MHD WAVE BREAKING IN THE OUTER PLASMASPHERE

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Abstract. Empirical models of the average magnetospheric magnetic field, plasma density, and temperature distributions are used to construct a model of the distribution of MHD wave mode speeds within the magnetosphere. Although the MHD wave speeds in general have a small dependence on wavelength of the magnetic field intensity or the plasma properties, considerable structure and variability is found which will lead to interesting "optical" effects on the propagation of low frequency waves. A persistent feature of the derived optical structure, which is qualitatively insensitive to known variability of the field or plasma, is a pronounced minimum of the wave speeds in the outer plasmasphere, i.e., a magnetospheric "shoal." This feature does not map along magnetic field lines, but is confined to the equatorial region, leading to a positive radial gradient of wave speeds near synchronous orbit. The breaking of earthward propagating disturbances in this region may play an essential role in the formation of the substorm injection boundary and in the creation of equatorially trapped warm ion distributions.

Introduction

The propagation of magnetohydrodynamic (MHD) waves within space plasmas has long been recognized as the means by which the energy and momentum associated with transient phenomena are transported (Southwood and Hughes, 1983, and references therein). Within a system like the terrestrial magnetosphere, the plasma and magnetic field are highly nonuniform, and this leads in general to a very inhomogeneous medium for propagation of such waves. Burton et al. (1970) reported observations of magnetic field and plasma densities for four passes of OGO 5 through the plasmapause region, calculating Alfvén speed profiles. We report here an attempt to construct a three-dimensional model of the typical magnetospheric distribution of Alfvén and fast mode wave speeds, based on the extensive statistical data bases now available. Our goal is to identify persistent or characteristic features of these distributions which have potentially important effects on the dissipation of such waves or their access to various regions of space. This approach implies an emphasis on propagation at wavelengths which are smaller than the magnetospheric system, in contrast to the emphasis on long wavelengths and standing modes found in much of the literature on magnetic pulsations.

MHD Wave Optics of the Magnetosphere

Magnetic Field

We have used the Mead and Fairfield (1975) empirical magnetic field model, which provides four different states of magnetotail stretching: superquiet, quiet, disturbed, and superdisturbed. Reference is made to the original publication for field line maps, intensity distributions, and other documentation concerning this model. Though we have used all four activity levels, here we only show results for the quiet model. Other magnetic field models can easily be substituted, but this model was considered to be adequate within the region of interest, taken to be that region within a radial distance of 12 Re geocentric.

Plasma Parameters

Obtaining a credible model of the plasma mass density and effective temperature (required for the ion acoustic speed) is a more difficult matter. The plasma electron density can be measured fairly reliably by noting the frequency of the upper hybrid resonance in passively sensed plasma waves (Persoon et al., 1983), or by an active plasma wave sounding technique (Higel and Wu, 1984). However, measurement of the ion mass composition and temperature, and electron temperature, is dependent at present upon accurate measurements of the low-energy "core" plasma ions and electrons. These measurements are very difficult to make at the low densities typical at high altitudes due to photoelectron emission by spacecraft surfaces and the consequent positive floating potential relative to the plasma. At present, good information on these parameters exists only at densities greater than approximately 100 cm⁻³, i.e., in the plasmapause and plasmasheet regions. Even with this restriction, statistical information on electron temperature is not routinely available.

In order to proceed, a plasma distribution model was constructed using as much empirical information as possible, with extrapolation of the plasmasheath held to a minimum, as described below. The resulting density and temperature models for quiet conditions are shown in Figure 1 as equatorial plane and midnight meridian cross sections.

The plasma electron density is tied to the observations from GEOS 2 at geosynchronous orbit reported by Higel and Wu (1984). These observations have been reduced to mean states for three levels of activity: quiet, average, and disturbed. Results shown here correspond to the average conditions. The radial distribution of auroral plasma density uses the observations of Berchem and Etcheto (1981) from ISEE, indicating a power law dependence on radius close to Earth. We have taken the power law in the form \( R^{(3.5 + 0.5 \sin(2\pi MLT/24))} \) so as to reflect the closer approach to diffusive equilibrium which prevails in the dusk bulge region. This \( R \) dependence is modified by an exponential dropoff beyond a variable \( R \) value. The scale length of the exponential increases with the \( R \) at which the exponential drop begins. The exponential drop continues until an asymptotic number density is reached which is representative of near-Earth plasma sheet values at midnight, i.e., 1.0 cm⁻³ (Strangeway and Kaye, 1986), and is somewhat higher at noon with a smooth variation over intervening local times. This asymptotic plasma sheet density falls slowly with radius (as \( R^{-\lambda} \)). The position at which the exponential drop begins is located for a particular local time so as to produce the correct number density at geosynchronous orbit as specified by the GEOS 2 data set. A crude indication of the magnetopause position was introduced by invoking a typical magnetosheath density outside the boundary indicated in Figure 1, panels A and C.

The plasma composition is taken from Horwitz et al. (1986), i.e., approximately 75% H⁺, 20% He⁺, and 5% O⁺. In the present model, evidence for heavy ion enhancements relative to these numbers in the plasmasheet region (Roberts et al., unpublished manuscript) and in the near-Earth plasma sheet (Strangeway and Kaye, 1986) has not been factored in. The net result is a mean ionic mass of 2 amu. It is anticipated that more sophistication in this parameter will be incorporated in future versions of this model, when statistical data bases can provide the proper justification. Such adjustments will tend to reduce the wave speeds derived for these regions.

The "effective temperature" plays two roles in this study. It is used to define the sound speed contribution to the MHD fast mode speed. As well, it could be used to control the plasma scale height along magnetic field lines, along which good thermal conduction is

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assumed to maintain isothermal conditions. In practice, the combination of low mean ionic mass and centrifugal reduction of gravity in the nearly corotating frame of the low-energy plasma in the plasmasphere leads to very small gradients of plasma density along magnetic field lines (Gallagher et al., 1987). This assertion is of course suspect in the topside ionosphere at the feet of the field lines, where a large density gradient exists. However, this region is outside the region of present interest (above 0.5 \( R_E \) altitude) and will not be dealt with here.

Since electron temperatures representing the full energy density of the plasma are not readily available throughout the magnetosphere, we assign “effective” temperature equal to the ion temperature with guidance from the work of Comfort (1986), Gallagher et al. (1987), and Garrett and DeForest (1979). The temperature rises gradually with radius from a base of 2500 K deep in the plasmasphere, then rises inversely with the exponential density drop described above until an asymptotic value is reached which is representative of plasmasheet temperatures, i.e., \( 1.16 \times 10^4 \) K. No attempt has been made to enforce pressure equilibrium with the \( J \times B \) force implied by the magnetic field model; however, a realistic pressure gradient results from this uniform temperature and the assumed weak radial dependence of the density outside the plasmasphere.

Number densities away from the equatorial plane are derived as follows: At locations earthward of a field line taken to mark the outer boundary of the plasma sheet, including the entire plasmasphere, the equatorially assigned temperature and density values are simply mapped along magnetic field lines to low altitude where the topside ionosphere is not modeled. This procedure leads to very low densities at low altitudes in the region conjugate to the plasma sheet, producing an “auroral plasma cavity” consistent with the observations of Persoon et al. (1987). In the polar region outside the plasma sheet boundary field line, the results of Persoon et al. (1983) are used to provide densities according to a simple expression with a power law in radius. Clearly the detailed structure of the F layer ionosphere is not addressed by these techniques. However, the agreement with auroral plasma cavity values observed by Persoon et al. (1987) confirms the plausibility of this procedure. As a further development of this work, we plan to incorporate either fully three-dimensional empirical data or a physical model of the field-aligned plasma distribution.

Wave Velocities

The results for Alfvén waves [computed as \( V_A = B/(4 \pi NM)^{1/2} \) with \( M \) the mean ionic mass] and magnetoacoustic or fast mode waves [computed as \( V_{FM} = (V_A^2 + V_S^2)^{1/2} \), where \( V_S = (kT_{eff}/M)^{1/2} \)] are shown in Figure 2. Note that we have used the ion acoustic speed.
corresponding to an assumption of isothermal conditions or unity ratio of specific heats. Both equatorial and midnight meridional sections are shown. The only region of significant difference between Alfvén and fast mode speeds is found in the plasma sheet, where the effective temperature is very high and the magnetic field is depressed by tail stretching. Examining first the equatorial section we find a deep minimum in both wave speeds which extends around all local times, but is deepest between 5 and 6 RE at early evening local times. In this region the fast mode speed drops as low as 150 km s\(^{-1}\), whereas typical values throughout the rest of the equatorial magnetosphere are near 1000 km s\(^{-1}\).

Examining the corresponding meridional distribution, we find that the deep minimum does not extend very far out of the equatorial plane along geomagnetic field lines, being confined to a region within about ± 20° of the equator. In fact the Alfvén speed rises very rapidly passing out of the plasmasphere at higher latitudes, and the “horns” of the plasma sheet form regions of extremely high Alfvén speed, much higher than values found in the central plasma sheet.

**Discussion**

The purpose of this study has been to compute the distribution of MHD wave speeds throughout the near-Earth magnetosphere. To do this we have defined a crude empirical model of the plasma distribution within the magnetosphere which is nevertheless plausible as a mean state. We wish to draw particular attention to the deep minimum of the wave propagation speeds found roughly between 4-6 RE in the outer plasmasphere or inner plasma sheet rather independent of local time. For the particular model plotted, the deepest minimum is found in the evening near 5-6 RE, though the existence of this feature is relatively independent of the activity level chosen for the plasma and field models. The basic velocity minimum feature is simply a consequence of the approximate L\(^{-3}\) magnetic field variation and the L\(^{-3}\) to L\(^{-4}\) variation of the plasma density in the inner plasmasphere, coupled with the rapid drop in plasma density, drop in magnetic field intensity, and rise in plasma temperature in the plasma sheet.

Acoustic waves in a gas obey a wave equation which is essentially identical to that describing shallow water waves. Therefore, we can make use of our experience and intuition with water waves in anticipating the behavior of magnetosonic waves. Of course, the index of refraction can be very anisotropic, and the fast mode should be coupled to the Alfvén mode, so the situation is more complex. However, we expect accelerated steepening and breaking of waves when they propagate into regions of decreasing wave phase speed, as would be the case for compressional waves propagating earthward through the synchronous orbit region at any local time. By analogy with water wave behavior, we propose to refer to the wave speed minimum region as the magnetospheric “shoal.” The term “beach” might be appropriate, as well, and has been used to describe the propagation of plasma waves into regions of slower phase speed. In
the present case the region of low speed is embedded within regions of higher speed, so that the analogy with a "shoal" seems preferable.

Breaking of an acoustic wave corresponds to the transient formation of a shock, with the implication that bulk flow energy flux is converted into thermal energy of the gas as it is processed by the shock. Since the acoustic wave would be occurring in a collisionless plasma, all the phenomena associated with collisionless astrophysical shocks are expected to occur, including the creation of highly unstable ion distributions and very energetic particles. The wave breaking and shock formation would be expected just as the wave enters the region of increasing core plasma density and decreasing hot plasma, i.e., just as it begins to compress and displace earthward the plasma sheet inner boundary. Though this situation is clearly very complex, the physics of it is a logical extension of recent advances in our understanding of steady collisionless shocks. The net effect on the plasma would include some combination of boundary displacement and local heating along the displaced boundary. Since the outer plasmasphere is known to consist of ionospheric plasma, we should expect the plasma heated by wave breaking to be primarily ionospheric.

Evidence for these effects in substorm events is summarized by Moore (1986) and references cited therein. It seems plausible as well that the concentration of wave heating of the low-energy plasma observed near the equator in the outer plasmasphere (Olsen et al., 1987) is associated with the breaking of wave disturbances propagating from the outer magnetosphere.

Conclusions

We have constructed an empirically-based model of the plasma and magnetic field parameters for the purpose of specifying the distribution of MHD wave phase speeds throughout the magnetosphere, exclusive of the magnetotail. In this paper we have pointed out a major feature of the resulting wave speed distribution, a torus-shaped minimum or magnetospheric "shoal." This feature is a characteristic of the statistically averaged configuration of the magnetosphere, though it may be more or less pronounced for plausible instantaneous states of the magnetosphere. The feature is not entirely symmetric and leads to equatorial contours of equal wave speed which spiral outward in the evening sector similar to the substorm injection boundary inferred from spacecraft data (Mcllwain, 1974). We assert that the existence of such a feature implies that magnetospheric waves will be subject to well-known wave phenomena such as refraction, reflection, and breaking, depending upon their wavelengths in relation to the scale of the feature. We suggest that these wave phenomena are of general importance to magnetospheric dynamics, specifically in the formation of substorm-injected plasma populations which produce bright diffuse aurora and the ring current.

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