Drag and Propulsion of Large Satellites in the Ionosphere: An Alfvén Propulsion Engine in Space

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Abstract. There is a motionally induced charge separation in a conductor moving across magnetic field lines. This charge may be conducted away, resulting in a dc current flow through the conductor if it moves through a plasma. The generation of Alfvén waves is a mechanism particularly effective for circulating the charge for very large conductors moving in or above the earth's ionosphere. This mechanism is studied in this paper and when applied to the analysis of the orbit of the Echo satellite is found to give rise to a significant damping of the motion as mechanical energy is converted to that of Alfvén radiation. The calculated drag is comparable to that observed for the orbit of Echo 1 and attributed in earlier studies entirely to the mechanical drag of considerable nonionized atmospheric density. Perturbations in electron density associated with this current flow may in appropriate circumstances be detectable even thousands of kilometers away from such a high altitude satellite. The drag can be changed to a propulsion mechanism when a source of electrical power is available on the satellite. Up to fifty per cent of the expended power is available for pushing a space vehicle across an ambient magnetic field.

1. INTRODUCTION

A conductor moving across a magnetic field \( B_0 \) in a vacuum will have an induced charge separation sufficient to cancel the electric field \( E = (v \times B_0)/c \) seen by a co-moving observer. When the surrounding medium is a plasma, there exist possible mechanisms for charge to be conducted away, resulting in a dc current flowing through the conductor. In this paper we consider the circulation of charge by means of the generation of Alfvén waves, a mechanism which is particularly effective for very large conductors moving in or above the earth's ionosphere. When applied to a study of the Echo satellite it gives rise to a significant damping of the orbit as mechanical energy is converted to that of Alfvén radiation. The calculated drag is equal to that observed for the orbit of Echo 1 and attributed in earlier studies entirely to the mechanical drag of considerable nonionized atmospheric density. In an appropriately designed satellite the drag force can be altered by variations of an internal resistance, or the associated dc current flow can be tapped as a battery. With a source of electrical power the drag force can be converted to a propulsion mechanism: the satellite pushes on the earth's magnetic field without any emission of propellant.

We exploit the qualitative analogy between a collisionless plasma in a magnetic field and a series of transmission lines parallel to the magnetic field. The moving conductor with its motionally induced charge separation is, in a sense, in successive contact with different transmission lines as it moves. It induces an impulse (Alfvén wave) traveling along the magnetic field which carries a charge separation and essentially completes the circuit in which the moving conductor is a dc battery.

Initially we shall idealize the conductor so that it not only has no internal resistance but also no work function to prevent the outward flow of electrons in response to an appropriately directed electric field normal to its surface. The proposed mechanism is appropriately modified for real conductors for which the work function is significant.

Alfvén waves, magnetohydrodynamic disturbances of frequency much less than the ion cyclotron frequency \( \Omega_i = eB_0/M_i c \), propagate one-dimensionally along the direction \( B_0 \). A small disturbance persists with constant amplitude or until damped by collisions among elec-
Fig. 1. Conductor moving with velocity $v_c$ in the $x$ direction perpendicular to magnetic field lines $B_0$ along the $z$ direction leading to a charge separation and motional electric field $E$ in the $y$ direction.

We have not found the solution offered here in earlier studies of related problems; these have been two-dimensional, or involved only motion parallel to $B_0$ or posed boundary conditions that the plasma was not disturbed at large distances from the conductor, none of which are valid for our solution. Moreover, we find in the rest frame of the moving conductor a steady dc current rather than a static situation.

2. IDEAL CONDUCTOR IN A LOSSLESS PLASMA

For the idealized conductor (see section 1) moving through a collisionless plasma in a direction perpendicular to the field lines (Figure 1), the Alfvén disturbance extends out in wings making an angle $\alpha$ with the field lines such that

$$\tan \alpha = \frac{v_c}{v_a}$$

where $v_c$ = speed of the conductor, and $v_a = \text{Alfvén speed}$. The motional electric field

$$E = \frac{(v_c \times B_0)}{c}$$

in the conductor is cancelled by a charge separation as illustrated in Figure 1. In the collisionless plasma, with infinite conductivity along the $B_0$ or $z$ direction, but with zero conductivity perpendicular to $B_0$, current wings as shown in Figure 2 are created along with a charge separation as required to maintain a constant $E$ field along the wing equal to that at the conductor. Anomalous radar echoes obtained from ionospheric disturbances associated with Echo 1 may also be related to these predicted Alfvén ‘wings,’ although further studies are desirable to confirm their presence. We comment more on this in section 5.

Let us first analyze the fields produced by the motion of an ideal conductor through a plasma with an impressed magnetic field $B_0$ in order to establish the appropriate regime of parameters and the nature of the Alfvén disturbance. We make the following approximations:

1. The conductor moving through the plasma is ‘ideal’—i.e., it has zero internal resistance and work function; it maintains the charge separation required to produce the field

$$E = \frac{-(v_c \times B_0)}{c}$$

(1)

In the opposite case of a conductor with internal resistance moving through a medium of higher conductivity, the charges would be ‘bled’ from

Fig. 2. Alfvén wings generated by an ideal conductor in a collisionless plasma.
the conductor by the medium at a more rapid rate than could be maintained, and the field \( \mathbf{E} \) would decrease. Conditions for the applicability of the 'ideal' conductor limit are discussed in the following.

2. A linearized solution is obtained; i.e., we retain terms linear in the Alfvén fields only. This requires in particular that the magnetic field \( \mathbf{h} \) associated with the Alfvén disturbance be small alongside the constant applied magnetic field \( \mathbf{B}_0 \).

Since we consider only Alfvén disturbances, we shall consider only low frequency waves such that

\[ \omega < \Omega_i = eB_0/M_i c \]

\[ \approx 4000 \text{ cps} \left( B_0/0.4 \text{ gauss} \right) \left( M_p/M_i \right) \]  

where \( \Omega_i \), the ionic gyrofrequency, is given numerically in terms of the ion to proton mass ratio \( M_i/M_p \) and of the field strength \( B_0 \) divided by a mean value at the earth's surface. This means \( \omega \lesssim 2000 \text{ cps} \) for 1000 to 2000-km altitude orbits in the protonosphere and \( \omega \lesssim 200 \text{ cps} \) for orbits of altitudes as low as 160 km in the \( F \) layer. For this reason we are limited to a study of large conductors, since the typical frequency radiated by a moving source of dimension \( L \) along its velocity and speed \( v_c \) is

\[ \omega \sim v_c/L \]  

and for \( v_c \sim 7 \times 10^4 \text{ cm/sec} \) for a satellite in earth orbit and \( v < 1000 \text{ cps} \), \( L > 10 \text{ meters} \).

Electron and ion collision frequencies, \( \nu_e \) and \( \nu_i \) in the ionosphere, are shown as functions of altitude in the curves of Figure 3 taken from Johnson [1961] and must be included in a detailed study, since they may be comparable with \( \omega \). They are, however, negligible relative to the gyrofrequencies, i.e.

\[ \nu_e/\Omega_i < 10^{-2} \quad \nu_i/\Omega_i < 10^{-4} \]  

and the ratios (4) are neglected throughout. In this regime the dielectric behavior of the magnetized plasma is given by the tensor \( \epsilon_{ij} \) defined by

\[ D_i = \epsilon_{ij} E_j = E_i + 4\pi j_i/\omega \]  

for the \( \omega \) frequency component:

\[ \epsilon_{ij} = \epsilon_{xx} \approx 1 + \frac{4\pi \sigma_0}{\omega} \left( 1 + i\omega \tau_e \right) \]  

where \( \sigma_0 = n_e e^2 \tau_e/m \) is the specific electrical conductivity of the plasma, \( \tau_e = 1/\nu_e \), \( \tau_i = 1/\nu_i \), and \( n_e = n \) is the electron (ion) number density in the neutral plasma. \( \sigma_0 \) is the conductivity parallel to the magnetic field or in the absence of a magnetic field and as a function of altitude varies roughly as shown in Figure 4 taken from Johnson [1961].

In the limit of a lossless, i.e., collision-free, plasma,

\[ \epsilon_0 \approx 1 - \omega_p^2/\omega^2 \]  

where \( \omega_p = \left( 4\pi n e^2/m \right)^{1/2} \) is the plasma frequency, and
Specific electrical conductivity of the ionosphere (zero field conductivity) versus altitude; taken from Johnson [1961].

\[ \varepsilon_\perp \approx 1 + \frac{4\pi n M_i c^2}{B_0^2} \]

\[ = 1 + \frac{4\pi \rho_i c^2}{B_0^2} = 1 + \frac{c^2}{v_a^2} \]  

(8)

defines the Alfvén velocity in terms of the magnetic field strength \( B_0 \) and the ionic mass density \( \rho_i = n M_i \). Numerically the plasma frequency varies from \( \omega_e \approx 6 \times 10^6 \) cps at altitudes of a few hundred km corresponding to electron densities of up to \( 10^6/cm^3 \), to \( \omega_e \approx 4 \times 10^4 \) cps at altitudes of 1600 km and electron densities of \( \sim 5 \times 10^3/cm^3 \). The Alfvén velocities rise from values of \( v_a \approx 2 \times 10^7 \) cm/sec at 300-km altitude to \( v_a \approx 10^6 \) cm/sec at 1600-km altitude. The value at 1600 km is not accurately known, owing to uncertainties in the ionic composition at the high altitudes. We return to this point later. The ratio of parallel to transverse components of \( \varepsilon \) is very large for the parameters of interest to us,

\[ |\varepsilon_\parallel/\varepsilon_\perp| > 10^3 \]  

(9)

Essentially the electrons are wrapped around the magnetic field lines, free to move only along the \( B_0 \) direction, as are the ions. This large ratio reflects propagation only in the direction parallel to \( B_0 \).

We want now to solve the Maxwell equations

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

\[ \nabla \times \mathbf{B} = \frac{\mathbf{D}}{\epsilon_0} + 4\pi \mathbf{j}_e/c \]

\[ \nabla \cdot \mathbf{D} = 4\pi \rho_e \]

\[ \nabla \cdot \mathbf{B} = 0 \]  

(10)

with \( \mathbf{D} \) and \( \mathbf{E} \) related by (5) and (6) and with \( \mathbf{j}_e \) and \( \rho_e \) representing the current and charge source provided by the moving conductor. The current in the moving conductor of Figure 1 flows mainly in the \( y \) direction to maintain the charge separation. We work in a linearized approximation in the field strengths \( \mathbf{E} \) and \( \mathbf{h} = \mathbf{B} - \mathbf{B}_0 \); \( \mathbf{h} = \mathbf{B} \).

Fourier transforming (10) we have

\[ \mathbf{k} \times \mathbf{E}(k_\perp, k_\parallel, \omega) = \frac{\omega \mathbf{h}}{c} (k_\perp, k_\parallel, \omega) \]  

(11a)

\[ \mathbf{k} \times \mathbf{h} = -\frac{\omega \mathbf{D}}{c} - \frac{4\pi i}{c} j_e(k_\perp, k_\parallel) \delta(\omega - k \cdot \mathbf{v}_a) \]  

(11b)

\[ \varepsilon_\parallel k_\perp E_\parallel + \varepsilon_\perp k_\parallel \cdot \mathbf{E}_\parallel = 4\pi \rho_e \]  

(11c)

Taking the curl of (11a), inserting (11b) and (11c), and defining \( \mathbf{E}_\parallel = \mathbf{E}_\parallel^r + \mathbf{E}_\parallel^s \), where
\[ k_\perp \cdot \mathbf{E}_\perp = k_\perp E_\perp \] and \[ k_\perp \times \mathbf{E}_\perp = k_\perp \times \mathbf{E}_\perp \] gives

\[
\left( \frac{\omega^2}{c^2} - k_\perp^2 - k_\parallel^2 \right) E\perp = \frac{4\pi i}{c^2} \omega \delta(\omega - k \cdot \mathbf{v}) (\mathbf{j}_e(k_\perp, k_\parallel) \cdot \mathbf{k}_\perp) / k_\perp
\]

\[
= -\frac{4\pi i}{c^2} \omega \delta(\omega - k \cdot \mathbf{v}) \left| \mathbf{j}_e \times \mathbf{k}_\perp \right| / k_\perp
\]

\[
\left( \frac{\omega^2}{c^2} - k_\perp^2 - k_\parallel^2 \right) E\perp = 0
\]

(12a)

\[
\mathbf{h}_\perp = \frac{c(\mathbf{k} \times \mathbf{E})_\perp / (\mathbf{k} \cdot \mathbf{v}_e)}{c^2 - \omega^2}
\]

\[
\mathbf{h}_\parallel = \frac{c(\mathbf{k} \times \mathbf{E})_\parallel / (\mathbf{k} \cdot \mathbf{v}_e)}{c^2 - \omega^2}
\]

(12b)

(12c)

(12d)

General features of the solution can be identified in (12). For a plasma with low transverse conductivity which approaches the behavior of a lossless medium satisfying (9), (12a) reduces to a one-dimensional wave equation for a transverse electric field propagating along the \( \mathbf{B}_0 \) direction with velocity \( c/\omega \) according to (8) and with frequency

\[ \omega = k \cdot \mathbf{v}_e \]  

(13)

and longitudinal wavelength

\[ \lambda_1 = \frac{1}{k_\parallel} = \frac{v_\parallel}{c} = \frac{v_e}{k \cdot \mathbf{v}_e} \]  

(14)

This is the Alfvén wave. According to (13), for the dominant transverse wave numbers \( k_\perp \sim 1/L \), where \( L \) is the dimension of the current source along the direction of motion, the frequency is \( \omega \sim v_e/L \), as was claimed earlier in (3). There is, in addition, according to (12b) an isotropic disturbance which is a solution of the ordinary d’Alembertian wave equation, also with Alfvén velocity \( v_e \). For a steady source current generated by a moving conductor with \( v_e < v_a \), the solution is a localized disturbance falling off as \( 1/r^2 \) at large distances from the source; this is the case applying here. When, however, \( v_e > v_a \), a radiation pattern is created that is the analog of the Cerenkov radiation for a charge passing through a medium at speeds greater than the light velocity in the medium.

In this lossless plasma approximation, the equations for \( \mathbf{E} \) and \( \mathbf{h} \) read

\[
\left( \frac{1}{\omega^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} \right)(\nabla \perp \cdot \mathbf{E}_\perp)
\]

\[ = -\frac{4\pi}{c^2} \frac{\partial}{\partial t} \nabla \perp \cdot \mathbf{j}_e(x - v_e t, y, z) \]  

(15a)

\[
\left( \frac{1}{\omega^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} \right)(\nabla \perp \times \mathbf{E}_\perp)
\]

\[ = -\frac{4\pi}{c^2} \frac{\partial}{\partial t} \nabla \perp \times \mathbf{j}_e(x - v_e t, y, z) \]  

(15b)

with \( E_\parallel = 0 \). Equation 15b describes the net radiation of energy into a magnetoionic wave with velocity \( v_e \) only if \( v_e > v_a \). As we noted, this radiation is analogous to the Cerenkov radiation, or sonic boom. Equation 15a radiates energy out along the direction of \( \mathbf{B}_0 \) for all values of \( v_e \) if the idealized conductor moves across the magnetic field lines and plucks them like violin strings. This solution satisfies the equation

\[ \mathbf{B}_0 \cdot (\nabla \perp \times \mathbf{E}_\perp) = 0 \]  

(16)

so that a two-dimensional slice perpendicular to \( \mathbf{B}_0 \) gives the electric field from a two-dimensional charge distribution \( (\nabla \perp \cdot \mathbf{E}_\perp) \). According to (15a), the Alfvén fields are functions only of the variables

\[ y \quad \text{and} \quad \left( x - v_e t \pm \frac{v_e}{v_a} |z| \right) \]

and propagate as described earlier in Figures 1 and 2. The wings move out from the conductor with \( v_a \) along \( \mathbf{B}_0 \) and form an angle \( \alpha \) such that

\[ \tan \alpha = v_e/v_a (\ll 1) \]

as illustrated there.

For the magnitudes of the fields, we know from the first of (10) that the tangential component of \( \mathbf{E} \) is continuous across the conductor surface and is fixed by the motional electric field in the conductor according to (1):

\[ \mathbf{E} = -(v_e \times \mathbf{B}_0)/c \]  

(17)
Since we are dealing with long wavelength modes of field excitation and by (14)

\[ \frac{1}{k} \gg \frac{1}{k_L} \sim L \]

the detailed three-dimensional geometric shape of the moving conductor is of no relevance. We can write an exact solution for an I-bar conductor described as in Figure 5 by two flat rectangular plates of dimensions L and N, separated by a distance M, and charged to maintain a potential difference in the y direction of \( V = \frac{v_c}{c} B_0 M \) (18)

The solution consists of growing parallel wings with a charge sufficient to keep the difference in potential between the wings always \( V \). The curless part of \( E_{\perp} \), which is the solution of (15a), has the functional form

\[ E_{\perp} = \left( x - \frac{v_c t}{v_a} \right) \left( \frac{v_c}{v_a} \right) |y|, y \]

and is the solution of the two-dimensional problem corresponding to Figure 5, i.e. two plates of width L and separation M making an angle \( \alpha = \tan^{-1} \left( \frac{v_c}{v_a} \right) \) with \( B_0 \). A constant potential difference \( V \) is maintained between the wings, so that \( E_{\parallel} = 0 \), and a constant current \( j_x \) flows through the I bar in the y direction. Only out along the horizontal arms of the I bar in the x direction does the current \( j_x \) decrease (i.e., \( \nabla \cdot j_{\perp} \neq 0 \)) as current flows into the wings. The vertical current in the source must be exactly sufficient to supply the charge needed for the two growing wings tips, i.e.

\[ I_{\perp} \approx \frac{E_{\perp} L^2}{2 \pi v_a} = \frac{v_c L}{2 \pi v_a} B_0 \]

We note that the field \( E_{\perp} \) that moves down the tube in the \( B_0 \) direction is not particularly different if the moving source is the I bar or a solid plate of the same dimensions. Only the fringing fields near the vertical edges are different, as is indicated in Figure 6. In particular the power radiated into the Alfvén-type waves is essentially the same in both cases; the effective radiating area need not be filled with conductor.

For the magnetic field, (15c) and (17) give

\[ h_\perp = \left( \frac{v_c}{v_a} \right) B_0 \]

between the plates plus correction terms near the edges from the fringing fields plus corrections vanishing as one moves out into the radiation zone far from the body due to local currents. A small ratio of the Alfvén field \( |h| \) to the earth’s field \( B_0 \) is a necessary criterion for the linear approximation we have made. For satellites in earth orbit, we have

\[ \frac{|h|}{B_0} = \frac{v_c}{v_a} \approx 4 \times 10^{-2} \text{ at 200-km altitude} \]

\[ \approx 10^{-3} \text{ at 1600-km altitude} \]

always very small next to unity.

The sufficiency condition for the validity of the neglect of higher-order terms depends on the interval over which we need to be sure of an accurate solution. For computations of the current flow through the moving satellite, its drag, and the power radiated into the Alfvén waves, the solution should be valid out to a distance of about 1 wavelength/2\( \pi \) \( \sim v_c/\omega \) away from the conductor. Over this interval \( h_\perp \) may cause a change in direction for the flow of charge away from the z axis by an angle \( \sim h_\perp /B_0 \) and a resultant transverse displacement of \( (h_\perp v_a)/(B_0 \omega) \). For this not to exceed the transverse dimension of the Alfvén wave, we need

\[ h_\perp v_a \gg B_0 \omega /k_\perp = R_\perp v_a \]
or

\[ h_l / B_0 \gg v_e / v_a \]  \hspace{1cm} (20)

Only for an ideal conductor with no internal resistance moving in a collisionless plasma in which the inertia of the electrons is negligible can the left-hand side of (20) gain even equality with the right-hand side.

For an irregularly shaped three-dimensional ideal conductor for which \( v_e / v_a \ll 1 \), only the projection onto the \( x-y \) plane is relevant: the wings and their charge distribution form a cylindrical tube inside of which there is a uniform electric field which, in the co-moving frame, exactly cancels \( v_e \times B_0 / c \). Thus in the moving frame the net \( E \) is zero everywhere in the conductor and on its surface, except along the line where the conductor touches the extended 'paper thin' cylindrical tube which carries the charge separation.

As far as the motion of the charged plasma is concerned, each two-dimensional slice (at constant \( z \)) of the Alfvén tube (wing) moves as if through an incompressible fluid. In the co-moving frame the electric field \( v_e \times B_0 / c \) together with that of an infinitesimally thin charge distribution looks like that in Figure 7. The drift velocity \( v \) of ions in the crossed electric and magnetic fields is

\[ v = \frac{eE \times B_0}{B_0^2} \rightarrow -v_e \]

far from conductor. As we have seen in (15), for \( v_e < v_a \), \( \nabla \times E = 0 \) far from the conductor and \( E_{tan} = 0 \) on the surface of the tube. Therefore \( \nabla \cdot v = 0 \) and \( v_n = 0 \) on the tube surface.

3. VOLTAGE, POWER, IMPEDANCE, AND CURRENT EXPRESSIONS

From the strength of the magnetic field (19) we readily compute the power radiated in the Alfvén disturbance to be

\[ P = \frac{1}{2} \frac{E \cdot E_0}{B_0^2} \]

Fig. 6. Fringing fields differing for \( I \) bar or solid plate conductors.

Fig. 7. Flow of plasma around Alfvén tube in co-moving frame.
\[ P = \frac{1}{4\pi} h^2 2(ML) v_a \]

\[ = \frac{B_{0}^2 v_{a}^2}{2\pi v_a} (ML) \]

where \(2MLv_a\) is the volume filled per second by an energy density \(h^2/4\pi\); the factor 2 takes into account the existence of wings extending in both directions along \(B_0\). Combining with (18) for the potential difference between the top and bottom wing, we compute the current flow:

\[ I = \frac{P}{V} = \left( \frac{cv_a}{v_e} \right) \left( \frac{B_0}{2\pi} \right) L \]

The effective impedance of the plasma for this current flow is then defined as

\[ Z = \frac{V}{I} = 2\pi \frac{v_e}{c} (M/L) \]

In terms of these familiar quantities of electrical circuit theory, the Alfvén wings can be interpreted as one-dimensional open-ended transmission lines of impedance \(Z\) across which a potential \(V\) is applied. In this ideal limit of a lossless medium, there is an infinite resistance between the upper and lower line (or Alfvén wing) and zero resistance along them. Corrections due to collisions leading to a finite transverse ionic conductivity are computed in the next section.

Finally we compute, using Gauss’s law, the electronic surface charge density in the wing as

\[ \Sigma_{e} = \frac{D}{4\pi} = \frac{1}{4\pi} \frac{c^2 v_a}{e} B_0 = \left( \frac{cv_a}{v_e} \right) B_0 \frac{4\pi}{e} \text{ esu/cm}^2 \]

The large electronic surface charge density arises from the enormous dielectric constants \(\varepsilon_\perp \sim 1 + c^2/v_a^2\) (with values between \(10^4\) and \(10^9\)). The total charge density given by \(\text{div} \cdot \mathbf{E}\) is smaller by \((v_e/c)^4\); i.e., almost all the electronic charge density is cancelled by the ions. This large polarizability of the heavy plasma ions will always result in an ionic displacement much smaller than the wing separation as long as the vertical dimension of the body, \(M\), satisfies

\[ v_e/M \ll \Omega_i \]

4. Real Plasma and Conductor Corrections

We turn now to the important practical factors modifying our discussion when applied to real conductors moving through a real plasma such as the ionosphere:

Effect of collisions in the plasma.
Effect of a finite work function limiting the electron current flow out through the surface of a conductor.
Effect of finite internal resistance in the conductor.

Finally we present some realistic drag calculations, as well as some novel speculations on how to take advantage of these magnetohydrodynamic ionospheric disturbances for orbit control (acceleration or deceleration and orientation) and for power generation or storage.

Effect of collisions in the plasma. From the left-hand side of (12a) we extract the dispersion relation for wave propagation in the medium

\[ k_{\perp}^2 = \varepsilon_\perp \frac{\omega^2}{c^2} - k_{\parallel}^2 \varepsilon_\parallel \]

In the lossless limit described in (7), (8), and (9), this reduced to

\[ k_1 = \omega/v_a \]

appropriate to a pure Alfvén wave. Retaining the damping terms in (6), we find the corrections to (26) are appreciable if \(1/\omega r\), or

\[ k_{\perp}^2 c^2/\omega^2 \varepsilon_1 = c^2/\varepsilon_\parallel v_e^2 \]

become comparable to unity. The damping of our Alfvén waves due to the first factor is always very small for the range of parameters of interest; as is seen from Figure 3, \(1/\omega r_i < 10^{-1}\) at about 160 km, and \(<10^{-2}\) above altitudes of 300–400 km, for frequencies in the hundreds of cycles per second range, corresponding to conductors with dimensions \(L \sim v_e/\omega \sim 50\) meters. Neglecting \(1/\omega r_i \ll 1\) and introducing (6) into (25), we include the effect of the term in (27):

\[ k_{\perp}^2 = \frac{\omega^2}{v_a} \left[ 1 + \frac{k_{\perp}^2 c^2}{\varepsilon_\parallel} \left( 1 - i/\omega r_i \right) \right] \]

In (28) the effects of electron inertia and electron collisions are both taken into account. The correction factor arising from electron inertia is small for large conductors in 160–500 km orbits, since \(\omega_p \sim 5 \times 10^6\) eps and

\[ \frac{k_{\perp}^2 c^2}{\omega_p^2} = \frac{(k_{\perp}^2 + k_{\parallel}^2)c^2}{\omega_p^2} = \frac{\varepsilon_\perp}{\varepsilon_\parallel} + \frac{k_{\parallel}^2 c^2}{\omega_p^2} < 1 \]
for $\omega < 10^3$ cps and for $1/k_z \sim M > 10$ meters.

This means that the wings cannot be considered to have zero thickness, but will spread to a thickness of perhaps ten meters. The imaginary (damping) correction from electron collisions is also small for these conditions when $1/k_z \geq 10$ meters according to Figure 3.

For higher altitude orbits above 1000 km, the damping term $i/\omega_r$ in (28) is $< 1$ in magnitude for $\omega \geq 10$ cps, corresponding to conductor dimensions no larger than $L \sim 1000$ meters (for the Echo 1 with $L \sim 30$ meters at an altitude of 1600 km, the correction is less than a few per cent). The real factor in (28), however, leads to a major change in the radiated Alfvén wave unless $1/k_z \geq 75$ meters; the simple Alfvén solution propagating in one dimension only applies only to the modes with $k_z \leq 1/75$ meters and with frequency $\omega \geq 10$ cps, and we restrict our attention to these. Thus the wing thickness for these parameters will, if geometrically possible, spread to more than 75 meters in order to carry the required currents, despite the electron inertia and the reduced density of electrons above 1000 km.

For an object such as Echo 1 with effective dimensions of $L \sim M \sim 30$ meters for the current source as defined in Figure 5, this means a reduction in the field strength, since only the modes with $k_z < 1/75$ meters $\sim L^{-1}/2.5$ can be described by the Alfvén solution discussed here. Since the power spectrum is flat in the transverse wave number $k_z = (k_1, k_2)$ out to $\sim 1/L$, the power radiated in the Alfvén disturbance must be reduced by a factor $\sim (30/75)^6 \sim 1/6$ from the value computed in (21).

We have presented these numbers in some detail in order to make clear the very approximate and qualitative nature of our numerical results and to show their sensitivity to the various atmospheric parameters, variable in time and imperfectly known, that have been used.

A useful analogy that may help provide a physical picture of this behavior is suggested in the low frequency limit such that $\omega_r \rightarrow 0$ and $\omega_r \rightarrow 0$. Then (28) can be rewritten as

$$k_z^2 = \frac{\omega^2}{v_s^2} - \frac{4\pi i\omega}{\sigma^i} - k_{z,0}^2 \frac{\sigma_{z,0}^i}{\sigma_0}$$

where $\sigma_0 = ne^2 r_e/m$ is the free field conductivity due to electrons in the plasma, as defined earlier, and $\sigma^i = ne^2 r_e/M_i(\Omega_i r_i)^2$ is the low frequency limit of the ionic conductivity perpendicular to the magnetic field direction when $\Omega_i r_i \gg 1$. The finite transverse ionic conductivity between the top and bottom layers of the Alfvén wings in Figures 1 and 2 allows the damping and termination of the disturbance. Viewing these wings as a simple transmission line, these terms are, respectively, the resistance $\propto (\sigma^i)^{-1}$ and the capacitance $\propto (k_z^2 \sigma_{z,0}/\sigma_0)^{-1}$ between the lines, i.e. the impedance $Z_z$ in Figure 8 in addition to the inductance $Z_z \propto v_s/c^2$ along the line through which the current flows.

Even if the moving conductor has dimensions such that inertial term of (27) is smaller than 1, the fact that it is nonzero means that in the dispersion law

$$k_z \sim \omega/v_s \left(\omega/k_z \right)^2 \frac{\omega^2}{2v_s^2}$$

the second term on the right-hand side plays a role at large $z$: the Alfvén pattern spreads in the $xy$ plane as it propagates in the $z$ direction. For subsequent calculations of currents and drag through the conductor, it is sufficient that the proposed solution be adequate out to $z \sim v_s/\omega \sim (v_s/v_e) L$. (In the transmission line analogy, the moving conductor remains in contact with a given transmission line for a time $L/v_e$ during which the signal has traveled a distance $v_e L/v_e$ in the $z$ direction. What happens after that does not affect the forces and currents in the conductor.) From (28') the perpendicular spreading of $E_z$ is such that for $\omega = v_e k_z$

$$v_z k_z^3 \frac{c^2}{2\omega^2 v_s^2} \lesssim 1$$

Therefore the Alfvén wave spreads ultimately to a lateral dimension $d$ which increases as $z^{1.3}$. For $k_z \sim 1/d$, $v_e \sim 10^8$ cm/sec, $v_s \sim 10^8$ cm/sec, and $\omega^2 \sim 10^{14}$ sec$^{-2}$, corresponding to $n_e \sim 5 \times 10^7$ cm$^{-3}$, we have

$$d \gtrsim 10^8 z^{1/3} \textrm{ cm}$$

Fig. 8. Transmission line analogy for the Alfvén wings.
At a distance of $z \sim 10^4$ km, $d \gtrsim 200$ meters, which is an increase in size of less than a factor of 10 for a conductor such as Echo 1 of size $\sim 30$ meters. By conservation of energy, there is a corresponding reduction by a factor $\sim 10$ in the field strengths $E_x$ and $h_x$ of the Alfvén wave at these distances. This result suggests that, for very high altitude satellites where collisions are not dominating, the wings may be detectable out to very great distances. Further, as the Alfvén wave penetrates to lower altitudes where the ion density increases and $v_a$ decreases, energy conservation implies that in the WKB approximation $D_1E_1v_a$ remains approximately constant. Therefore the charge separation associated with the wave increases as $v_a^{-1.2}$. In this estimate we have neglected the possible growth of the transverse dimension of the Alfvén wave because of the neglect of higher-order terms in (11).

**Effect of finite work function limiting the current flow through the surface of a conductor.** In the upper wing of the Alfvén disturbance in Figure 2, a negative current flows out from the conductor, while in the lower wing the negative flow is toward the conductor. In our solution we assume this flow is maintained indefinitely without change in charge density. The plasma electrons are drawn into the conductor surface, neutralizing the electric field of the ions. In order to maintain the charge separation associated with the wave increases as $v_a^{-1.2}$. In this estimate we have neglected the possible growth of the transverse dimension of the Alfvén wave because of the neglect of higher-order terms in (11). To overcome the work function at the conductor surface so that the electron current can flow. The alternative to this is for the current to be carried by the ions into the surface of the conductor at the top, and the corresponding reduction of the conductivity $\sigma_v$ by the mass ratio $m/M_i \lesssim 5 \times 10^{-4}$ would mean the failure of our simple Alfvén solution for the scale of sizes that we have considered and the dominance of the resistive and capacitive corrections discussed in the preceding section.

The proposed solution is valid only if the current can be maintained by electrons. Since cold emission is totally negligible, we turn to the sun's radiation and calculate the possible electron current which could be maintained by considering the Alfvén wave emission together with the photovoltaic emission during the daylight hours only. The flux of photons from the sun deposits $0.140 \, \text{watt/cm}^2$ as the total irradiance above the atmosphere at the earth's mean distance from the sun. This corresponds to the total radiation [Johnson, 1961] from a blackbody at a temperature of $5800^\circ \text{K}$ (although the spectrum is close to that of a blackbody at $6000^\circ \text{K}$). The numbers of incident photons per cm$^2$ with energies above 2, 3, and 4 electron volts are $1.2 \times 10^7$, $3 \times 10^6$, and $3.5 \times 10^5$, respectively. The corresponding maximum photovoltaic currents for these work functions are 20, 5, and 0.6 milliampere/cm$^2$, respectively, where $f$ is the efficiency defined as the number of electrons emitted per incident photon. For the physical parameters of interest here and with readily achieved efficiencies $f$, these photovoltaic currents are large enough to maintain the Alfvén disturbance and are nowhere near the space charge limit of Child's Law [Harnwell, 1949]. If we consider Echo 1, for example, in a 1600-km orbit where $v_a \sim 10^8$ cm/sec, with effective dimensions of $L \sim M \sim 30$ meters, we obtain the following values from (19)-(24) for the voltage, power, current, impedance, and surface charge density associated with the radiation of the Alfvén wings:

- $P = 3$ watts
- $V \sim 3$ volts
- $I \sim 1/2$ amp in each wing
- $Z \sim 6$ ohms in each wing
- $\Sigma_{\text{el}} \sim 8 \times 10^4$ electrons/cm$^2$

An average value of 0.2 gauss at this altitude for the perpendicular component of the earth's magnetic field was assumed in writing these numbers. Since $1/k_i > 75$ meters for the Alfvén solution to be valid according to (28), for $\omega_p \sim 4 \times 10^8$ cps, the power of 3 watts must be reduced by a factor of $(30/75)^2 \sim 1/6$ to $P = 1/2$ watt. The current is correspondingly reduced by a factor of $\sim 1/2.5$, according to (22), to a total value of $I \sim 0.2$ amp.

The Echo 1 balloon has a good conducting surface consisting of a few microns of aluminum evaporated on to a mylar base, and, if we take a very conservative (i.e. high) value of 4 volts for the work function and an effective emitting surface area for electrons of $L \delta M$, where $L$ is the width and $\delta M < M$ the thickness of a
wing in the notation of Figure 9, we find that
the incident flux of $3.5 \times 10^4$ photons/cm$^2$ with
energies $> 4$ ev produces the required 0.2-amp
current if the efficiency $f$ is not less than

$$f = 1.2 \times 10^{-3}/\delta M \text{(meters)}$$

For a wing thickness of $\delta M \lesssim M/2 \sim 15$ meters,
the required efficiency of $f \sim 10^{-4}$ is even less
than expected. Since work functions and photo-
electric efficiencies are relatively sensitive to the
'history' of a surface, we do not probe this
question here in further detail.

Electrons can also be removed from the
surface by collisions with positive ions that are
neutralized at the surface. These ions collide
with the surface by virtue of the motion of the
conductor (see Figure 7). The maximum current
density so obtained at 1600 km is $\sim n_i e v_i \sim 10^{-2}$
$\mu$amp, much less than the currents maintained
by photoelectrons in daylight. The possible
electron ejection from impacts of the ions
(sputtering) and of the more copious neutral
atoms will also increase the electron emission.

The current of 0.2 amp/4 $\times 10^8$ cm$^2 <$ 0.1
$\mu$amp/cm$^2$ is well below the space charge limit.
To show this, we compute the thickness of the
ion sheath surrounding the conducting surface of the Echo 1
in order to neutralize the electric field in the plasma. It is, by Gauss' law,

$$l = \frac{E}{2 \times 4\pi n_{\text{ions}} e} = \frac{v_i B}{8\pi n_{\text{ions}}} \approx 10^{-1} \text{ cm}$$

where we used $n_{\text{ions}} \sim n_{\text{electrons}} \sim 5 \times 10^9$/cm$^3$ at 1600 km altitude. Thus we have a potential
drop of 3 volts occurring in a distance of 0.1
cm from the surface of Echo, and through this
drop a current of hundreds of microamperes can
flow before being limited by space charge
according to Child's law.

In (31) we compute a surface charge density
of $\sum e_i = 8 \times 10^8$ electrons/cm$^2$ in the wing.
For a wing thickness of $\delta M \sim 15$ meters, the
 corresponding volume density is

$$8 \times 10^5/1.5 \times 10^9 = 530 \text{ electrons/cm}^3 \text{ (32)}$$

which adds $\sim 10\%$ to the ambient density.
There is thus a sizeable perturbation in the
ionosphere which can be observable by radar.

Effect of finite internal resistance in the con-
ductor. According to (22), the electric field
$-v_i/c B_0$ of the moving conductor causes a
current flow of magnitude

$$I = (c v_i/c)(B_0/2\pi)L \text{ (33)}$$

This current flow and the electric field are
maintained if the conductivity of the conductor
itself (so far assumed to be a perfect conductor
with $\sigma_e = \infty$) is high enough. By Ohm's law the
flow of current in the conductor in the $y$ direction
in Figure 5 is

$$I_e = N L \sigma_e E = N L \sigma_e (v_i/c)B_0 \text{ (34)}$$

The criterion for there to be no reduction of
power or current flow due to internal resistance
is obtained by combining (33) and (34) to get

$$\sigma_e > \frac{c \nu_i B_0/v_i \cdot 2\pi L}{N L (v_i/c)B_0}
\text{ (35)}$$

For Echo 1, with $v_i \sim 10^8$ cm/sec and $N \approx 30$
the requirement is

$$\sigma_e > 6 \times 10^7 \text{ esu} \approx 6 \times 10^{-14} \text{ abmho/cm}$$
Since this is well below the specific conductivity of an aluminum layer a few microns thick, no significant correction need be made. For conductors at lower altitude orbits, the criterion (35) becomes somewhat more severe as $v_e$ drops to $\sim 2 \times 10^7$ cm/sec at 160-500 km altitude. As is clear from the Maxwell equations, the current flow is reduced by a factor

$$\frac{1}{1 + \left( c^2 / 2 \pi v_e n_e \sigma \right) }$$

(36)
due to internal resistance of the conductor if it is appreciable. In simple terms of Ohm's law, (36) says merely that the effective resistance is increased from the value given in (23) because of adding the internal and plasma resistances in series.

5. DAMPING OF ECHO 1 ORBIT

The detailed orbit of the Echo 1 satellite shows a complicated variation of perigee, apogee, and eccentricity values with time due to a combination of factors. This problem has been studied by Jastrow and Pearse [1957] and by Shapiro and Jones [1960], who include effects of electrostatic charging of the body in orbit (negligible), solar pressure, and atmospheric drag. This latter factor plays a significant role even at tiny densities above 1000 km, because of the very abnormally large ratio of surface area to mass of Echo 1 ($\pi R^2 \approx 6 \times 10^6$ cm$^2$; weight $\approx 150$ pounds). In fact the detailed analysis of Echo's orbit has yielded a 'measurement' of the mass density in the upper ionosphere. To account for the observed drag, after allowance for other calculable factors such as solar pressure, the needed mass density [Shapiro and Jones, 1960] is given as $\rho_{\text{mass}} = 1.2 \times 10^{-18}$ gm/cm$^3$ at 1600 km. The corresponding power dissipation (in the approximation that $v_e = 7 \times 10^6$ cm/sec is large compared with molecular and ion velocities; with $T^* \sim 15000^\circ \text{K}$ and effective molecular mass of $M \sim 8M_p$, the average molecular velocity is $v_M \sim 2 \times 10^6$ cm/sec) is

$$P_{\text{atm-drag}} = (\pi R^2) \rho_{\text{mass}} v_e^3 \sim \frac{1}{2} \text{ watt}$$

(37)

Backscatter radar measurements made at Lima, Peru, in 1962 have found $\sim 5 \times 10^4$ electrons/cm$^2$ at the 1600 km altitude [Bowles, 1964; Bowles et al., 1962]. The corresponding ion mass density is $\sim 3 \times 10^{-20}$ gm/cm$^3$ for a molecular mass of $4M_p$ (for He$^+$) and $\sim 10^{-29}$ gm/cc if the ions are H$^+$. Thus there can be, at most, a few per cent ionization at these altitudes if all the Echo 1 damping comes from neutral atoms as in (37). However, the Alfvén mode damping which requires no neutral atoms may be sufficient to give the needed damping from the observed electron (ion) density $n_e \sim 5 \times 10^9$/cc.

We need a dissipation of $\sim \frac{1}{2}$ watt to replace $P_{\text{atm-drag}}$ in the analysis of the Echo 1 orbit. In (31) and the subsequent discussion, we found a magnetohydrodynamic breaking from radiation into the Alfvén mode of $P \sim \frac{1}{2}$ watt, very close to the required damping. Since we must rely on the sun to provide the energy leading to photoelectric emission and to maintain the current, we should further reduce $P$ by another factor of 2 to a time averaged value of $\sim \frac{1}{4}$ watt, as Echo 1 is in the daytime sky only one-half of the time. What we have achieved here, then, is a new drag mechanism which is of the right order of magnitude to explain the observed orbit parameters of Echo 1 without the requirement of a high value of the ionospheric mass density relative to ion density.

As we follow Echo 1 to lower altitude orbits, the frictional drag increases in proportion to the mass density and dominates the Alfvén drag by the time we are down to altitudes $\leq 1000$ km.

Direct experimental confirmation of the ideas presented here is highly desirable, particularly in view of the very qualitative nature of our results as applied to Echo 1, which is somewhat too small to meet the criteria for the Alfvén regime of parameters. This confirmation could be achieved by observation of the Alfvén wings by specular reflection of radar from the surface charge layer computed in (32).

As we saw in (30), an ionospheric disturbance is predicted extending perhaps many hundreds of kilometers along field lines in both directions from Echo 1. This may explain why Echo 1 transits were seen in instances when the radar cross section of the body itself was too small to be seen above the instrumental noise [Tiuri and Kraus, 1963]. In these same measurements ionospheric disturbances of duration $\pm 20$ minutes before and after transit of Echo 1 above the radar sighting were often recorded. To explain this in the present context, the Alfvén wings would need to extend as a detectable charge separation along a magnetic field line in one
direction for a distance between 5000 and 10,000 kilometers.

Passive detection at sea level is not feasible because of critical reflection at the $D$ level. Alfvén waves generated by a source whose geometrical size is not $\geq 1$ km will not give a detectable magnetic anomaly at sea level (Drell, Foley, and Ruderman, to be published).

6. Speculations on Orbit Maneuverability and Power Storage Using an Alfvén Propulsion Engine in Space

For Echo I the power level generated in the Alfvén disturbance was measured in watts. From (21) it is evident that large conductors ($L \sim M \sim 100$ meters) at lower altitudes ($\sim 160-500$ km with $v_a \sim 2 \times 10^7$ cm/sec and $B_0 \sim 0.4$ gauss) can dissipate power at the level of kilowatts when crossing field lines; for example,

\[ L \sim M = 100 \text{ meters} \]
\[ v_a = 2 \times 10^7 \text{ cm/sec} \]
\[ B_0 = 0.4 \text{ gauss} \]

lead, by (19), (21), (22), and (23), to

\[ P_{\text{Alfvén}} = 8(L/100 \text{ m})(M/100 \text{ m}) \text{ kw} = 8 \text{ kw} \]
\[ V = (v_a/c)B_0 M = 30(M/100 \text{ m}) \text{ volts} \]
\[ = 30 \text{ volts} \]
\[ I = 130(L/100 \text{ m}) \text{ amp} \]
\[ = 130 \text{ amp in each wing} \] (38)
\[ Z = 0.23 M/L \text{ ohm} = 0.23 \text{ ohm} \]

According to (28), we are comfortably within the Alfvén regime with this choice of parameters. The surface charge density in a wing is, by (24),

\[ \Sigma_{ch} \approx 2 \text{ esu} = 4 \times 10^8 \text{ electrons/cm}^2 \]

For a wing thickness of $\sim 10$ meters, this gives a volume density of $\sim 4 \times 10^8$ electrons/cc, or about 10 times ambient at these altitudes. The corresponding current flow through the conducting surface of area $10 \times 100$ m$^2 = 10^5$ cm$^2$ into each wing is $13 \mu\text{amp/cm}^2 = 8 \times 10^{12}$ electrons/cm$^2$. Since metallic conducting surfaces with work functions in the range 3-4 volts and with photoelectric efficiencies of $\sim 1\%$ are well within the range of comfortable technology, there is no difficulty maintaining this current flow with the sun as the source of photoelectrons. (For 24-hour operation, we could resort to hot wire filaments or some other active device.) Once again, this current of $\sim 13 \mu\text{amp/cm}^2$ is well below the Child’s law limit for emission through a potential drop of 30 volts occurring in a fraction of a centimeter.

The high level of power generated by the Alfvén disturbance invites speculation on ways of making practical use of it. If used passively as a controllable drag mechanism it can serve (1) as a means of converting satellite kinetic energy to electrical power, (2) as a way of bringing satellites to lower altitudes without propellant, and (3) to adjust satellite attitudes by exploiting torques. If used in conjunction with an on-board source of electrical power (viz, solar panels or small nuclear reactors), it can serve (4) to counteract atmospheric drag effects on satellites and even propel them to higher altitudes, (5) as a means of storing energy by converting it temporarily to satellite kinetic energy, and (6) to maneuver the position and attitude of a spacecraft (without propellant) by pushing on the earth’s magnetic field.

Most simple is the question of utilizing the power flowing in the current producing the Alfvén fields. A low impedance motor with internal resistance of $\sim 0.23$ ohm could tap $\frac{1}{2} \times 8 \text{ kw} = 2$ kw of power for use in a satellite at the expense of its kinetic energy. This might be generated by flying a 'kite,' as illustrated in Figure 10, consisting of two 100-meter-long conducting slabs each of 5- to 10-meter diameter, connected by a conducting rod of 100-meter length oriented perpendicular to $B_0$ through which the current flows. The surface of this conducting rod is insulated from the outside world, and in series with it is connected the low impedance motor delivering the power. Figure 10 shows a schematic design. The rigidity must be sufficient to withstand some tens of pounds of force distributed over 100-meter rods and surfaces. A number of photosensitive slabs might be desirable to minimize effects of rotation about the vertical conductor. For short time power needs, we must compare with a very efficient small gasoline engine which can deliver $\sim 1$ kw of power for an hour by burning $\sim 1$ pound of fuel. For more sustained use, 500 pounds of solar cells would give an equivalent power.
Fig. 10. A schematic arrangement for the Alfvén propulsion engine.

Short-circuiting the junction between $M$ and $M'$ of Figure 10 maximizes the drag (equivalent to 8 kw of power or a drop of 40 km per day for a 10^4-pound satellite.) Opening the junction between $M$ and $M'$ reduces the drag by an order of magnitude, since the effective area for Alfvén wave radiation is reduced from $(M + M')L$ to $2(\Delta M)L$. Changing the relative sizes of $M$ and $M'$ gives a torque about the earth’s magnetic field direction and thus rotates the satellite about $B_0$. Rotations about the vertical occur if the center of mass of the satellite is slightly displaced from the vertical line current $M -- M'$. Other geometries offer a variety of similar possibilities.

If a source of electrical power is available on the satellite, the direction of the drag currents can be reversed and the drag converted to a push. The advantage of the Alfvén propulsion engine over a small rocket engine lies in those circumstances in which the source of power does not involve the consumption of a heavy fuel (as in the case of a gasoline engine) which could just as well be used as a propellant. Rather, the Alfvén engine as a propulsion mechanism would be most useful for power which originates from solar panels or a nuclear generator.

Among the problems in using a manned orbiting laboratory is that the atmospheric drag is a major source of power dissipation, setting a lower limit of 100 miles on the altitude. It is estimated that the drag will lower the orbit of a 20,000-pound manned orbiting laboratory at this altitude by ~15 km/day. This represents drag force of

$$\frac{dp}{dt} = \frac{1}{2} mg \frac{dh/dt}{v_c}$$

where $h$ is the altitude and $dh/dt = 15$ km/day ~ 15 cm/sec, and a power dissipation

$$P_{\text{drag}} = \frac{1}{2} mg \frac{dh}{dt} \approx 7 \text{ kw}$$

To compensate for this drag, we propose flying the ‘kite,’ but with a power source aboard to drive the current backward, thereby gaining the 5-10 kw power required to neutralize the drag loss and to maintain the altitude of the satellite with a small increase of total weight in orbit. Higher power levels could also be utilized to make the satellite sail to higher altitudes, the gain being ~2 km/day for each steady input of ~2 kw (available, for example, from about 500 pounds of solar cells).

When the current flow through the conductor is reversed by an impressed voltage, the drag force $I \times B$ reverses its direction, leading to an acceleration. The normal Alfvén wave gave a power dissipation (drag)

$$P_{\text{Alfvén}} = \frac{B_0^2 v_c^2}{2\pi v_a} (ML) = I \frac{MB_0v_c}{c}$$

where $I$ is the current flow driven by the potential difference $v_a BM/c$. An impressed voltage $V_f$ in the opposite direction (say between $M$ and $M'$ of Figure 9) gives a rate of increase of satellite kinetic energy

$$P_K = P_{\text{Alfvén}} V_f - (v_c/c)B_0M$$

A sufficiency condition for the validity of our approximations (in this case, that of linearity over a distance $\chi$ from the conductor) restricts $V_f$ to $2v_c B_0M/c$. It is not yet clear whether the region of nonlinearity increases or decreases the linear drag or propulsion. Thus a power source of 60 volts and 16 kw can give half its power to generating Alfvén waves and half to increasing the kinetic energy of the system described in (38). This is sufficient to lift the manned orbiting laboratory about 15 km per day. Clearly this is also a way to store electrical or solar panel energy.
by using it to lift a satellite, but the storage and subsequent reconversion are at most each 50% efficient.

We may speculate on this kind of mechanism as a propulsion engine for flight to further reaches of space. For interplanetary travel, typical parameters are $B_0 \sim 10^{-4}$ gauss and densities $\sim 10^{-23}$ g/cm$^3$, leading to Alfvén speeds of

$$v_a \sim 10^6 \text{ cm/sec} = 10 \text{ km/sec}$$

To be in the Alfvén regime we require

$$\omega < \Omega_s = eB/Mc$$

$$\approx 10^{-1} \text{ cps for } B \sim 10^{-5} \text{ gauss}$$

The corresponding conductor dimension is

$$L \sim v_a/\omega > 10^7 \text{ cm} = 100 \text{ km}$$

For our solution we have the presumed requirement

$$h/B = v_a/v_e = v_e/10^6 \text{ cm/sec} < 1$$

Then for $v_e \approx v_a$

$$P_{\text{Max}} = (B_0^2/2\pi)v_e(ML)$$

$$= (1.6 \times 10^{-11}) \times (10^6)(10^{10})$$

$$\times (ML/\text{km}^2) \times 10^{-7} \text{ watt}$$

$$\approx 0.02(ML/\text{km}^2) \text{ watt} < \frac{1}{2} \text{ kw}$$

either for the conversion of electrical energy to satellite kinetic energy or vice versa.

The conversion of energy from gravitational attraction near another planet to electrical energy is a reasonable possibility if both a reasonable magnetic field and ionospheric plasma are present.

The phenomenon discussed in this paper can be used to determine ionic mass densities in regions of known field strength, as both the Alfvén field strength and the power dissipation are proportional to $\rho^{1/2}$:

$$h = (v_e/v_a)B = \sqrt{4\pi\rho v_e} \propto \sqrt{\rho}$$

$$P_{\text{Alfvén}} \propto (B_0^2/v_e) \propto \sqrt{\rho}$$

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Additional note. M. J. Lighthill has discussed the theory of Alfvén radiation from objects of smaller dimension, for which the unidirectional mode discussed here (the vorticity wave, in his terminology) is not radiated [Lighthill, 1960a, b]. We wish to thank Professor Lighthill for a very friendly and illuminating letter discussing these questions.

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