Io's Interaction With the Plasma Torus

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A new model for the interaction of Io with the dense corotating plasma of the Io torus is described which involves pickup and Alfvén waves carrying a field-aligned current. Pickup refers to the process whereby ions freshly created near Io are accelerated by the corotational electric field. It is shown that the pickup current is connected to a field-aligned current carried by an Alfvén wave. The combination of currents allows for a self-consistent determination of the electric field in Io's vicinity. Using a simplified solution for the electric field, we calculate the total pickup rate, field-aligned current, enhancement of plasma density in Io's vicinity, thermal energy injected into the torus, UV radiation power, and inertial drag on the magnetosphere. All observations made by Voyager are compatible with an electron impact ionization mechanism and a neutral SO$_2$ Io atmosphere with a density of 10$^9$ cm$^{-3}$.

1. INTRODUCTION

Since the discovery by Bigg [1964] of the control that Io, Jupiter's innermost Galilean moon, has over the Jovian decametric radiation (DAM) many new observations have emphasized the important role that Io plays in the dynamics of the Jovian magnetosphere-ionosphere system. Io is, indeed, one of the most fascinating astronomical objects. Not only does its surface reveal ongoing geological activity (such as volcanism) on an unprecedented scale, but Io also seems to be the major source of neutrals (like sodium) and the heavy ion plasma in the Jovian magnetosphere. This plasma determines the magnetic field topology, which in turn controls the dynamics of energetic charged particles, some of which escape the Jovian magnetosphere and subsequently propagate as far as the earth. It may be said (however, with tongue in cheek slightly) that Io's influence extends out to the earth and that in a subtle way the earth's environment is subject to processes going on at Io. This is quite a surprising and exciting thought. Yet although the fact of a strong interaction of Io with the Jovian magnetosphere has been known for a long time, no unified theory exists which ties together all observations. In fact, several theories of the interaction contradict at least some aspects of the data. For example, the DAM control has often been explained as being due to field-aligned currents driven by the large emf across Io due to Io's motion relative to the magnetosphere plasma (see, for example, Smith [1976] and references therein). However, these models require a large Pederson conductivity for the bulk of Io and its ionosphere. This requires the density of Io's atmosphere to be quite large ($N \approx 10^{11}$ cm$^{-3}$). On the other hand, the extended neutral sodium cloud around Io has been related to sputtering of sodium from Io's surface by energetic charged particles penetrating Io's atmosphere [Matson et al., 1974]. A dense atmosphere would absorb the energetic particles before they hit the surface and would also prevent the sodium atoms from escaping from Io. The sputtering mechanism requires a thin atmosphere ($N \approx 10^9$ cm$^{-3}$). Furthermore, if Io were an ideal conductor and carried the magnetic field with it, charged particles would drift around Io, and only extremely energetic particles ($E > 10$ MeV) would impact the surface of Io [Schulz and Eviatar, 1977; Thomesen, 1979]. Yet it is observed that Io absorbs charged particles, in particular, low-energy particles [see Thomesen, 1979, and references therein], indicating that Io's flux tube (IFT) is not frozen into Io but moves in relation to the moon. On the other hand, Ness et al. [1979] observed magnetic field perturbations near IFT which indicate that large field-aligned currents flow along IFT ($I_1 = 5 \times 10^6$ A). A further problem is posed by the existence of a dense ionosphere at Io [Kliore et al., 1974, 1975] with a maximum electron density of $5 \times 10^9$ cm$^{-3}$.

It seems that a new model for the Io interaction is required. The ingredients for the model proposed in this paper have been separately discussed and published by various authors [e.g., Drelt et al., 1965; Goertz, 1973; Cloutier et al., 1978; Smith and Goertz, 1978; Ip and Axford, 1979; Neubauer, 1980; G. L. Siscoe and A. Eviatar, private communication, 1979]. The model is based on two basic features: pickup and Alfvén waves carrying a field-aligned current. The combination of these two mechanisms yields a very powerful framework into which many observations can be fit without any ad hoc assumptions.

Pickup refers to the process whereby ions freshly created in the vicinity of Io are accelerated by the corotational electric field until their guiding centers acquire the corotation speed [Cloutier and Daniell, 1973]. Ions could be created by photoionization, in which case the ionization rate depends only on the neutral atmospheric density. They could also result from electron impact ionization by the ambient plasma electrons moving into the atmosphere of Io. In this case the ionization rate depends on the electron density itself. We will assume that throughout the pickup region an exospheric model is valid in the sense that the collision frequency is much less than the ion gyrofrequency. For a thin atmosphere ($N < 10^{10}$ cm$^{-3}$) this is well satisfied. We will describe the pickup process in more detail in section 2, where we will show that the pickup mechanism produces a current perpendicular to the local field line which is related to the ionization rate and the local electric field.

If an exospheric model is not valid, Pederson and Hall currents must be added. To keep the discussion general, we include them at first but show later that they are unimportant. For a steady state situation these currents have to be closed by field-aligned currents. The magnitude of the field-aligned current determines the local electric field in a self-consistent manner. If, for example, no parallel current were allowed to flow, the pickup current would eventually reduce the corotational electric field by polarization of the plasma. We will show in section 2 that the magnitude of the field-aligned cur-
current is determined by the Alfvén wave conductance (or, better, impedance) and the local electric field gradients. We then derive a differential equation for the electric field which allows, at least in principle, for a completely self-consistent determination of the electric field in Io's vicinity.

In section 3 we will use a simplified solution to calculate the total pickup rate, field-aligned current, enhancement of plasma density in Io's vicinity, and thermal energy injected into the magnetosphere by Io. In section 4 we will use the result of recent Voyager observations to determine the one unknown quantity in the model, namely, the ionization rate. We will see that observations indicate that ionization by electron impact is quite plausible. In section 5 we will show that the model is also consistent with the motion of plasma in the outer Jovian magnetosphere. In section 6 we will discuss the propagation of Alfvén waves through the Io plasma torus.

2. ELECTRIC FIELD IN THE VICINITY OF IO

We assume with Ip and Axford [1979], G. L. Siscoe and A. Eviatar (private communication, 1979), and Kumar [1979] that the atmosphere of Io consists mainly of SO$_2$. This is quite reasonable in light of the strong volcanic activity on Io. When SO$_2$ molecules are dissociated and ionized, the resulting S$^+$ and O$^+$ ions move under the influence of the local electric and magnetic field. The newly created charged particles perform a cycloid motion which can be represented by a superposition of a guiding center motion at a velocity (in cgs units)

$$V_s = -\frac{E \times B}{B^2} \frac{c}{c}$$

and a gyromotion with a gyrospeed

$$V_\perp = |V_s|$$

The guiding center of the ions is displaced by one gyroradius in the direction of the electric field and that of the electrons opposite to the electric field. If a pair of ion-electron pairs are created per unit time, a current flows in the direction of the electric field which is given by

$$J_\perp = \sum_i n_i \gamma_i R_p E / E = \sum_i m_i c^2 \gamma_i E / B^2$$

This pickup current exerts a force on the plasma which is given by

$$F = J_\perp / c \times B = \sum_i n_i m_i E / B^2 c$$

and balances exactly the change of plasma momentum. In addition to this pickup current there is the normal conductive current

$$J_\perp = \sigma_\perp E + \sigma_\parallel E \times B / B$$

where $\sigma_\perp$ and $\sigma_\parallel$ are the Pederson and Hall conductivities, respectively. We will see later that for the model atmosphere to be used later an exospheric approximation is quite valid and this current can be neglected. We do, however, wish to formulate the problem as generally as possible now and introduce simplifications later.

It should be noted that the electric field $E$ in (3) and (4) is the local electric field, which must be determined self-consistently. In order to do that we need to know all currents which flow in the vicinity of Io. In addition to the above currents there is a polarization current due to the distortion of the electric field near Io. The polarization current is given by

$$J_p = \frac{1}{4\pi} \frac{c^2}{V_\perp^2} \frac{dE}{dt}$$

where

$$V_\perp^2 = \sum_i \frac{B^2}{4\pi n_i m_i}$$

The only additional current of interest is the field-aligned current. It may be thought that the parallel current has to close through the Jovian ionosphere and is thus determined by Jupiter's ionospheric conductivity. However, on a field line upstream from Io the pickup, conduction, and polarization current are zero, and no parallel current flows. Only when the field line penetrates Io's atmosphere will a current begin to flow. This current is carried by an Alfvén wave propagating from Io toward Jupiter's ionosphere [e.g., Goertz, 1973; Drell et al., 1965; Goertz and Boswell, 1979; Neubauer, 1980]. Only after the Alfvén wave has reached the ionosphere and, after reflection, returns to Io can the local currents be influenced by the resistive characteristics of the Jovian ionosphere. Before the discovery of the dense plasma torus at Io's orbit [Bridge et al., 1979; Warwick et al., 1979] the Alfvén return trip travel time was believed to be small. Goertz and Defft [1973] estimated the round trip time as 60 s, and even if the local electric field near Io is the full corotational electric field, the Alfvén waves would return to Io and establish a steady current pattern in which the ionospheric resistance determines the total current. However, in the torus with an average ion mass ($m$) the Alfvén velocity is only $670/(m)^{1/2}$ km/s ($n_e = 4 \cdot 10^3$), and it takes $470(m)^{1/2}$ s to propagate 1 $R_J$. Thus if Io is in the torus, a round trip Alfvén travel time could be as large as 1000 s when we assume that the torus consists mainly of singly charged sulfur and oxygen ions. Only if the electric field near Io is reduced to one thirteenth of its full corotational value $E_0$ ($E_0 = (\nabla \times B)/c = 0.1 V/m$) will the Alfvén wave return to Io. If $E > E_0/13$, the return Alfvén wave will not return to Io, and the resistive characteristics of the Jovian ionosphere will not be able to influence the currents in Io's vicinity (see Neubauer [1980] for a more detailed discussion of this point). In fact, it takes more than one round trip time to establish a steady state current pattern in which the Jovian conductivity controls $j_\parallel$. Thus we believe that $j_\parallel$ is related to Jupiter's ionospheric conductance $\sigma_\parallel$, but is solely controlled by the current characteristic of the Alfvén wave.

The parallel current density carried by the Alfvén waves is given by Goertz and Boswell [1979] and Neubauer [1980]:

$$J = \pm \frac{c^2}{4\pi V_\perp} \nabla \cdot E \frac{B}{B}$$

where the plus/minus sign refers to the two waves propagating away from Io (northward and southward). Thus from the continuity equation of charge we get

$$4\pi \frac{dE}{dt}(0) = \frac{d}{dt} (\nabla \cdot E) = \nabla \cdot \left[ \frac{c^2}{C^2} \sum \frac{\gamma_i}{\langle n_i \rangle} E + 4\pi \frac{V_\perp^2}{C^2} \sigma_\perp E \\
+ 4\pi \frac{V_\perp^2}{C^2} \sigma_\parallel E \times B + \frac{dE}{dt} \pm V_\perp \nabla \cdot E \frac{B}{B} \right]$$
Fig. 1. Variation of the 'dielectric constant' as a function of radius. The dashed curve is the form used in this paper, namely, $\epsilon = 1 + 3H/V_A\tau_p$.

For a uniform magnetic field the divergence of the Hall current is zero. We write $\tau_p^{-1} = n/(\sigma_\infty V_A/c^2)$ and obtain

$$\nabla \cdot \left( \frac{c^2}{V_A^2} \left[ \frac{1}{\tau_p} E_\perp + \frac{dE_\perp}{dt} \left( 1 - \frac{V_A^2}{c^2} \right) \right] \right) = 0$$

We see that the time scale for changing the electric field is very small because $c^2/V_A^2 \gg 1$ and we can neglect the partial time derivative of the electric field. We now assume that the Alfvén velocity does not vary appreciably in the near vicinity of Io. We can then write (for $V_A^2/c^2 \ll 1$)

$$\nabla \cdot (E_\perp/\tau_p) + \frac{d}{dt} \nabla \cdot E_\perp \pm V_A \frac{d}{dz} \nabla \cdot E_\parallel = 0$$

The last two terms can be combined as the gradient of $\nabla \cdot E_\perp$ along the Alfvén characteristic

$$S = \frac{V_\perp \pm V_A B/B}{V_A}$$

[see Neubauer, 1980], and we can finally write

$$\nabla \cdot \left( \frac{E_\perp}{\tau_p} + \frac{V_A}{\tau_p} \frac{d}{ds} E_\perp \right) = 0$$

(7)

It should be noted that Neubauer allows for a plasma flow component along the unperturbed magnetic field $B_0$ (the angle $\theta$ in his equations can be nonzero). We do not take such a flow into consideration.

Integrating (7) along the Alfvén characteristic yields a simple Laplace equation

$$\nabla \cdot (E_\perp \epsilon) = 0$$

(8)

where the formal 'dielectric' constant $\epsilon$ is given by

$$\epsilon = 1 + \int \frac{ds}{V_A \tau_p}$$

(9)

The boundary condition for (8) is that far away from Io the electric field $E_\perp$ approaches the corotation electric field $E_\parallel$:

$$E_\parallel = B \times V_\infty/c$$

(10)

If the physical process of ionization is known and $n_\infty$ can be calculated, the electric field near Io is in principle completely determined by (8)–(9).
plified version of the problem, we can immediately write the solution for the electrostatic potential as

$$\Phi = -E_r \sin \phi \quad 0 < r < R_{io} + H/2$$

$$\Phi = -E_{or} \sin \phi + \left( E_o - E_i \right) \frac{(R_{io} + H/2)^2}{r} \sin \phi$$  \hspace{1cm} (11)

$$R_{io} + H/2 < r$$

$$E_i = E_o \frac{2}{2 + 3H/V_o \tau_p}$$

The equipotentials are shown in Figure 2. We can at once draw one interesting conclusion from this figure. Except for extremely high energy particles, charged particles drift along equipotentials [Schulz and Eviatar, 1977]. Since $E_i \neq 0$, a non-zero fraction of particles will drift into Io. Thus Io acts as an absorber of charged particles. However, the absorption cross section of Io is not its surface area but is reduced by a factor $2/(2 + 3H/V_o \tau_p)$. Such a reduction is required by the observed energetic particle absorption effects of Io [Thomsen, 1979]. We also see that the back of Io is not shielded from the drifting plasma.

We are now in a position to calculate a number of observable quantities from the model. Along a drift path $p$ the electron density will increase as (the maximum electron density occurs on the downstream edge of Io)

$$\frac{d}{dp} \int n_e \cdot \nu \cdot da = \sum_i \int n_i \cdot da$$  \hspace{1cm} (12)

where

$$\frac{dp}{dt} = \nu = \frac{E \times B}{B^2} \cdot c$$

$$da = dt \times d1_\perp$$

Since the drift occurs along equipotentials, we have

$$E_L \cdot d1_\perp = E_{or} d1_0$$  \hspace{1cm} (13)

and we get

$$\frac{d_n}{dp} = \frac{\sum_i n_i E_0}{V_o E_{or}} \frac{(n)}{2 + 3H/V_o \tau_p}$$  \hspace{1cm} (14)

Integrating (6) over $l_\perp$ for the parallel line current density we get

$$\frac{d1_{||}}{dp} = \frac{c^2}{4\pi} \frac{E_o}{V_{ao}} \left[ 1 - \frac{E_i}{E_o} \right] \left( \frac{n_o}{n_0} \right)^{1/2}$$  \hspace{1cm} (15)

which for $V_{ao} = 200 \text{ km/s}$ (corresponding to a density of 2000 cm$^{-3}$ and an average ion mass of 20 amu) becomes

$$\frac{d1_{||}}{dp} = \frac{0.8 \times 10^8}{R_{io}} \frac{n_o}{n_0} \left[ 1 - \frac{2}{2 + 3H/V_o \tau_p} \right]$$  \hspace{1cm} (16)

We can calculate the total number of ions added to the torus as

$$N_i = \int n_i d1_{||} \cdot dz = n_i 4\pi R_{io}^2 H = 4\pi R_{io}^2 \frac{(n)}{\tau_p}$$  \hspace{1cm} (17)

As the newly created ions drift around Io, their gyroenergy remains constant, although the guiding center speed changes in response to the changing electric field along the drift path. The constancy of the gyroenergy follows from the conservation of the first adiabatic invariant (there are small changes of $V_i$ associated with magnetic and electric field gradients in the vicinity of Io, but they can be shown to be small). Thus the total gyroenergy (or heat) added to the torus by the ion species $i$ is

$$P_{int} = \int n_i m_i E_o^2 c^2 \cdot d1_{||} \cdot dz = n_i E_{or} \frac{E_i}{E_o} \frac{4\pi R_{io}^2 H}{2}$$  \hspace{1cm} (18)

where $E_{or}$ is the corotational energy of the species. Thus the average energy of an ion is

$$E_i = E_{or} \left( \frac{E_o}{E_i} \right)^2 = E_{or} \left( \frac{2}{2 + 3H/V_o \tau_p} \right)^2$$  \hspace{1cm} (19)

The total power dissipated in Io is equal to

$$P = \int J_{||} \cdot E \cdot d1_{||} \cdot dz = 2 \sum_i P_i$$

Half of this power is carried away by the thermal energy of the ions, and half is carried away by their guiding center energy.

4. COMPARISON WITH OBSERVATIONS

For electron impact ionization by the ambient magnetospheric electrons which have a temperature $T_e \sim 8 \text{ eV}$ [Broadfoot et al., 1979] the ionization rate is

$$n_i = 0.3 \times 10^{-8} n_o N_o \text{ cm$^{-3}$ s$^{-1}$}$$  \hspace{1cm} (20)

where $N_o$ is the number of neutrals. Using the usual formulae for Pederson conductivity and scale height, we find

$$n_i = \frac{1}{\tau_p} \left[ 1 + 3\sigma_o \times 10^6 (gh)^{1/2} \right]$$  \hspace{1cm} (21)

where $\sigma_o$ is the ion-neutral collision cross section ($5 \times 10^{-15}$ cm$^2$) and $g$ is the gravitational acceleration at the surface of Io. The second term in brackets is due to the Pederson current and is small.

To obtain some estimates for the observable quantities, we adopt the following nominal values:

Io atmosphere

$$n_o = 10^9 \text{ cm$^{-3}$} \quad H = 0.8 \times 10^7 \text{ cm/s}$$

This scale height and the value of $N_o$ represents the exospheric model of Kumar [1979] with an exospheric SO$_2$ temperature of 1100$^\circ$K and a density of $2 \times 10^9$ cm$^{-3}$ at the base of the exosphere. These numbers are meant only as guides and must not be taken too literally. Not enough is known about the atmosphere of Io at present to give more reliable values.

Torus plasma

$$n_o = 2 \times 10^3 \text{ cm$^{-3}$} \quad m_i = 20 \text{ amu} \quad V_{ao} = 2 \times 10^6 \text{ cm/s}$$

Again these values are meant only as guides but are representative of the measurements [Bridge et al., 1979]. These values yield $\tau_p = 1$. From this we obtain $\Delta n_o = 8 \times 10^9$ cm$^{-3}$ for the additional electron density by integrating (14) over an average path length of 3$H$. This is only slightly larger than the maximum values observed by Pioneer 10 [Kliore et al., 1974]. How-
ever, there are reasons to believe that the torus density was less in 1973 than in 1979 [Broadfoot et al., 1979], and this disagreement should not be taken too seriously, especially when one considers the crudeness of the model.

However, the model would not be compatible with a maximum electron density in the Io ionosphere that is much less than this value for the conditions that existed in 1979.

Using this value in (16), we obtain $I_9 = 3.7 \times 10^6$ A for the total field-aligned current flowing along the surface of the Io flux tube. This is in good agreement with the value obtained by Ness et al. [1979], who estimate $I_9$ to be between $10^6$ and $5 \times 10^6$ A.

For the ion production rate we get $N_{+e} = 10^{28}$ s$^{-1}$ and $N_{0+} = 2 \times 10^{28}$ s$^{-1}$, where we have used $N_0 = 2N_e = 10^9$ cm$^{-3}$.

Finally, we obtain $(\epsilon_{e+}) = 66$ eV and $(\epsilon_{e+}) = 33$ eV for the average energy of the ions injected into the torus. To compare these values with the observed ion temperatures and the observed UV radiation, we have to calculate the energy budget in the torus. The ions will collisionally heat the electrons, which in turn will lose their energy through UV radiation. However, the ions do not stay in the torus for ever but diffuse out of the torus. The mean lifetime of the ions can be obtained by dividing the total number of ions in the torus by the ion production rate:

$$\tau_i = n_i V_{torus}/N_i = 10^8$$

Then for the energy budget of the torus ions and electrons we have

$$n_i' dT_i/dt = P_i - n_i' \frac{T_i}{\tau_i} - \left[ \nu_{ei}(T_i - T_e) + \sum \nu_{ii}(T_i - T_i) \right] n_i'$$

where $n_i'$ is the number of the $i$th ion in the entire volume of the torus. The second term expresses energy loss associated with particle loss, or

$$\tau_i \frac{dT_i}{dt} = (\epsilon_i) - T_i - \left[ \nu_{ei}(T_i - T_e) + \nu_{ii}(T_i - T_i) \right] \tau_i$$

where $\nu_{ei}$ is the electron-ion collision frequency and $\nu_{ii}$ is an ion-ion collision frequency. Using as a characteristic electron temperature $T_e = 8$ eV, we have

$$\nu_{ei}/\nu_{ei}^{-1} = 5 \times 10^6 \quad \nu_{ii}/\nu_{ii}^{-1} = 2.5 \times 10^9$$

For these values we get for the equilibrium ion temperatures $T_{+e} = 51$ eV and $T_{0+} = 34$ eV. These are quite compatible with the observed temperatures [Bridge et al., 1979]. The total power radiated by the electron is $P = 6 \times 10^{11}$ W. This is factor of 3 smaller than the estimate of Broadfoot et al. [1979]. It should be realized that a density of $n_0 = 2 \times 10^9$ cm$^{-3}$ is an underestimate for the torus density, which may well be a factor of 2 higher. Power is also carried away by the Alfvén waves. Some of this power is dissipated in the torus (see discussion below) and heats the torus plasma and hence increases the total UV radiation power (see section 6 and Figure 4). We feel that at this stage it is unnecessary to search for values that would yield improved agreement with the observations. We believe that the model is quite successful in explaining a large number of different observations.

Consequences for the Outer Magnetosphere

The ion injection rate calculated above is considerably larger than the injection rate from the Jovian ionosphere [Goertz, 1973, 1976; Siscoe, 1978]. Io is apparently the major source of plasma in the outer Jovian magnetosphere. The radial transport of Io-injected ions proceeds via diffusion [Siscoe, 1978]. We have estimated the mean lifetime of ions in the torus as $10^8$ s. In that time the ions diffuse a distance of $1 R_j$ (the torus radius). Thus we require a diffusion coefficient of roughly $D_{\text{diff}} = 10^{-8} R_j^2/s$. This is somewhat larger than the maximum value of the diffusion coefficient estimated by Thomsen et al. [1977] from the energetic particle observations near Io (see also Thomsen [1979]). An enhanced diffusion coefficient at the outer edge of the torus is not surprising, because the outer edge of the torus is unstable against the flute instability, which should result in enhanced diffusive transport across magnetic field lines.

The inner edge of the torus is not flute unstable, and the diffusion coefficient should be smaller there. In order to transport the same number of particles inward as outward the density gradient should be steeper at the inner edge of the torus than at the outside. This is indeed observed [Bridge et al., 1979; Warwick et al., 1979]. It should be noted here that the injection rate calculated above will maintain the plasma torus at the observed extent and density provided that the diffusion coefficient is not larger than the one estimated above.

The plasma transported away from Io will cause an inertial drag on the Jovian ionosphere [Hill, 1979]. Hill calculates the rate of slowing down of the Jovian ionosphere by equating the rate of increase of angular momentum by the outward mass transport to the torque that can be provided by the Jovian ionosphere. He derives a differential equation which relates the change of rotation rate to the mass injection rate and the height-integrated conductivity of the Jovian ionosphere. We have used his equation and integrated it, using as a boundary
condition that the plasma at \( L = 6 \) is rigidly rotating with the planet. (Hill has used this condition at \( L = 1 \).) We also have inward mass transport for \( L < 6 \). Figure 3 shows the results of that calculation superimposed on the observation of McNutt et al. [1979]. It seems that

\[
L_o = (\sigma \Sigma_j R_i^2 B_j^2 / M)^{1/6} = 20
\]
gives a good agreement with the data. Using the mass injection rate \( \dot{M} \) estimated above, we find that \( \Sigma_j = 0.06 \) mho. This is somewhat less than the value suggested by Hill [1979]. For \( \Sigma_j = 0.1 \) mho we would get \( L_o = 23 \), which would also be quite compatible with the data. Again, we feel that the crudeness of the model does not warrant a more detailed search of the parameter space.

6. THE COUPLING OF ALFVEN WAVES TO AND THEIR TRANSMISSION THROUGH THE TORUS

So far we have only analyzed the energy contained in the newly created ions. There is an additional power contained in the Alfvén wave which propagates away from Io. This is not simply related to the (mechanical) energy of the ions but must be calculated separately. The plasma density in the torus varies quite rapidly along the Io flux tube, and we must calculate the Alfvén power which can be transmitted through the torus. From the usual MHD equations one obtains the wave equation for the perturbation of the plasma flow velocity:

\[
\frac{\partial^2 \varphi}{\partial z^2} = V_A(z)^2 \frac{\partial^2 \varphi}{\partial \tau^2}
\]

where

\[
\varphi = \varphi_0 - \frac{E \times B}{B^2} c
\]

and

\[
V_A^2(z) = B_0^2 / 4 \pi \mu m(z)
\]

Deift and Goertz [1973] have discussed the conditions for the validity of this equation and show that near the torus (called lopshere by them) it is applicable.

We calculate only the power contained in the Alfvén wave propagating into a decreasing density. We model the density variation along the IFT as

\[
\frac{1}{V_A^2} = \frac{1}{V_{A0}^2} + \left[ \frac{1}{V_{A0}^2} - \frac{1}{V_{A0}^2} \right] e^{-2z/\zeta_0}
\]

where \( V_{A0} \) is the Alfvén velocity at Io and \( V_{A0} \) is an Alfvén velocity outside the torus. Fourier transforming (24) and introducing the new variable yield

\[
x = \left[ \frac{1}{V_{A0}^2} - \frac{1}{V_{A0}^2} \right] \frac{\omega \zeta_0 e^{-2z/\zeta_0}}{V_{A0}^2}
\]

We can rewrite (24) as a Bessel differential equation for the Fourier transform of \( \varphi \):

\[
x^2 \frac{\partial^2 \varphi(\omega)}{\partial x^2} + x \frac{\partial \varphi(\omega)}{\partial x} + (\omega^2 + \nu^2) \varphi(\omega) = 0
\]

\[

\nu^2 = -\frac{\omega^2 \zeta_0}{V_{A0}^2} = -c_2^2 \frac{x_0^2}{V_{A0}^2}
\]

The solution is

\[
\varphi(x, t) = \int_0^\infty g(x = 0, \omega) \frac{J_{\nu}(x_0 \omega) - (1 / V_{A0}^2)^{1/2} e^{-\omega \zeta_0}}{J_{\nu}(x_0 \omega) - (1 / V_{A0}^2)^{1/2}} \omega \omega d\omega
\]

with

\[
g(x = 0, \omega) = \int_0^\infty \varphi(x = 0, \omega) e^{-\omega \zeta_0} d\omega
\]

To obtain the power flux \( F \) we have to calculate the magnetic field perturbation from

\[
\frac{\partial b}{\partial z} = B_0 \frac{\partial b}{\partial z}
\]

\[
F = -\frac{1}{8\pi} \text{Re} \left( B_0 b^* \right)
\]

For an arbitrary \( \varphi(x = 0, \omega) \) the flux cannot be represented in closed form. For a sinusoidal perturbation \( \varphi(x = 0, \omega) = \varphi_0 \sin \omega t \) we can write

\[
F(x) = \frac{B_0^2}{8\pi V_{A1}} \frac{\omega_0^2}{V_{A0}} e^{-2x/\zeta_0} \text{Re} \left[ \frac{J_{\nu}(c_1 \varphi_0 e^{-2x/\zeta_0})}{J_{\nu}(c_1 \varphi_0)} \left( \frac{J_{\nu}(c_1 \varphi_0 e^{-2x/\zeta_0})}{i J_{\nu}(c_1 \varphi_0)} \right)^* \right]
\]

with \( c_1 = \omega / V_{A1} \). From the discussion above we realize that

\[
\varphi_0 = \varphi_0 \left( 1 - E / E_0 \right) \frac{\epsilon - 1}{\epsilon + 1} = 5.6 \times 10^7 \frac{\epsilon - 1}{\epsilon + 1} \text{ cm/s}
\]

\[
\omega = \frac{\pi \mu_0 E_i}{2R_{Io} E_0} \left( \frac{\pi \mu_0 / R_{Io}}{\epsilon + 1} \right) \approx \frac{10^{-1}}{1 + \epsilon}
\]

The scale height for the variation of the Alfvén velocity in the torus is [Siscoe and Chen, 1977]

\[
\zeta_0 = 2R_0 = 1.4 \times 10^{10} \text{ cm}
\]

The density along Io's orbit varies from a maximum value of a little more than \( 3 \times 10^7 \text{ cm}^{-3} \) when Io is in the magnetic equatorial plane to a minimum value of \( 10^3 \text{ cm}^{-3} \) when Io's magnetic latitude is a maximum (10°). The Alfvén velocity at Io's position varies correspondingly from 163 to 283 km/s (where we have assumed a constant average atomic mass of 20 amu). The Alfvén velocity outside the torus is much larger. If we take an asymptotic density of 10 cm\(^{-3}\), we get \( V_{A0} = 2830 \text{ km/s/} (m) = 20 \text{ amu or } V_{A0} = 1.27 \times 10^4 \text{ km/s/} (m) = 1 \text{ amu asymptotically.}

We take as a representative value \( V_{A0} = 10^4 \text{ km/s.} \) Since \( V_{A0} \gg V_{A0} \text{ in any case, the exact value is not of great importance. From (9), (20), and (21) we find an approximate relation for } \epsilon:

\[
\epsilon = 1 + 7.4 \times 10^{-2} n^{1/2}
\]

and we see that everywhere along Io's orbit, \( c_1 \zeta_0 \gg 1 \). Thus near Io a WKB approximation would be valid. However, as \( z \) increases, the Alfvén speed becomes large, and the WKB approximation should becomes less applicable. In particular, we do not expect equipartition between kinetic and magnetic energy everywhere along the IFT [Deift and Goertz, 1973]. It is this fact which necessitates the rather formidable treatment of the Alfvén wave propagation through the torus. Since we are
interested mainly in the power at \( z \to \infty \), we can use a small argument expansion for the Bessel function in the numerator of (32):

\[
J_{n} \left( c \sqrt{2z_0} \right) = \frac{1}{\Gamma(n+1)} - \cos \left( c \left( z_0 - 1/4 \pi - 1/2 \nu \pi \right) \right)
\]

Using standard forms, we can finally write for the flux at \( z \to \infty \)

\[
F(z \to \infty) = \frac{B_0^2 \nu_0^2}{4\pi V_A} \sinh \left( \nu c_2 z_0 \right) \frac{\sinh \left( \nu c_2 z_0 \right)}{\nu^2} \cos^2 \left( c_2 z_0 - \pi/4 \right)
\] (36)

and using similar expansions, we obtain for the flux at \( z = 0 \)

\[
F(z = 0) = F(z \to \infty) + \frac{1}{c_1} \frac{B_0^2 \nu_0^2}{4\pi V_A} \]

\( (37) \)

The second term in (37) represents the flux lost in the torus due to internal reflections. This flux is trapped in the torus and will eventually be dissipated by heating the torus plasma. To obtain the total power in the Alfven waves, we multiply the flux with the surface area of Io. This is also valid for \( F(z \to \infty) \) because we have assumed that the magnetic field along IFI in the torus does not change. Figure 4 shows the variation of \( P \), the power transmitted through the torus, and \( P_d \), the power trapped and eventually dissipated in the torus, as a function of plasma density at Io's orbit. We also show the thermal power carried away by the newly created ions. The power lost in reflection by the Alfven waves as they propagate through the torus is small in comparison to the power transmitted through the torus. However, the Alfven waves may be Landau damped by thermal electrons [Hasegawa and Chen, 1976]. An analysis of this effect would be beyond the scope of this paper, and we will address the question of Landau damping in a future paper. Contrary to the assertion of Neubauer [1980] we believe that internal reflections in the torus are negligible. However, if some of the Alfven power is reflected at the Jovian ionosphere and trapped in the magnetosphere, it may lead to a significant additional heating. It should be noted that when the torus is dense \( (n_0 > 10^7 \text{ cm}^{-3}) \), most of the energy is carried by the Alfven waves. It is quite tempting to associate this energy with the energy necessary to generate the Io-controlled decametric radiation [Goertz, 1973] which radiates a nominal power of \( 10^8 \text{ W} \) [Smith, 1976; Warwick, 1970]. However, it is not clear that the total Alfven wave power is injected into the Jovian ionosphere where the decametric radiation is generated. Because the Alfven velocity changes rapidly near the Jovian ionosphere, we expect reflection to occur. In a future paper we will explore the consequences of our model for the Io-controlled decametric radiation.

7. Summary

We have described a new model for the interaction of Io with the Jovian magnetosphere. Io is assumed to have an exospheric \( \text{SO}_2 \) atmosphere which is maintained by volcanic eruptions. The average neutral atmosphere is \( 10^9 \text{ cm}^{-3} \) and has a scale height of 80 km. These are representative values which yield good agreement with various observations but are by no means sacrosanct. The corotating torus plasma produces \( \text{S}^+ \) and \( \text{O}^+ \) ions by electron impact ionization. Newly created ions are displaced in the direction of the local electric field. The displacement represents a current—the pickup current. This current is connected to a field-aligned current carried by an Alfven wave. The Alfven wave also carries a perpendicular current which is related to the change of the torus plasma velocity. We derive a differential equation for the electric field in Io's vicinity and solve it for certain idealized situations. We show that the electric field in Io's atmosphere is smaller than the corotational electric field and hence the torus plasma slows down as it approaches Io. It is perhaps instructive to consider the energy flow in this model. The torus plasma loses energy by slowing down as it approaches Io and by ionizing the atmosphere of Io. Part of this energy goes into the Alfven waves, part into stretching the field lines (distortion of the magnetic field), and part into acceleration of the newly created ions. In Io's wake the magnetic field lines straighten out, and the total plasma is accelerated. Behind Io the plasma is denser (new ions are added) and hotter. Furthermore, energy is propagated toward Jupiter in the form of an Alfven wave. Thus there is a net energy drain on Io of the order of \( 10^{12} \text{ W} \). It is clear that the interaction will slow Io down (as seen from a corotation frame of reference) or reduce Io's angular momentum about Jupiter. However, the reduction of the orbital period of Io due to a power drain of \( 10^{12} \text{ W} \) is 2 orders of magnitude below present detectability [Smith, 1976].

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