Polarization of the Auroral Electrojet

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We consider an idealized model of electrojet polarization. Precipitation from the inner edge of the electron plasma sheet creates a density maximum in the auroral-oval ionosphere, which in turn leads to Hall and Pedersen conductance maxima. We then assume that a uniform westward convection electric field is imposed on the lower ionosphere before polarization. Field-aligned currents must flow into the ionosphere equatorward and out of the ionosphere poleward of the Hall conductance maximum. As the convection field and ionospheric density increase during the substorm growth phase, the field-aligned current densities should eventually reach an instability threshold beyond which anomalous resistance should produce field-aligned electric fields. The partial blockage of the field-aligned currents produces an equatorward electric field and therefore a partial Cowling conductivity in the lower ionosphere. Rough numerical estimates indicate that the expected field-aligned currents can exceed the stability threshold estimated by Kindel and Kennel (1971), that 1- to 5-kv field-aligned potential drops correspond to significant electrojet enhancement, and that the required energy dissipation of field-aligned currents in the topside ionosphere, a few ergs/cm² sec column, suggests significant topside modification following auroral breakup.

The growth phase of magnetospheric substorms commences with a southward shift in the interplanetary magnetic field [Fairfield and Cahill, 1966; Nishida, 1968a, b; 1971; Hirshberg and Colburn, 1969; Aubry et al., 1970; Arnoldy, 1971], which intensifies field-cutting at the nose of the magnetosphere and internal magnetospheric convection [Dungey, 1961; Levy et al., 1964; Axford et al., 1965]. Mozer and Manka [1971] and Mozer [1971] have observed, before breakup in the nightside auroral oval, a gradual buildup of the westward convection electric field, which drives enhanced ionospheric currents [Oguti, 1968; McPherron, 1970].

In the geomagnetic tail the magnetic field increases by as much as 50% during the growth phase [Fairfield and Ness, 1970; Camidge and Rostoker, 1970; Russell et al., 1971a; Aubry and McPherron, 1971], and the plasma sheet thins [Hones, 1970; Hones et al., 1971]. Coroniti and Kennel [1972a, b] have argued that these tail changes are consistent with increased flaring of the tail magnetopause due primarily to a reduction in size of the dayside magnetosphere and to an increase in tail flux [Aubry et al., 1970]. Other consequences of enhanced tail flaring stress are an increase in plasma-sheet plasma pressure and an earthward motion of the tail currents and electron plasma-sheet inner edge [Siscoe and Cummings, 1969].

In this paper we investigate the response of the nightside auroral oval to the enhancement of convection and the intensification of electron precipitation from the plasma sheet. For typical growth-phase parameters we find that the field-aligned currents flowing into and out of the auroral oval can exceed the threshold for topside ionospheric current instabilities [Kindel and Kennel, 1971]. A direct consequence of any resulting topside anomalous resistance to the current flow is that the ionospheric electric field polarizes into a Cowling current electrojet configuration. The possibility that the electrojet is a Cowling current has been discussed by Fukushima [1969].

We assume that the ionospheric plasma density of the nightside oval is maintained solely by the precipitation of plasma-sheet electrons. The nightside oval is then bounded by the last closed tail field line and bounded at its equatorward edge by the inner boundary of the electron plasma sheet [Vasyliunas, 1968]. In a flaring tail the plasma-sheet...
pressure decreases as the geocentric distance increases, the decrease implying that the electron precipitation heat flux should decrease poleward. When there is convection, electrons are adiabatically compressed and heated as they flow from the plasma sheet into the dipole field; hence the precipitation heat flux should maximize at the equatorward edge of the oval. Since dissociative recombination is the major ion loss mechanism in the lower ionosphere, the ionospheric plasma density is proportional to the square root of the electron heat flux. Hence the ionospheric density and the height-integrated conductivities (the conductances) should increase proceeding equatorward, reach a maximum, and then fall off rapidly at the equatorward edge of the auroral oval. Since inward flow raises the mean electron energy, electron precipitation at the equatorward edge of the oval should penetrate to deeper atmospheric layers [Rees, 1963]. Hence the ratio of Hall conductance to Pedersen conductance should maximize as well.

During growth phase the observed predominantly westward electric field near midnight drives a westward Pedersen and a poleward Hall current. Pedersen currents are never divergence free in the ionosphere and must be fed by field-aligned currents into the ionosphere on the morning side and out on the evening side. The N-S gradient of the Hall conductance implies that the poleward Hall current also may not be divergence free in the ionosphere and may therefore be fed by field-aligned currents flowing into the ionosphere at the equatorward edge and by outward currents poleward of the maximum of the auroral-oval ion density (for a general discussion see Heppner et al. [1971]). Considerations of symmetry suggest that the Pedersen current, and therefore the electric field, ought to maximize in the local midnight sector; hence the poleward Hall current should also maximize near midnight. The field-aligned current density feeding the Hall current will maximize in the region of the sharpest N-S gradient of Hall conductance, which occurs at the equatorward edge of the oval when the sharp inner boundary of the electron plasma sheet is mapped into the ionosphere [Vasyliunas, 1965]. The arguments discussed above and the facts that E-W ionospheric scale lengths are longer than N-S ionospheric scale lengths and that the Hall conductance is ordinarily larger than the Pedersen conductance indicate that the maximum field-aligned current density should be found at the equatorward edge of the auroral oval in the midnight sector.

The buildup of the convection electric field coupled with the enhanced Hall conductance from increased electron precipitation indicates that the field-aligned currents should intensify during the substorm growth phase. Since the conductivity parallel to the magnetic field is ordinarily quite large, the field-aligned currents feeding the auroral oval ordinarily flow freely, i.e., without large parallel potential drops. They may have been observed as transverse magnetic perturbations at 1100 km over the auroral oval [Zmuda et al., 1967; 1970] and as E-W magnetic-field perturbations near the boundaries of the plasma sheet [Russell et al., 1971a]. Field-aligned currents probably also flow in and out along the system of homogeneous arcs often found during the growth phase. In this paper we will conceptually average over such arc systems and consider only the large-scale features of the auroral-oval current and electric-field distribution, a limitation that suits a comparison with Mozer's electric-field measurements.

The ability of the magnetic-field lines to carry parallel currents is not unlimited. According to Kindel and Kennel [1971], parallel current densities of a few times 10^8 e cm^-2 sec^-1 will lead to electrostatic-wave instabilities in the topside ionosphere generally above 1000-km altitude. The arguments discussed above indicate that, as the growth phase develops, a field line near the equatorward edge of the auroral oval in the local midnight sector would most likely be the first to become unstable. The nonlinear saturation of current-driven instabilities leads to an 'anomalous' parallel resistance and therefore to the development of parallel electric fields. Then the field-aligned currents feeding the auroral-oval Hall current would be partly blocked. The lower ionosphere must then polarize, the polarization creating an equatorward electric field whose Pedersen current reduces the net poleward current in the lower ionosphere. According to Mozer [1971], such an equatorward electric-field shift is a characteristic feature of auroral breakup. The equatorward polarization field, equivalent to a partial Cowling conductance, can then strongly enhance the west-
ward current when the Hall conductance exceeds the Pedersen conductance.

Thus, following the strict logic of the growth-phase development, we have arrived at salient features of substorm breakup: an equatorward electric-field shift and an enhancement of the auroral electrojet. However, we have left a 'chicken-egg' cause and effect relationship unresolved, since anything in the geomagnetic tail that suddenly enhances earthward convection would also, by the logic discussed above, polarize the auroral-oval ionosphere. In effect, it is unclear whether tail magnetic-field collapse causes electrojet polarization or whether the change in convection boundary conditions implied by electrojet polarization triggers the tail field collapse.

In section 1 we undertake a simplified analysis of electrojet polarization. We assume the auroral-oval density enhancement to be uniform between sharp boundaries at the poleward and the equatorward edges; field-aligned currents then flow only at the edges of the oval. We assume a primary quasi-constant westward electric field imposed on the ionosphere by magnetospheric convection. If the convection electric field has a quasi-steady N-S component, the auroral-oval density enhancement should be rotated to be aligned perpendicular to the flow direction. We further assume that any polarization electric field that results from field-aligned resistance appears only in the ionosphere below the anomalous resistance region and does not map into the magnetosphere. For steady subsonic convection this assumption is reasonable, since the hot magnetospheric plasma will tend to discharge any polarization fields in space. However, for unsteady or rapid flows the field-line capacitance and inductance may permit a polarization electric field to be established in the magnetosphere. With these assumptions the entire lower ionosphere is treated as a lumped element in a circuit comprising the poleward Hall current, the field-aligned currents that flow through resistors, and the currents that arise from the polarization electric field. We then compute the parallel resistance for which the southward polarization and convection field components are roughly equal, the parallel resistance corresponding to Mozer's observations at breakup. Field-aligned potential drops of 5-10% of the E-W convection potential (a few kilovolts) are required. Thus, without solving the difficult nonlinear anomalous resistance problem, we can infer from Mozer's measurements and the present interpretation the required integrated anomalous resistance needed for polarization. It is interesting that the required potential drops are consistent with the energies of electron beams typically observed [Evans, 1968]. Direct rocket measurements of parallel electric fields in the lower ionosphere may also be indicative of anomalous field-aligned resistance [Mozer and Bruston, 1967]. Since the anomalous Joule heating in the topside amounts to several ergs/cm² see column, we would expect significant changes in topside structure following breakup.

The analysis in section 1 completely neglects all questions of spatial structure of the auroral-oval ionosphere. However, the gradient scale lengths determine the field-aligned current density and, therefore, whether or not topside current instabilities are possible. Thus we must create a model for the N-S ionospheric density profile, which, in turn, depends on the profile of electron precipitation. In the absence of a parallel electric field, the electron precipitation rate depends on the electron distribution in space and the pitch-angle scattering rate. Only in the limit of strong pitch-angle diffusion can the electron precipitation rate be conveniently estimated a priori. The observed isotropy of the electron precipitation fluxes into the auroral oval suggests they are often near the strong diffusion limit [Kennel, 1969]. In section 2 we couple strong diffusion precipitation and convection [Kennel, 1969] to find the spatial profile of the electron precipitation fluxes. Since this profile depends critically on the magnetic topology, we can uniquely model only the inner edge of the plasma sheet where the convection flow penetrates an essentially dipolar field. The electron heat flux can then be approximately related to the E-region ionospheric density and therefore to the Hall and Pedersen conductances. The idealizations involved in this model suggest that it may often err quantitatively; however, we hope no essential physical trends have been overlooked.

In section 3 the effects of parallel resistance are investigated. Here we treat the anomalous resistance as a small perturbation in the sense that runaway electron beams created by the parallel electric field produce no additional ioni-
zation in the lower ionosphere. This absence of additional ionization is certainly not the case for auroral arcs. However, we hope the large-scale structure is adequately treated. We find that typical growth-phase ionospheric conductances and electric fields can lead to parallel current densities that exceed the stability threshold. The threshold is exceeded first at the equatorward edge of the auroral oval and then, as the electric field increases, somewhat farther north. The polarization model has a broad region several hundred kilometers thick of westward electrojet, which may in fact be divided into two parts. There is a weak eastward electrojet equatorward of the main westward electrojet.

1. POLARIZATION OF A BLOCK IONOSPHERE

We treat the polarization of an idealized two-dimensional auroral oval in which the conductivity enhancement produced by plasma-sheet electron precipitation is sharply bounded at its northern and southern edges and uniform in between. When a uniform westward electric field is applied, field-aligned currents flow into the ionosphere at the southern edge and out at the northern edge. No field-aligned currents flow elsewhere. The field-aligned currents bounding the oval are assumed to produce anomalous parallel resistances that produce a polarization electric field in the lower ionosphere.

Consider a Cartesian coordinate system appropriate to the nightside northern auroral oval in which \( z \) points vertically upward, \( x \) southward, and \( y \) eastward. For simplicity the geomagnetic field is assumed to point in the \(-z\) direction. The oval ionosphere is assumed to be uniform in \( y \), sharply bounded at its northern and southern edges, and uniform over its N-S width \( w \). The height-integrated currents within the oval then obey

\[
I_x = \sum_P E_x + \sum_H E_y, \quad (1)
\]

\[
I_y = \sum_P E_y - \sum_H E_x, \quad (2)
\]

where \( \sum_P \) and \( \sum_H \) denote the height-integrated Pedersen and Hall conductances, respectively. For simplicity we assume that the net Pedersen resistance of the polar cap and subauroral regions is large enough that no N-S Pedersen currents can flow outside the oval. Furthermore, we define \( \Delta_P \) and \( \Delta_H \) as the difference between the Hall conductivities of the auroral oval and the subauroral region and the difference between the Hall conductivities of the oval and polar cap, respectively. \( I_s \) has a divergence in two dimensions, which requires field-aligned currents \( I_s \) and \( I_x \) at the southern and northern edges of the auroral oval, respectively,

\[
I_s = \Sigma_P E_x + E_y \Delta_P + \Delta_H E_y \quad (3)
\]

\[
I_x = -\Sigma_P E_y - \Delta_N E_x \quad (4)
\]

Note that, if \( \Delta_P = \Delta_H \), then \( I_s = -I_x \). Furthermore, when the oval is unpolarized (\( E_x = 0 \)), a westward convection field (\( E_x < 0 \)) implies \( I_s < 0, I_x > 0 \) (field-aligned currents in at the southern edge of the oval and out at the northern edge). We now treat the whole auroral-oval bottomside ionosphere as an element in a circuit involving the field-aligned currents. We assume that, when \( |I_s| \) and \( |I_x| \) exceed certain thresholds, they will become unstable somewhere in the topside ionosphere and produce integrated anomalous resistances \( R_s \) and \( R_x \). Furthermore, we assume that for subsonic flow the field-line capacitance can be neglected and that any polarization electric field in space is discharged by the hot highly conducting magnetospheric plasma. Therefore \( E_x \) exists only below the anomalous resistance region. The condition that the potential drops across \( R_s \) and \( R_x \) just balance the potential across the auroral oval due to any polarization \( E_x \) is

\[
E_x w = R_N I_N - R_s I_s \quad (5)
\]

\( E_x \) is assumed to be imposed by magnetospheric convection and uniform in \( x \). Finally, we define \( P = w/\Sigma_P \), the integrated Pedersen resistance across the auroral oval in the N-S direction, \( w \) being the width of the oval.

After some algebra, we arrive at the following relations:

\[
I_s = E_x \left[ \Delta_P + (R_N/P)(\Delta_P - \Delta_H) \right] / \left[ 1 + (R_s + R_N)/P \right] \quad (6)
\]

\[
I_x = -E_x \left[ \Delta_P + (R_N/P)(\Delta_P - \Delta_H) \right] / \left[ 1 + (R_s + R_N)/P \right] \quad (7)
\]

\[
I_x = E_x \left[ \sigma_H \Delta_P + R_N \Delta_H \right] / \left[ P + R_s + R_N \right] \quad (8)
\]

\[
I_y = E_x \left[ \sigma_P + \sigma_H (R_s \Delta_P + R_N \Delta_H) \right] / \left[ \sigma_P P + R_s + R_N \right] \quad (9)
\]
If \( \Delta_s = \Delta_n = 0 \), i.e., the ionosphere is completely uniform, there are no field-aligned currents \((I_s = I_n = 0)\), there is no polarization \(E_s\), and, consequently, there is no enhancement of the electrojet, \(I_s = \Sigma_P E_P\). Next consider the case where \(\Delta_s \neq 0\) and \(\Delta_n \neq 0\) and where the products \(E_s \Delta_s\) and \(E_n \Delta_n\) do not produce field-aligned currents above the threshold for instability. Then \(R_s\) and \(R_n\) will be effectively 0. Since the field-aligned currents required to feed the auroral-oval Hall current flow freely along field lines between the ionosphere and outer space, there is no polarization \(E_n\) and no electrojet enhancement. Finally, we can recover the classical Cowling conductivity by assuming at least one field-aligned resistance, for example, \(R_s\), to be very large. In this instance,

\[
I_s \to 0
\]

\[
I_n \to E_n(\Delta_s - \Delta_n)
\]

\[
I_s \to E_s(\Sigma_H - \Delta_s)
\]

\[
I_n \to E_n[\Sigma_P + (\Sigma_H \Delta_s/\Sigma_P)]
\]

\[
E_s/E_n \to -\Delta_s/\Sigma_P
\]

In the limit \(\Delta_s \approx \Sigma_H \approx \Delta_n\), \(I_s = I_n = I = 0\), \(E_s/E_n = -\Sigma_H/\Sigma_P\), and \(I_n = E_n[\Sigma_P + (\Sigma_H \Delta_s/\Sigma_P)]\). Here the Cowling conductivity provides a strong electrojet when \(\Sigma_H/\Sigma_P \gg 1\); this condition usually holds for the nightside oval [Bostrom, 1964]. Clearly, anomalous resistance becomes important, and the transition from Pedersen conductivity to Cowling conductivity for the electrojet occurs when \((R_s + R_n)/P\) grows to be of the order of 1.

Mozer's [1971] electric-field measurements indicate that a southward electric field whose magnitude is roughly equal to the original westward electric field develops during substorm breakup. If we interpret the southward field shift as being due to the development of anomalous field-aligned resistances, we may use the observational condition \(|E_s/E_n| \sim 1\) to infer several properties of the anomalous resistances without invoking detailed kinetic-theory solutions for the saturation of current-driven instabilities in the topside ionosphere. In the following discussion we will assume for simplicity that \(\Delta = \Delta_s = \Delta_n\) and \(R_s = R_n = R\).

Let us define \(E_s/E_n = -1\) as polarization onset, which occurs when

\[
R = \frac{P}{2} \frac{1}{(\Delta/\Sigma_P) - 1} = \frac{1}{2} \frac{w}{\Sigma_H - \Sigma_P}
\]

where \(\Delta \approx \Sigma_H\). \(E_s/E_n \geq -1\) is possible only if \(\Sigma_H > \Sigma_P\). When \(\Sigma_H \gg \Sigma_P\), polarization occurs when each parallel resistance is roughly equal to half the N-S integrated Hall resistance. For polarization onset the bottomside currents are given by

\[
I_s = (\Delta - \Sigma_H)E_s \quad I_n = E_n(\Delta + \Sigma_P)
\]

Thus the electrojet is enhanced by a factor of \(1 + (\Delta/\Sigma_P)\) relative to the unpolarized state before breakup.

The magnitude of the field-aligned potential drops is given by

\[
\Delta V_1 = RI = \frac{|E_n| \Sigma_P R}{1 + (2R/P)}
\]

which for polarization onset reduces to \(\Delta V_1 = |E_s|w/2\). If we estimate that \(|E_n| = \Phi/l\), where \(\Phi\) is the E-W potential across the auroral oval and \(l\) is the length of the auroral oval in the E-W direction, then

\[
\Delta V_1/\Phi \approx w/2l
\]

Since \(w/2l \approx 0.05-0.1\), a field-aligned potential drop of roughly 5-10% of the electromotive force (emf) along the oval can produce polarization and an electrojet. The total energy dissipation per unit length of the field-aligned currents in one resistor at polarization onset is given by

\[
W_1 = l^2R = (|E_s|^2/2)(\Sigma_H - \Sigma_P)w
\]

and the ratio of topside to total bottomside energy dissipation at polarization onset is

\[
W_1/W_\perp = (\Delta - \Sigma_P)/2\Sigma_P
\]

where both parallel resistors have been counted. When \(\Delta \gg 3\Sigma_P\), more energy is dissipated in parallel currents than in perpendicular currents.

Let us now insert some characteristic values of the parameters involved in equations 16–21 to test the plausibility of these estimates. We assume \(w \approx 600\) km (a 6° auroral oval) and \(l \approx 6000\) km (an oval that extends from local evening to local dawn). Then, if the emf \(\Phi\) at breakup is 120 kv, we find \(\Delta V_1 \approx 6\) kv. Since this value is comparable to the characteristic energy of monoenergetic particle beams observed during...
breakup [Evans, 1968], it does not seem unreasonable. When $\Delta - \Sigma_T \approx 2 \times 10^{14}$ esu = 20 mhos, the dissipation in the two parallel resistors is from (20) $1.6 \times 10^{4}$ ergs/cm sec, which, if the electrojet extends over 6000 km, amounts to $8 \times 10^{17}$ ergs/sec in all. If the field-aligned currents are actually distributed and the energy dissipation $W_e$ is more or less uniform over the width $w \approx 600$ km, the field-aligned current dissipation per square centimeter column is roughly $2 \text{ ergs/cm}^2 \text{ sec}$. Such dissipation in the low-density topside ionosphere should lead to a gross change in its structure at substorm breakup.

Thus this simple idealized model of electrojet polarization leads to the following general conclusions. First, when $A/\Sigma_T > 1$, small field-aligned potential drops correspond to significant electrojet enhancements. Second, the observation $\Sigma_p/\Sigma_T \approx 1$ implies that the energy dissipation of field-aligned currents after breakup is comparable to that of the Pedersen currents in the lower ionosphere. Hence significant heating of the topside ionosphere should occur when the electrojet is enhanced by polarization.

2. STRONG DIFFUSION ELECTRON PRECIPITATION PROFILE

Here we construct a model for the latitudinal distribution of the energetic electrons precipitating from the plasma sheet into the nightside auroral-oval ionosphere. We assume that the electron fluxes are maintained nearly isotropic in pitch angle, not an unreasonable assumption since the precipitating electrons are often observed to be isotropic [Kennel, 1969]. By combining strong diffusion precipitation with convection, a spatial profile of electron precipitation may be deduced. This profile depends strongly on the magnetic-field topology, which can only be uniquely specified for the inner edge of the plasma sheet where convection carries the plasma-sheet electrons into a more or less dipolar field.

A solution for the penetration of hot precipitating electrons into a dipole field has been obtained by Kennel [1969]

$$n(L) = \frac{n(L^*)}{n(L_T)} = \left( \frac{L_T}{L} \right)^4 \cdot \exp \left\{ \frac{-3\delta}{22} \left[ \left( \frac{L_T}{L} \right)^{32/3} - 1 \right] \right\} \quad (22)$$

where $n(L)$ is the electron density, $L$ is the $L$ shell, and $L_T$ is the largest $L$ parameter in the magnetic midnight meridian plane where the magnetic field may reasonably be considered dipolar. Equation 22 is restricted to the magnetic midnight meridian; $L_T$, which may be estimated by the procedure of Siscoe and Cummings [1969], is typically 8–10, depending on the strength of the geomagnetic tail field. The parameter $\delta$ describes the relative strength of convection and strong diffusion electron precipitation.

$$\delta = \frac{L_T}{L} \quad L = (CE/B)T_{\text{min}}|_{L-L_T} \quad (23)$$

where $E$ is the convection field, which is assumed westward and constant in space and time in the equatorial plane, $CE/B$ is the equatorial plane convection speed, and $T_{\text{min}}$ is the electron minimum lifetime. For a dipole field the electron minimum lifetime is roughly $L'/[E_r(\text{kev})]^{1/2}$. Instead of considering the variation of $T_{\text{min}}$ with energy $E_r$, the solution discussed above assumes that electrons are lost on a characteristic time scale equal to the minimum lifetime of an electron at the thermal energy; i.e., $T_{\text{min}} = L'/[T_r(\text{kev})]^{1/2}$. For an adiabatic gas with $\gamma = 5/3$ (appropriate to pitch-angle isotropy with small precipitation losses), $T_r$ scales as

$$\frac{T_r(L)}{T_r(L_T)} = \left( \frac{L_T}{L} \right)^{17/4} = \left( \frac{L_T}{L} \right)^{8/3} \quad (24)$$

Equation 24 should be approximately valid until the flow carries the plasma beyond the density maximum, where precipitation energy losses become significant. The maximum of $n(L)$ occurs at $L/L_T = (\delta/4)^{1/3}$; the maximum hot electron density $n^*$ is

$$n^*(L) = \left( \frac{4}{\delta} \right)^{6/11} \exp \left\{ \frac{3\delta}{22} - \frac{6}{11} \right\} \quad (25)$$

Within $L < L_T(\delta/4)^{1/3}$, $n$ decreases rapidly; this region of decrease is the inner edge of the electron plasma sheet. When the electron precipitation flux exceeds the proton precipitation flux, a return current of cold electrons must flow from the ionosphere to maintain charge neutrality. Vasyliunas [1968] has suggested that the cold and hot electrons mix in less than a flow time and thereby lower the hot electron temperature. This effect has not been included here; it would weaken the gradient of hot
electron density at the inner edge and decrease the electron temperature. Two other quantities of interest are the omnidirection particle flux $J$, which is twice the precipitation flux, and the electron precipitation heat flux $F$. For Maxwellian energy distributions these fluxes would scale as

\[
J = \left( \frac{n}{4} \right) \left( \frac{T_e}{m_e} \right)^{1/2}
\]

\[
F = \left( \frac{8}{\pi m_e} \right)^{1/2} \frac{T_e}{L}^{3/2}
\]

where $m_e$ is the electron mass. We will use these scalings as illustrations.

Inserting (22) and (24) into (26), we arrive at

\[
\frac{J(L)}{J(L_0)} = \left( \frac{L}{L_0} \right)^{16/3} \exp \left\{ -\frac{3\delta}{25} \left[ \left( \frac{L}{L_0} \right)^{22/3} - 1 \right] \right\}
\]

\[
\frac{F(L)}{F(L_0)} = \left( \frac{L}{L_0} \right)^{8} \exp \left\{ -\frac{3\delta}{25} \left[ \left( \frac{L}{L_0} \right)^{22/3} - 1 \right] \right\}
\]

$J$ and $F$ have maximums at $(L/L_0) = \left\{ 38/16 \right\}^{16/3}$ and at $(L/L_0) = \left\{ 8/8 \right\}^{12/11}$, respectively; the magnitudes of the maximums are given by

\[
\frac{J^*}{J(L_0)} = \left( \frac{16}{3\delta} \right)^{6/11} \exp \left\{ \frac{3\delta}{25} - \frac{8}{11} \right\}
\]

\[
\frac{F^*}{F(L_0)} = \left( \frac{8}{\delta} \right)^{12/11} \exp \left\{ \frac{3\delta}{25} - \frac{12}{11} \right\}
\]

The electron temperature at the heat flux maximum is given by $T^*_e = T_e(L_0) \cdot (8/8)^{3/2}$. The maximums of the successively higher moments fall increasingly close to the earth because of convection plasma heating, undiluted here by mixing with cold ionospheric plasma. Furthermore, since $\delta \propto E^{-1}$, the successively higher moments have maximums whose magnitudes are increasingly strong functions of $E$. In particular, the maximum heat flux varies approximately as the convection electric field. Thus, owing to betatron heating, convection enhancements significantly increase the electron energy deposited in the auroral-oval ionosphere. However, the locations of the hot number density, flux, and heat flux maximums are weakly dependent on $E$ and vary as $E^{-1/2}$. Thus equatorward motions of the auroral oval are primarily due to decreases in $L_e$ caused by increasing geomagnetic tail fields.

The above solutions may be mapped on the auroral-oval ionosphere by using the relation

\[
\frac{\sin^2 \theta_T}{\sin^2 \theta} \approx \frac{\theta_T^2}{\theta^2} \approx \frac{x_T^2}{x^2} = \frac{L}{L_T}
\]

where $\theta$ is the magnetic colatitude, $\theta_T$ is the colatitude corresponding to the last dipolar tube of force, $L = L_T$. The linear distance from the geomagnetic pole in the midnight meridian plane is $x = R_0 \theta$. The approximation $\theta \ll 1$ is reasonably accurate for $L > 6$. From (31) the heat flux $F(L)$ maps as

\[
\frac{F(x)}{F(x_T)} = \left( \frac{x}{x_T} \right)^{16} \exp \left\{ -\frac{3\delta}{25} \left[ \left( \frac{x}{x_T} \right)^{44/3} - 1 \right] \right\}
\]

3. POLARIZATION OF AURORAL-OVAL IONOSPHERE

Basic equations. Let us assume that the conducting $E$ region of the ionosphere has an effective height $h$, that the mean electron and ion densities are denoted by $N_e$ and $N_i$, respectively, and that the field-aligned current density above the $E$ region is denoted by $j_i$. Then rough equations for $N_e$ and $N_i$ may be constructed,

\[
\frac{\partial N_e}{\partial t} + \frac{\partial}{\partial x} (N_e V_{D,e}) + \alpha N_e^2 = \frac{F(x)}{kh} + \frac{j_i}{eh}
\]

\[
\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial x} (N_i V_{D,i}) + \alpha N_i^2 = \frac{F(x)}{kh}
\]

where $V_{D,e}$ and $V_{D,i}$ are the electron and ion drifts in the N-S ($x$) direction and E-W spatial gradients have been neglected. Here $\alpha$ is a volume dissociative recombination coefficient, $F$ is the electron precipitation heat flux, the ionization efficiency $k$ is equal to 35 ev/ion pair, and $j_i$ has been assumed to be carried completely by electrons. Furthermore, we assume that run-away electrons accelerated by a field-aligned potential do not carry any of the parallel current, and thus $j_i$ does not ionize the lower ionosphere.

We define the height-integrated current $I_z$ to be roughly $I_z = N_e eh(V_{D,i} - V_{D,e})$ where charge
Neutrality has been assumed. Subtracting (33) from (34), we arrive at a current continuity equation
\[ \partial I_x/\partial z = -j_0 \] (35)

We may relate \( I_x \) to the perpendicular electric fields by the conductivity law
\[ I_x = \sigma \mathbf{E}_x + \mathbf{N}(P \mathbf{E}_x + \mathbf{HE}_y) \] (36)

where in the second form of (36) the Pedersen and Hall conductances have been assumed to be proportional to the mean E-region ionospheric density.

Henceforth, the convection field \( \mathbf{E}_y \) will be assumed to be given, and \( \mathbf{E}_x \) will be assumed to arise from polarization created by field-aligned potential drops in the topside ionosphere. Since the electric field in the ionosphere is curl free to a very good approximation, we have \( \partial \mathbf{E}_y/\partial z = \partial \mathbf{E}_x/\partial z \). Then by defining \( \mathbf{E} = -\nabla \phi \), \( \phi_i \sim \int_{z_1}^{\infty} \mathbf{E}_x \, dz \), where \( z_1 \) denotes the top of the E region, and by integrating we find
\[ \int_{z_1}^{\infty} \partial \mathbf{E}_x/\partial z \, dz = E_x(\infty) - E_x(z_1) = \frac{\partial \phi_i}{\partial x} \] (37)

Then, again assuming that no polarization \( E_y \) exists in the magnetosphere so that \( E_y(\infty) = 0 \), we arrive at a relation between the polarization field in the lower ionosphere and the field-aligned potential drop
\[ E_x = \partial \phi_i / \partial x \] (38)

We now construct a simplified model for the effects of anomalous resistance. We expect \( E_i = \eta j_0 \), where \( \eta \) is the anomalous resistance. Similarly, since \( E_i = -\partial \phi_i / \partial x \), we also expect
\[ \phi_i = -r(x)j_0 \] (39)

where \( r(x) \) is the height-integrated resistance. In general, \( r \) will depend on \( |j_0| \). For example, the results of Kindel and Kennel [1971] indicate that \( r \) will remain zero until \( |j_0| \) exceeds a certain threshold of the order of \( 3 \times 10^6 \) el/cm² sec. Similarly, the laboratory results of Hamberger and Jancarik [1972] indicate that \( r \) increases significantly each time \( |j_0| \) increases to exceed the threshold of a stronger current instability, for example, the transition from ion acoustic to electron-ion beam instability.

Combining (38) and (39), we find
\[ E_x = -(\partial / \partial x)(\eta j_0) \] (40)

By using (35),
\[ E_x = \partial / \partial x[r \partial I_x / \partial x] \] (41)

Finally, by combining with (36) we find
\[ I_x = N\left( P \partial / \partial x r \partial I_x / \partial x + \mathbf{HE}_y \right) \] (42)

**Limit of small anomalous resistance.** We will examine the structure of these equations by assuming that \( r \) may be considered a small perturbation, which probably corresponds to the initial stages of electrojet polarization. In the absence of good information and for simplicity we assume \( r \) to be independent of \( x \). Since \( V_D \), is proportional to \( E_y \), a small quantity, (34) reduces in steady state and lowest order to
\[ N(x) = N(x) = \left( F(x)/k_b h \right)^{1/2} \] (43)

In the lowest order, (36) reduces to
\[ I_x = N(x) \mathbf{HE}_y \] (44)

so that \( j_0 \) is proportional to the derivative of the ionospheric density (from equation 36),
\[ j_1 = -\partial I_x / \partial x = -\mathbf{HE}_y \partial N / \partial x \] (45)

and the polarization field is proportional to the second derivative of the ionospheric density (from equation 41),
\[ E_x = r \partial^2 I_x / \partial x^2 = r \mathbf{HE}_y \partial^2 N / \partial x^2 \] (46)

We have consistently assumed \( \mathbf{E}_y \) to be independent of \( x \), an assumption corresponding to the assumptions that \( \mathbf{E} \) is curl free and that spatial variations in the E-W \( y \) direction may be neglected.

We may now substitute \( F(x) \) (from equation 32) into (43)–(46) to find the required spatial dependences,
\[ N(x) = \left( \frac{x}{x_T} \right)^8 \exp \left\{ -\frac{3\delta}{44} \left( \frac{x}{x_T} \right)^{4/3} - 1 \right\} \] (47)
\[ J_1(x) = 8 \left( \frac{x}{x_T} \right)^7 \left[ 1 - \frac{\delta}{8} \left( \frac{x}{x_T} \right)^{4/3} \right] \cdot \exp \left\{ -\frac{3\delta}{44} \left( \frac{x}{x_T} \right)^{4/3} - 1 \right\} \] (48)
where \( J_0(x) \) is the number flux of electrons carrying the parallel current.

Equations 47 and 48 indicate that the ionospheric density has a single maximum at \((x/x_r) \approx (8/3)^{2/4}\) of magnitude

\[
\frac{N(x)}{N(x_r)} = \left(\frac{8}{3}\right)^{6/11} \exp \left\{ \frac{3\delta}{44} - \frac{6}{11} \right\}
\]

(50)

Equatorward of the density maximum \((x > x_r(8/3)^{2/4})\), \( J_1(x)/J_0 \) is negative; for westward \( E_0 \) the negative corresponds to field-aligned currents flowing into the ionosphere, whereas poleward of the density maximum, the field-aligned currents are flowing out of the ionosphere. The current density maxima occur at the zeroes of (49) or approximately at the points \( \delta(x/x_r) \approx 2 \) and \( 88/3 \). Substituting in (48), we may compare the magnitudes of the poleward \( [\delta(x/x_r)^{2/4} \approx 2] \) and equatorward parallel current maxima. The maximum parallel current density flowing into the ionosphere at the equatorward edge of the auroral oval is roughly twice the maximum current density flowing out of the ionosphere poleward of the density maximum. Thus, as \( E_0 \) increases, the equatorward field-aligned currents will exceed the stability threshold first.

Equation 49 indicates that the polarization field \( E_0 \) has two zeroes at the field-aligned current density maxima. Consequently, this model contains three distinct polarization field regions. At \( x/x_r = 1 \) and at large values of \((x/x_r)^{2/4} \) \( E_0 \) is positive and corresponds to northward polarization and the eastward electrojet. The middle region, where the polarization field is southward and the electrojet is westward, corresponds to the normal electrojet.

**Estimate of physical magnitudes.** We now consider the normalization of \( N(x) \), \( J_1(x) \), and \( E_0(x) \) at \( x = x_r \), which corresponds to the last dipolar field line. Clearly,

\[
N(x_r) = \left[ F(x_r)/k ah \right]^{1/2}
\]

(51)

where, by defining the Hall conductance \( \Sigma_H(x_r) \)

\[
\frac{E_0(x)}{E_0} = 56 \left( \frac{x}{x_r} \right)^6 \left[ 1 - \frac{89}{168} \delta \left( \frac{x}{x_r} \right)^{44/3} \right] + \frac{\delta^2}{56} \left( \frac{x}{x_r} \right)^{88/3} \exp \left\{ -\frac{3\delta}{44} \left[ \left( \frac{x}{x_r} \right)^{44/3} - 1 \right] \right\}
\]

(49)

where \( J_0(x) \) is the number flux of electrons carrying the parallel current.

Equations 47 and 48 indicate that the ionospheric density has a single maximum at \((x/x_r) \approx (8/3)^{2/4}\) of magnitude

\[
\frac{N(x)}{N(x_r)} = \left(\frac{8}{3}\right)^{6/11} \exp \left\{ \frac{3\delta}{44} - \frac{6}{11} \right\}
\]

We now estimate the input conditions at \( L = L_r(x = x_r) \). The flow solution (22) scales with two parameters, \( L_r \) and \( \delta \), and estimates the minimum lifetime to be

\[
T_{\text{min}} = \frac{\pi L R_E}{B_0} \frac{B_r}{B(L_r)} = \frac{B_r}{B_0} \frac{\pi L^4 R_E}{a_s}
\]

(52)

\[
E_0 = r \frac{J_0}{x_r} \frac{\Sigma_H(x_r) E_v}{x_r^2}
\]

(53)

where \( B_r/B_0 \) is the ratio of the magnetic field in the auroral-oval ionosphere to the equatorial field, \( R_E \) is the earth radius, and \( a_s \) is the plasma-sheet electron thermal speed at \( L = L_r \). Scaling the electric field in space \( E_s \) to the ionospheric field \( E_0 \) by the approximate mapping relation,

\[
E_s = E_0 \left[ \frac{B(L_r)}{B_r} \right]^{1/2} = E_0 \left[ \frac{B_r}{B_r} \right]^{1/2} L_r^{-3/2}
\]

we find

\[
1 - \frac{\delta}{L_r} = \frac{\pi c E_0}{a_s B_r} \left( \frac{B_r}{B(L_r)} \right)^{3/2} = \frac{\pi c E_0}{a_s B_r} \left( \frac{B_r}{B_r} \right)^{3/2} L_r^{3/2}
\]

(54)

We may also scale \( J_0 \) and \( E_0 \) to the parameters \( \delta \) and \( L_r \). Using \( x_r = R_E L_r^{3/2} \) and (54), we find

\[
J_0 = \frac{\Sigma_H(x_r)a_s B_r}{\pi c R_E} \left( \frac{B_r}{B_r} \right)^{3/2} \frac{1}{\delta L_r^4}
\]

(55)

and

\[
E_0 = \frac{\tau \Sigma_H(x_r) a_s B_r}{\pi c R_E^2} \left( \frac{B_r}{B_r} \right)^{3/2} \frac{1}{\delta L_r^{7/2}}
\]

(56)

where \( J_0 \) has been written in precipitation units. Occasionally it is convenient to measure \( E_s(x) \) in units of the convection field \( E_s \); if \( E_s(x)/E_s = q(x) \) given by (49), then

\[
E_s = \frac{q \Sigma_H(x_r) L_r}{x_r^2} = \frac{q \Sigma_H(x_r) L_r}{R_E^2}
\]

(57)

In the absence of good models for the inner plasma sheet, we can only make plausible estimates for the quantities (equation 51 and
For example, if we choose the plasma-sheet number density \(n(L_T) = 0.3/cm^3\) and \(T(x_T) = 1\) kev, then \(a = 1.33 \times 10^4\) cm/sec and \(F(x_T) = 1\) erg/cm² sec. Then choosing \(k = 35\) ev/ion pair, \(h \approx 30\) km, and \(\alpha = 2 \times 10^{-7}/cm^3\) sec, we find \(N(x_T) = 1.7 \times 10^{4}/cm^3\).

On this basis \(\Sigma_H(x_T) = 1.55 \times 10^{14}\) esu \(\approx 17\) mhos. Then using \(B_R = 6.4 \times 10^4\) cm, \(B_i/B_R = 5/3\), we find

\[
\delta = (3.3 \times 10^{-5})/E_\delta L_T^{9/2}
\]

\[
J_0 = \frac{1.6 \times 10^{11}}{L_T^4 \delta} = 5 \times 10^{13}E_\delta(L_T)^{1/2}
\]

\[
E_\delta = 1.2 \times 10^{-7} \frac{r}{\delta L_T^{7/2}} \text{ esu/cm}
\]

\[
= \frac{3.6r}{\delta L_T^{7/2}} \text{ mV/m}
\]

where \(E_\delta\) is measured in electrostatic units per centimeter.

Assuming that \(a\) does not vary significantly, we can estimate a plausible range for the parameter \(\delta\). Let us consider two extreme cases. At quiet times \(E_\delta\) might be 10 mV/m and \(L_T\) might be 10, whereas during a developed growth phase \(E_\delta\) could increase to 50 mV/m and \(L_T\) decrease to 8. For these extremes \(\delta = 0.3\) and \(\delta = 0.2\). A plausible range is thus \(\delta = 1.0\) to 0.1 for very quiet to very disturbed conditions. Increasing \(E_\delta\) for fixed \(L_T\) decreases \(\delta\); decreasing \(L_T\) increases \(\delta\). These two effects tend to compensate each other somewhat in the growth phase. Although the shape parameter \(\delta\) may not vary strongly during the growth phase, (59) indicates that \(J_0\), which scales the field-aligned current distribution, ought to increase, since \(E_\delta\) increases and \((L_T)^{1/2}\) decreases only slightly during the growth phase. Thus, if the field-aligned currents are stable initially, a sufficiently long growth phase will increase them until they become unstable. The scalings (equations 58–60) do not reflect the fact that \(F(x_T)\) and, consequently, \(N(x_T)\) and \(\Sigma_H(x_T)\) may also increase because of compression of the plasma sheet. In fact, none of the rough scalings we have been able to deduce contribute to reducing the field-aligned current densities.

**Summary of results.** Figure 1 schematically summarizes the geometrical configuration under consideration. Figures 2, 3, and 4 describe in normalized units the variations of \(N(z)\), \(J_1(z)\), and \(E_\delta(x)\) derived from (47)–(49) for various values of \(\delta\). A value of a few tenths for \(\delta\) seems reasonable. The ionospheric density in Figure 2 rises to an increasingly larger and sharper maximum as \(\delta\) decreases. For \(\delta = 0.2\), \(N_{\max}/N(x_T) \approx 4\); for \(N(x_T) \approx 1.7 \times 10^4/cm^3\) this relation implies that \(N_{\max} \approx 7 \times 10^4/cm^3\), \(F_{\max} \approx 16\) ergs/cm² sec, and \(\Sigma_{H_{\max}} \approx 68\) mhos. The equatorward edge of the auroral oval, defined as the distance over which \(N(x)/N(x_T)\) returns to 1, is of the order of 0.15 \(x_T\), which, for \(x_T = 2000\) km, is roughly 300 km.

Figure 3 gives the spatial profile of \(J_1(z)/J_0\) for several values of \(\delta\). The region poleward of the density maximum contains positive \(J_1\) (current out of the ionosphere), and the region equatorward contains negative \(J_1\) (current into the ionosphere). The maximum equatorward \(\left|J_1\right|\) is roughly twice the polarward maximum \(\left|J_1\right|\). We can estimate the threshold convection electric field \(E_\delta\), which will just begin to create anomalous resistance, by setting \(J_{\delta}(z) = 3 \times 10^9\) el/cm sec, the rough stability limit calculated by Kindel and Kennel [1971]. With \(J_0\) given by (59), \(L_T = 9\), \(\delta = 0.2\), and \(J_{\delta}(z) = 30 J_0\); thus \(E_\delta \approx 20\) mV/m will begin anomalous resistance and electric-field polarization of the ionosphere in rough agreement with the results of Moser [1971].

Figure 4 describes the polarization electric field \(E_\delta/E_{\delta}\) if a spatially uniform value of \(r\) is assumed. Actually, \(r\) should be small \((E_\delta \approx 0)\) except where \(J_1 > 3 \times 10^8\) el/cm² sec.

We can estimate the anomalous resistance required to polarize the ionosphere by requiring \(|E_\delta/E_{\delta}| \approx O(1)\), the order of magnitude observed by Moser [1971]. For \(\delta = 0.2\), \(\phi_{\max} = 360\) (equation 57). For \(L_T \approx 10\) and \(\Sigma_H(x_T) \approx 1.5 \times 10^{14}\), \(r = 7.4\) esu. For this value of \(r\), the maximum potential drop along the field lines is \(|\phi_{\delta}(z)| = |rJ_{\delta}(z)| \approx 150 E_\delta\) volts, where \(E_\delta\) is measured in millivolts per meter. For the polarization threshold field, \(E_\delta \approx 20\) mV/m, and then \(|\phi_{\delta}(z)|\) is of the order of 3 kv.

For \(\delta = 0.2\), \(N(z)/N_0\), and \(J_1(z)/J_0\), Figures 5 and 6 plot the electric-field polarization ratio \(E_\delta/rE_{\delta}\) and the total normalized electrojet current

\[
\frac{I_\delta(x)}{I_0} = \frac{N(x)}{N_0} \left[1 - \frac{\Sigma_\delta(x_T) E_\delta}{\Sigma_\delta(x_T) E_{\delta}}\right]
\]
where $I_w = \Sigma_x(x_T)E_y$. $\Sigma_x(x_T)/\Sigma_y(x_T)$ has been estimated as 3. In Figure 5, $E_y = 30$ mv/m, and $r = 1$ was assumed only in the spatial region where $|J_x| > 3 \times 10^8$ el/cm$^2$ sec; $r = 0$ outside this region. Thus the sharp discontinuity in $E_x$ and $I_x$ was produced. Westward electrojet polarization occurs only at the equatorward edge with $I_x$ enhanced by roughly 2–4 over its unpolarized level; the electrojet width is roughly 100 km. A very weak eastward electrojet occurs equatorward of the westward electrojet. In Figure 6 $E_y = 50$ mv/m, and $r = 2$ was assumed in the polarization regions ($|J_x| > 3 \times 10^8$ el/cm$^2$ sec). A strong westward electrojet with $I_x$ enhanced by 2–7 times its unpolarized level occurs at the equatorward edge. A weaker westward electrojet also occurs at about 150 km poleward of the equatorward electrojet, since the outward $J_x$ also exceeds the instability threshold. Kisabeth and Rostoker [1971] have observed separate equatorward and poleward electrojets during substorm expansion phase when, presumably, the convection electric field is large. However, the steady-state ionospheric density model assumed here is undoubtedly a poor approximation to expansion phase conditions; hence the double electrojet in Figure 6 should be considered as only indicative that multiple polarization and electrojet regions are possible when the field-aligned currents exceed instability threshold.

The detailed variations shown in Figures 5 and 6 are probably not trustworthy, primarily because $E_y(x)$ depends on the second derivative of the ionospheric density and, ultimately, on the second derivative of the plasma-sheet heat flux $F$. Here inaccuracies in the precipitation model are crucial. For example, near the poleward edge ($x \approx x_T$) the dipole magnetic-field model undoubtedly becomes inaccurate; at the equatorward edge the precipitation model again becomes questionable, owing, for example, to the breakdown of the adiabatic temperature law. Nevertheless, as long as the heat flux into the ionosphere has a maximum, the qualitative features of Figures 4 and 5 should be preserved.
Fig. 2. Normalized ionospheric electron density profiles computed from (47) for various values of the convection parameter $\delta$ (equation 23). The density $N(x)$ is normalized to $N(x_T)$, the electron density produced by the plasma-sheet electron heat flux at $x = x_T$. All distances $x$ are normalized to $x_T$, the distance from the geomagnetic pole of the last dipolar field line. If the last dipolar line corresponds to 20° colatitude, then $x_T \approx 2000$ km. A value of a few tenths for $\delta$ seems reasonable. Consequently, the ionospheric density increases by a factor of 2-4 in a few hundred kilometers and then diminishes rapidly. This density variation could be difficult to observe directly.

DISCUSSION

Although strictly speaking the analysis discussed above is severely limited by our assumed ionospheric model, many of its conclusions should have a wider qualitative validity. The electron precipitation heat flux is both observed [Frank and Ackerson, 1971] and theoretically expected [Kennel, 1969] to be spatially inhomogeneous within the nightside auroral oval. The oval ionospheric plasma density should, therefore, have spatial gradients, probably in both latitude and longitude, our single-density maximum model being the simplest possibility. The predominantly westward (near midnight) convection electric field drives a poleward Hall current, which, because of the inhomogeneous Hall conductance, cannot be divergence free in the ionosphere. Longitudinal gradients of the Pedersen conductance will also enhance the Pedersen current divergence. Field-aligned currents can flow into (out of) the ionosphere equatorward (poleward) of any density maximum to maintain the total current divergence free.

Such field-aligned currents may flow freely in space provided that near perfect electrical conduction exists along the field lines. The current circuit is presumably closed via as yet undetermined pressure-gradient drift currents in the magnetosphere. However, for large convection electric fields and/or sharp ionospheric density gradients, the field-aligned currents can exceed the threshold for instability in the topside ionosphere [Kindel and Kennel, 1971]. The unstable plasma turbulence should produce an anomalous resistance that partly blocks the current. A parallel electric field is then required for the flow of current. Since the Hall current divergence no longer flows freely, a southward (in the northern hemisphere) polarization electric field develops in the lower ionosphere to reduce the Hall current. The coupled convection and polarization electric fields drive a westward Cowling current electrojet, which, when $\Sigma_n/\Sigma_r$ is large, can greatly exceed the
unpolarized westward current. If several density maximums exist within the oval and if their associated field-aligned currents are unstable, multiple polarization regions and electrojets should occur.

Although admittedly an idealization, our single-density maximum model does delineate several physical trends in the ionospheric response to substorm growth phase. The gradual buildup of the convection electric field [Moser, 1971] enhances the ionospheric currents [McPherron, 1970] and therefore the field-aligned currents stemming from the divergence of the Hall current. The increased tail flaring stress results in the inward motion of the electron plasma-sheet inner edge [Siscoe and Cummings, 1969; Coroniti and Kennel, 1972b], which, since the electrons penetrate deeper into the dipole field, raises the electron precipitation heat flux. The resulting enhancement of the ionospheric conductances further intensifies the field-aligned and ionospheric currents. In our model the sharp inner-edge spatial gradient of the precipitation heat flux produces the largest field-aligned currents at the equatorward edge of the auroral oval. Thus the enhancement of convection and magnetosphere configurational changes during the growth phase drive the equatorward field-aligned currents toward instability. Even without an impulsive convective collapse of the tail, the field-aligned currents should exceed instability threshold sometime during the growth phase provided that the convection electric field and the ionospheric conductances increase. The subsequent equatorward directed polarization electric field and Cowling current westward electrojet are typical ground-based signatures of substorm expansion phase.

A rapid tail collapse would also drive the field-aligned currents to instability and produce electrojet polarization. Hence there may be two
types of substorm breakups: an adiabatic breakup that results solely from the gradual growth-phase changes in the magnetosphere and an impulsive breakup associated with a rapid convective tail collapse. Carefully timed correlation studies between ground and satellite observations are needed to distinguish whether one or both types of breakup occur.

Semiquantitative estimates based on the precipitation-ionospheric density model indicate that the typical growth phase westward of the electric fields of 20–30 mV/m measured by Mozer [1971] are sufficient to produce field-aligned current instability and electrojet polarization at the equatorward edge of the auroral oval. For larger electric fields \( E_x \sim 50 \text{ mV/m} \) a second westward electrojet may occur one to several hundred kilometers poleward of the equatorward electrojet. From the observed ratio of the polarization to convection electric field \( |E_p/E_x| \sim 0(1) \) [Mozer, 1971], the required parallel potential drop across the anomalous resistance region is of the order of 1–5 kV. The parallel electric field accelerates ions into the ionosphere at the equatorward edge. The anomalous Joule dissipation in the topside ionosphere is of the order of 1 erg/cm² sec, which is comparable to the total Pedersen dissipation. This value represents a non-negligible fraction of the incident electron precipitation heat flux and could considerably modify the topside ionosphere.

Our simplified precipitation–ionospheric density model is clearly inadequate for the expansion phase. First, the structure of auroral arcs and their effects on the ionospheric and field-aligned currents should be included. In fact,

![Image]
Recent rocket measurements of large field-aligned currents in arcs [Vondrak et al., 1971; Park and Cloutier, 1971] indicate that perhaps the entire divergence of the Hall current may flow inside the arcs. Thus arc structure may crucially affect electrojet polarization. Second, as in laboratory experiments, anomalous topside resistance probably produces run-away electrons. At the equatorward edge, run-away electrons are accelerated into space and possibly into the conjugate ionosphere, where they may substantially contribute to ionospheric ionization. Since the parallel potentials required for polarization are of the order of several kilovolts, the run-away electrons possibly constitute the several keV electron beams often observed during breakup. Third, the anomalous resistance region undoubtedly does not completely prevent the polarization electric field from penetrating into the magnetosphere, as is assumed here. The polarization electric field and any associated field-aligned divergence of the Cowling electrojet into space should modify the convection pattern and the magnetospheric current distribution. Thus the reaction of the magnetosphere to electrojet polarization needs to be evaluated. Finally, expansion phase is longitudinally asymmetric, so that the E-W ionospheric density gradients and current divergences must also be considered. Our analysis suggests that, when the field-aligned currents are near the instability threshold, local measurements of the electric field in the lower ionosphere may not yield an accurate picture of the over-all convection pattern, since local ionospheric conductance inhomogeneities will produce local electric-field polarizations.

During magnetic storms the plasma-sheet inner edge may penetrate far into the dipole, and large convection electric fields are expected [Coroniti and Kennel, 1972a, b]. Hence \( E_x \sim 8 \) and \( E_z \sim 6 \times 10^4 \) esu/cm (the largest polar-cap field observed by Cauffman and Gurnett [1971]) might be reasonable; from (58) and (59) 8 \( \approx 0.05 \) and \( J_z \approx 8 \times 10^8 \) el/cm² sec. The field-aligned currents are now unstable nearly everywhere in the oval, and electrojet polarization should now occur over virtually the entire nightside auroral oval. Should intense polar-cusp precipitation fluxes maintain a very high dayside auroral-oval Pedersen conductance during storms [Russell et al., 1971b; Frank, 1971], large electromagnetic inertia of dayside line-tying could prevent rapid changes in the convection rate [Coroniti and Kennel, 1972a], and thus, once polarization is established, the nightside oval may remain polarized. Hence almost continuous electrojet currents might be expected during the main phase. The intense Joule dissipation from the field-aligned current instabilities should then greatly modify the structure of the topside ionosphere and perhaps change the basic physics of ionosphere-magnetosphere interactions.

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