = 0$) for the case of $B_0 = 1.0$.

Fig. 9. The asymptotic quasi-static state for the case of $B_0 = 1.0$ and $M_A = 0.8$. 
4. DISCUSSION

1. When a sheared magnetic field and a sheared flow both exist at the magnetospheric boundary, the development of the instability at the boundary depends strongly upon the Alfvén Mach number. In the case $B_0 = 1.0$, $R_0 = 2.0$, and $R_1 = 0.1$ we find that the instability properties for distinct ranges of $M_A$ are intrinsically different from each other. If $M_A < 0.4$, the spontaneous tearing mode is dominated. For $0.4 \leq M_A \leq 1$ the tearing mode and K-H instability are "comparable" to each other. The coupling of these two processes leads to a modified type of tearing mode instability, the AQSS of which is composed of a magnetic island with a concentric fluid vortex. For $M_A > 1$, the K-H instability governs the evolution of the system. The vortical motion generated by the instability drives lines of force with opposite directions to meet each other, causing merging and reconnection of the field lines. The size, shape, and position of the magnetic island are all related to those of the fluid vortex. Time development in this case is much faster than that of the spontaneous tearing mode. The reconnection rate is much stronger as well. Apparently, the instability appearing in this situation is essentially different from the conventional tearing mode; the larger $M_A$ becomes, the more the K-H instability controls the system.

2. In our simulation a frame of reference is used in which $V_{1,2} = \pm V_0 \hat{e}_x$, where 1 and 2 represent the magnetosheath and magnetosphere, respectively. The plasmas are assumed to be incompressible, for simplicity [Fu and Lee, 1985; Liu and Hu, 1988; La Belle-Hamer et al., 1988]. The initial conditions of equations (7) and (8) are taken to model the sheared flow and the sheared component of the magnetic field at the dayside magnetopause. Weak and even moderate asymmetries in $B_y$, $V_y$, and $N_j (j = 1, 2)$ do not lead to results qualitatively different from those for symmetric condition. As the IMF has a southward component, $B_y$ and $B_2$ are not exactly antiparallel in general. However, in our two-dimensional model, the presence of $B_z$ and $V_z$ does not influence the time development of $B$ and $V$, as indicated by equations (1)–(6), and $B_0$
(see equation (8)) and $v_A$ measure only the sheared component of the initial magnetic field and Alfvén velocity at the magnetospheric boundary. Therefore when the two-dimensional approximation is roughly acceptable, i.e., the scale length of perturbations in the $y$ direction is much larger than that on the $(x, z)$ plane, we can apply our model to a wide region of the dayside magnetopause (not only near the meridian plane), provided the magnetosheath flow $v_s$ exceeds the threshold.

3. For typical magnetopause conditions of flux transfer events (FTEs) detected by ISEE 1 and 2, the plasma density is $\sim 40$ cm$^{-3}$ and the magnetosheath southward magnetic fields are $B \sim 30$ nT [Paschmann et al., 1982; Coroniti, 1985]. Taking $B_0 = 30$ nT and $n_0 = 40-20$ cm$^{-3}$, we obtain $v_A = 100-141$ km/s. This means that as long as the magnetosheath flow passes over the magnetosphere at $v_s > 200-282$ km/s, the VITM will take place. Furthermore, we estimate that it takes about 2 min to form the QASS for $B_0 = 1.0$ and $M_A = 2.0$. Therefore our simulation has confirmed the concept of vortex-induced magnetic reconnection at the magnetopause [Liu and Hu, 1988; Hu et al., 1988] and shows that the VITM can indeed present a possible mechanism for the formation of FTEs at the dayside magnetopause away from the stagnation point.

4. An important conclusion obtained in our simulation lies in the fact that the AQSS of the VITM is characterized by a large-scale magnetic island together with a concentric fluid vortex. A two-dimensional magnetic island corresponds to a magnetic vortical flux tube in a three-dimensional situation in the case of $B_0 \neq 0$. Thus if a FTE is formed through the VITM at the dayside magnetopause, its flux tube must be a current tube and a vortex tube as well. We have investigated the properties of the AQSS for VITM in detail and will present the results in a separate paper.

5. In obtaining the simulation results presented above, $R_m^{-1}$ and $R^{-1}$ are taken as 2.0 and 0.1, respectively. Corresponding dissipation coefficients can be examined as follows. The anomalous resistivity $\eta = R_m^{-1} v_0^2 v_A^{-1} \sim 8.2 \times 10^3 \Omega m$, which is an order less than the value required for fast reconnection at the dayside magnetopause [Coroniti, 1985]. Meanwhile, the cross-field anomalous diffusivity $D_L = \mu = R_m^{-1} v_0 v_A^{-1} \sim 3.3 \times 10^3 \text{ m}^2/\text{s}$, which is much lower than the value needed for the viscous interaction model of solar wind–magnetosphere coupling [Haerendel and Paschmann, 1982; Pu et al., 1986]. We have also investigated the cases of lower $R_m^{-1}$ and $R^{-1}$. The details of how the linear growth rate of the coupled instability varies with $M_A$ for $R_m^{-1} < 2.0$ differ from those for $R_m^{-1} = 2.0$ plotted in Figure 2a, and the magnitudes of $\gamma_{ITM}$ for the K-H modified type of tearing mode are not always greater than 1. However, the critical value of $M_A$ for generating the VITM is still found to be $\sim 1$, and the structures of magnetic islands and fluid vortices of the VITM for $R_m^{-1} < 2.0$ are also quite similar to those for $R_m^{-1} = 2.0$. Besides, the linear and nonlinear instability properties of the VITM for $R_m^{-1} < 2.0$ are not significantly different from those for the case of $R_m^{-1} = 2.0$. Figure 11 plots the variations of both the linear growth rate and the saturation time with different $R_m^{-1}$ when $B_0 = 1.0$ and $M_A = 2.0$. We see that as $R_m^{-1}$ reduces to $\sim 0.1$, $\gamma_{ITM}$ and $T_s$ are only a little higher and lower than the corresponding values for $R_m^{-1} = 2.0$. Figure 12 shows how $|B_{zM}|$ varies with $R_m^{-1}$ under the same conditions. It is seen that $|B_{zM}|$ for $R_m^{-1} \sim 0.1$ is only about 2 times larger than that for $R_m^{-1} = 2.0$. Calculations also show that the basic properties of the simulation results for $R^{-1} < 0.1$ are not substantially different from those for $R^{-1} = 0.1$ either.

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