Non Self-Similar Scaling of Plasma Sheet and Solar Wind Probability Distribution Functions of Magnetic Field Fluctuations

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Abstract

Solar wind magnetometer and plasma data and magnetometer data acquired by Cluster in the magnetospheric plasma sheet are employed to construct probability distribution functions (PDFs) of magnetic field fluctuations over various temporal and spatial scales. This technique, often used in analysis of laboratory plasmas, is used to look for intermittent plasma turbulence and non self-similar properties in the fluctuations. We examine the distribution of the magnetic field fluctuations for a single spacecraft and between pairs of correlated spacecraft time-series data in both the plasma sheet and the solar wind (fast and slow speed streams). We demonstrate that the plasma sheet fluctuations have a distribution consistent with intermittent turbulence and that they scale non self-similarly using two different methods [Sorriso-Valvo et al., 1999; Hnat et al., 2002]. In the solar wind both methods show that the fast solar wind magnetic field fluctuations have a PDF consistent with intermittent turbulence and scale non self-similarly, but the two methods give conflicting results for the slow solar wind scaling properties. Finally, we use the results of the two methods as well as kurtosis versus scale size to roughly determine the turbulent eddy scale size in both types of solar wind (about 500 RE) and the magnetospheric plasma sheet (between 0.9 and 1.5 RE).
1. Introduction

Electromagnetic energy is stored in the magnetotail lobes and transferred to the plasma sheet and the most likely mechanism for this transport is reconnection. The result of this reconnection is flows and magnetic field fluctuations that look turbulent. Understanding if these fluctuations are turbulent is important for understanding the physics of this transportation of lobe electromagnetic energy to the plasma sheet. However, measurements to characterize spatial and temporal variations independently and ascertain the role of PS turbulence in energy transport remain incomplete.

The presence of turbulence in the solar wind has been established by using different methods, some of which analyze probability distribution functions (PDFs) of flow and magnetic field fluctuations [Marsch and Tu, 1994; Sorriso Valvo et al., 1999; 2001]. Many of these same methods have been employed to demonstrate the presence of turbulent fluctuations in the magnetospheric plasma sheet [Angelopoulos et al., 1999; Borovsky et al., 1997; and Weygand et al., 2005]. Qualitatively, the PDFs in turbulent plasma are approximately Gaussian at large scales and become increasingly peaked, with high probability of large fluctuations in the wings at smaller scales. In terms of the kurtosis, which is the fourth moment of the distribution and is associated with intermittent turbulence [Angelopoulos et al., 1999], the PDFs have a large kurtosis (lepokurtic) at small scales and become mesokurtic at large scales (i.e., they approach a kurtosis of 3). Both Marsch and Tu [1994] and Sorriso-Valvo et al. [1999; 2001] demonstrated this variation of the PDFs with scale size using Helios data in the solar wind. The non-linear variation of the PDF with scale size is associated with non self-similar scaling related to turbulence and has been observed in turbulent fluid flows [Castaing et al., 1990].

Sorriso-Valvo et al. [1999; 2001] quantified the change in the PDFs by fitting them with a model developed by Castaing et al. [1990] that demonstrates how model parameters vary with scale size. The Castaing et al. [1990] model was developed from the classical Kolmogorov’s picture of turbulent energy cascade [Kolmogorov, 1941] and took into account that the energy transfer rate in the cascade at different scales is not strictly self-similar. Their model, which fitted a PDF of flow fluctuations $\delta \psi$ at a given
scale \( \tau \), is a convolution of a Gaussian and a log-normal distribution or equivalently is the sum of weighted Gaussian distributions with a log-normal distribution as the weighting function:

\[
\Pi_\lambda (d\Psi) = \frac{A(a_s)}{2\pi \lambda} \int_0^\infty \exp \left[ -\frac{(d\Psi)^2}{2\sigma^2} \left( 1 + a_s \frac{d\Psi / \sigma}{1 + (d\Psi)^2 / \sigma^2} \right) \right] \exp \left( -\frac{\ln^2(\sigma / \sigma_0)}{2\lambda^2} \right) d\sigma \quad (1)
\]

Here \( A(a_s) \) is a normalization constant, \( a_s \) is a skewness factor to account for asymmetry, \( \sigma_0 \) is the most probable value of \( \sigma \), \( \lambda \) is the width of the log-normal distribution, and \( \sigma \) is the standard deviation for a Gaussian distribution. The Castaing et al. [1990] model was first applied to high Reynolds number turbulent flow. PDFs of the flow fluctuations varied from peaked distributions with high probabilities in the wings at relatively small spatial separation to more Gaussian distributions at larger spatial separations. Castaing et al. [1990] were able to fit the distributions qualitatively out to about 10 standard deviations.

Using an approach similar to that of Sorriso-Valvo et al. [1999], Burlaga and Viñas [2004] examined the variation in the PDFs of the ACE plasma flow data, but instead of using the Castaing model they considered a generalized Tsallis distribution, which is frequently used in nonextensive statistics. In extensive statistics the total energy of the system is proportional to the size of the system and long rang interactions can be ignored. This is not necessarily the case in nonextensive statistics. The findings of their study were similar to those of Sorriso-Valvo et al. [1999]. In addition, this Burlaga and Viñas study showed that with increasing scale size, the kurtosis decreases and reaches a value of 3 at temporal separations of about 280 hr.

The fact that PDFs of fluctuations are non-Gaussian at small scales and do not scale self-similarly is thought to indicate the presence of intermittent turbulence. Hnat et al. [2002; 2003] took the concept of non self-similarity farther by demonstrating that PDFs of the solar wind magnetic field fluctuations do not rescale linearly, as is required for self-similarity. However, they were able to linearly rescale the ion density, energy density \( B^2 \) and \( \rho v^2 \) as well as the Poynting flux in the MHD limit. To show non self-similarity, they examined the change of the PDFs peak versus the scale size and determined that the scaling exponential is a non-linear function.
1.1 PDFs of Plasma Sheet Turbulence

Turbulence has been identified in the magnetospheric plasma sheet as well as in the solar wind. For plasma sheet studies different methods that have been used include statistical examinations of PDFs of flow and magnetic field and the fluctuations in these measurements [Borovsky et al., 1997; Angelopoulos et al., 1999; Petrukovich and Yermolaev, 2002; Weygand et al., 2005]. Most of these studies presented evidence of non-Gaussian PDFs of flow and magnetic field fluctuations from which they concluded that the plasma was turbulent. In particular, Borovsky et al. [1997], during 10 plasma sheet traversals, examined the distribution of $V_x$ and $V_y$ flow observations made by ISEE-2 spacecraft. The ISEE-2 two dimensional Fast Plasma Analyzer did not regularly obtain the $V_z$ component in the plasma distribution function. The distributions of the $V_x$ and $V_y$ components were well fit with an exponential function. Borovsky et al. attribute the high probability of large flow values in the wings (associated with bursty bulk flows (BBFs)) to eddy turbulence.

The flow distributions were also analyzed by Angelopoulos et al. [1999] and Petrukovich and Yermilaev [2002]. Angelopoulos et al. [1999] examined the distribution of $V_x$ and $V_y$ components of the flow for BBFs, non-BBFs, and the combined data set using Geotail/LEP data. The $V_z$ component was not examined and it is not clear why not. In addition to showing the non-Gaussian shape of the distributions, Angelopoulos et al. [1999] also fit the distributions with the Casting et al. [1990] model. They found the model was a good fit for the distributions of the BBF flows and the non-BBF flows, but not for the distribution of the two combined. Using the CORALL instrument on the Interball Tail spacecraft, Petrukovich and Yermilaev [2002] examined the $V_y$ and $V_z$ flow components. The $V_x$ component values were considered to be less accurate than the other two components because the ion flux measurements were not measured within $\pm 16^\circ$ of the sun pointing spin axis. This study did not attempt to fit the distribution with any model, but they did determine the kurtosis of the distributions and found values greater than 9 in all components.

Also of interest for this study is the Angelopoulos et al. [1999] observation of the non-Gaussian PDF of $V_y$ difference between two spatially separated spacecraft, Wind and Geotail. We believe this study is the first study in which two spacecraft were used to
identify a non-Gaussian PDF. The statistics of two point measurements are a reliable indicator of turbulent fluctuations because turbulence relates to the structure of fluctuations in space. Single spacecraft measurements provide statistics of temporal fluctuations, which give information on spatial fluctuations if the Taylor hypothesis applies. The assumption is valid only if the fluctuations evolve slowly with respect to the time required for the plasma to flow past the spacecraft (i.e., the fluctuations must be frozen into the flow).

As far as we are aware, only one other study has examined non self-similarity of PDFs of magnetic field fluctuations in the Earth’s plasma sheet. Weygand et al. [2005] used a single Cluster spacecraft to demonstrate qualitatively the variation of the PDFs from a highly peaked PDF with large probabilities in the wings at small scale sizes to Gaussian at large scale sizes. They support their observations by demonstrating that the kurtosis systematically decreases with increasing scale size and by indicating that the PDFs cannot be linearly rescaled with the method introduced by Hnat et al. [2002].

In this study, we use multiple spacecraft and two different analysis methods to show that the PDFs of the magnetic field fluctuations scale non self-similarly over a range of temporal and spatial separations in both the fast solar wind and the plasma sheet. In section 2 we describe the data used in this study. In section 3 we outline the procedure for constructing PDFs from a single spacecraft and from spacecraft pairs and describe how the two types of PDFs can be compared. The Sorriso-Valvo et al. [1999] method of showing non self-similarity is applied in section 4. In that section, we first follow previous investigators by using a single spacecraft to show non self-similarity in the solar wind. Then we demonstrate that analogous results can be obtained with multiple spacecraft over a range of spatial separations. Finally we use the multi-spacecraft aspect of Cluster to show that non self-similar scaling of PDFs is found within the plasma sheet under specific conditions. In section 5, we apply the method of Hnat et al. [2002] to demonstrate non self-similar properties of the fast solar wind and the plasma sheet. This method shows that the slow solar wind has self-similar scaling properties, a result that conflicts with the results of Sorriso-Valvo et al. [1999]. Finally, in section 6, we discuss the similarities and differences between the results of this study and previous studies.
2. Data Sources

Data from many spacecraft were used for this study. The magnetic field and plasma measurements within the plasma sheet were obtained from the Cluster spacecraft, the solar wind plasma and magnetic field data were obtained from the Advanced Composition Explorer (ACE), Cassini, Galileo, Geotail, IMP-8, and Wind. Cross calibration has been undertaken for the four Cluster spacecraft instruments but no such comparison has been applied to the instruments on the other spacecraft. Offsets that may exist among the instruments are small (i.e., a fraction of a nanoTesla) and are subtracted from the time series data during the data processing. These offsets are not significant to this study. The removal of the offsets does not change the results of this study.

The Cluster mission, supported jointly by the European Space Agency (ESA) and NASA, consists of four identical spacecraft, optimally in a tetrahedral configuration, with a perigee of 4 RE, an apogee of 19.6 RE, and a spin period of 4 s. These four spacecraft are providing the first three-dimensional measurements of large- and small-scale phenomena in the near-Earth environment [Escoubet et al., 1997]. Each Cluster spacecraft carries 11 instruments. This study uses data from the magnetometer (FGM) [Balogh et al., 1997] and the ion spectrometer (CIS) [Reme et al., 1997]. The Cluster spacecraft orbital plane precesses around the Earth annually. From 2001 to 2003 the Cluster spacecraft apogees were in the magnetotail between July and October. At apogee the spacecraft were in a nearly regular tetrahedron. In the magnetotail seasons of 2001 and 2002 the tetrahedron’s spacing was 1000 km and 5000 km (i.e., on the order of the inertial range for turbulence within the plasma sheet), respectively. The latter spacing is ideal for examining turbulent eddy scale sizes that are on the order of 5000 to 20,000 km [Neagu et al., 2002; Weygand et al., 2005]. From July to October 2003, Cluster obtained another series of plasma sheet crossings at an inter-spacecraft spacing of about 100 km (i.e., on the order of dissipation range).

Each Cluster spacecraft carries a boom mounted triaxial fluxgate magnetometer [Balogh et al., 1997]. Magnetic field vectors routinely are available at 22 Hz resolution (nominal mode). Both pre-flight and in-flight calibrations of the two magnetometers have been performed to produce carefully calibrated (and inter-calibrated) magnetic field data. The relative uncertainty in the data after calibration is at most 0.1 nT, an estimate
determined by examining the drift in the offset after calibration [Khurana, and Schwarzl, private communication, 2004]. The digital resolution of the magnetometer is on the order of 8 pT [Balogh et al., 1997].

The CIS instrument [Reme et al., 1997] plays a key role in identifying periods when Cluster enters the PS, which is characterized by high electron and ion temperatures as well as significant H⁺, He⁺, and O⁺ populations. CIS provides fundamental plasma parameters such as density, velocity vectors, pressure tensor, and heat flux. For this study the uncertainty in the density and temperature is not critical. In order to identify the plasma sheet we need only to know when the values significantly increase or decrease, times that identify when Cluster enters and exits the plasma sheet. The uncertainty of the velocity is approximately ± 2 km s⁻¹ in the x and y components, but approximately ±15 km s⁻¹ in the z direction [Kistler, private communication, 2004]. The Vz uncertainty was determined by assuming the mean Vz in the CPS is zero.

The twin ACE magnetometer instruments measure the local interplanetary magnetic field direction and magnitude in the upstream solar wind at the first libration point. The triaxial fluxgate magnetometers are capable of a very wide dynamic range of measurements from ±4 nT to ±65,536 nT per axis in eight discrete ranges with an uncertainty of about 0.025% in each range [Smith et al., 1998]. These magnetometers have a temporal resolution of six vectors per second. The Solar Wind Electron, Proton, and Alpha Monitor (SWEPAM) measures the solar wind plasma electron and ion fluxes with separate sensors [McComas et al., 1998]. Both sensors use electrostatic analyzers to provide plasma density, velocity, and thermal velocity measurements every 64 s.

Cassini passed by the Earth in mid-August, 1999 on the way to Saturn. Data from the triaxial fluxgate magnetometer at about 32 Hz resolution are used in this study. These data have an uncertainty of about 0.1% in the gain and 0.1° in the pointing direction [Dougherty et al., 2005].

The IMP-8 spacecraft orbits the Earth in a near circular orbit at about 35 RE with a spin rate of 24 rpm. The magnetic field instrument (MAG) consists of a boom-mounted triaxial flux-gate magnetometer designed to study the interplanetary and geomagnetic tail magnetic fields. On July 11, 1975, because of a range indicator problem, the MAG
experiment operation was frozen into the 36-nT range. The digitization accuracy in this range is about ±0.3 nT and the intrinsic sensor noise level is 0.03 nT RMS. See http://sd-www.jhuapl.edu/IMP/imp_merge_user_info.html for more information. The plasma data are obtained from the solar Faraday plasma cup (PLS). Measurements of the ion density, temperature and flow are made once every 20 s. See ftp://space.mit.edu/pub/plasma/imp/fine_res for additional data.

The Galileo spacecraft passed by the Earth twice while en route to Jupiter. For this study we used magnetic field measurements from the two triaxial fluxgate magnetometers. Data from the Earth fly-bys have a resolution of 5 Hz with an uncertainty of about ±0.03 nT in the range of ±512 nT common to the two sensor triads. [Kivelson et al., 1992]

Like IMP-8, Geotail orbits close to the Earth with an apogee of about 30 R_E located in the solar wind during the summer months. Magnetic field data were obtained from the triaxial fluxgate magnetometer (MGF) with a resolution of about 3 s and an uncertainty of about ±0.1 nT [Kokubun et al., 1994]. Solar wind plasma flow vectors were measured by the low energy particle (LEP) experiment with a resolution of about 12 s. [Mukai et al., 1994].

Our study used two instruments on the Wind spacecraft: the magnetometer (MFI) and the solar wind plasma instrument (3DP). The triaxial fluxgate magnetometers, mounted on a boom away from the spacecraft body, have a range ±65536 nT and temporal resolution of 3 s [Lepping et al., 1995] and provided IMF direction and magnitude. The Wind plasma instrument measures the solar wind ions and electrons with an energy resolution of ΔE/E of 0.2 [Lin et al., 1995] and a temporal resolution of 3 s. This instrument was used to determine the solar wind speed.

3. Procedure

In this section we outline the procedure used to construct the PDFs of magnetic field fluctuations with data from just one spacecraft or from two spacecraft and explain how we obtain the log-normal variance ($\lambda$) values from the Castaing et al. [1990] fit.
3.1 Single Spacecraft Probability Distribution Functions.

From single spacecraft measurements, one can construct the PDFs of temporal fluctuations of the magnetic field. The data used to calculate the magnetic field fluctuations have been filtered with a Savitsky-Golay filter [Orfanidis, 1996] to remove the large scale background features. This procedure is especially important for the plasma sheet data where the signature of the large scale structure of the magnetotail dominates some of the magnetic field components. Once the data have been filtered we calculate the fluctuations over time $\tau$ in the $ith$ component of the magnetic field from

$$\delta B_i(\tau, x) = B_i(t + \tau, x) - B_i(t, x).$$

The fluctuations are normalized to the standard deviation of the interval. PDFs are then constructed from the fluctuations and the PDF is fit with the Castaing et al. [1990] model (equation (1)) to obtain $\lambda^2$. A reduced chi-square test was performed to determine the goodness of the fit. All the PDFs in this study were well fit with the Castaing et al. model. To establish whether non self-similar scaling is present, we must determine how $\lambda^2$ varies with scale size.

3.2 Two Spacecraft Probability Distribution Functions

The use of PDFs from single spacecraft measurements for tests of turbulence is based on an assumption that the fluctuations in the solar wind are frozen into the flow. If the assumption is valid, the results obtained from measurements of temporal fluctuations should not differ from those obtained by making two-point measurements, i.e., if the Taylor hypothesis applies, then PDFs of $\delta B_i(\tau, x)$ should be the same as PDFs of $\delta B_i(t, v_{plasma} \tau)$. Observations of the solar wind at separations of several hundred $R_E$ demonstrate that this assumption is reasonable [Ridley, 2000]. Nonetheless, tests of the validity of using $\tau$ versus $\Delta x$ have not been demonstrated by using multiple spacecraft with widely differing spatial separations. We can directly measure the fluctuations across space; however, we still need to assume that the fluctuations are occurring in a homogeneous plasma, which we take to mean that the plasma that we are measuring flows from spacecraft$_1$ to spacecraft$_2$. This may be a reasonable assumption in the solar wind, but may not always be the case within the plasma sheet. After the application of the Savitsky-Golay filter the magnetic field fluctuations are determined as
\[ \partial B_i(\Delta x, t) = B_{2i}(x_i + \Delta x, t) - B_{1i}(x_i, t) \]  

where the scale \( \Delta x \) is a spatial separation between the two spacecraft and the first subscript on \( B \) identifies the spacecraft. From here the procedure is the same as section 3.1; PDFs of the normalized fluctuation are constructed and fit to obtain values for \( \lambda^2 \), but this time for a range of spatial separations.

To compare the results of the two procedures, the PDFs determined for temporal separations need to be mapped into the equivalent spatial separation PDFs. We assume that the Taylor hypothesis applies for the single spacecraft measurements and time and space are related through velocity \( \nu = (\Delta r_2 - \Delta r_1)/t \). The equivalent spatial separation is then found by using the mean flow measured at the first spacecraft and the temporal separation in the fluctuations.

4. Observations

4.1 Solar Wind Probability Distribution Functions

Figure 1 and Figure 2 show a decrease of \( \lambda^2 \) (the grey circles) with increasing temporal separation for the slow solar wind and the fast solar wind, respectively. The triangles identify the two sigma uncertainty in the value of \( \lambda^2 \). These results were obtained by using the Wind magnetic field measurements for single period of slow solar wind (< 450 km/s) from May 17 to May 26, 1998 and fast solar wind (> 500 km/s) on Feb 24, to Feb 29, 2000. These slow and fast solar wind ranges are similar to the ranges of solar wind defined in Sorriso-Valvo et al. [1999] and similar to those determined by the solar wind O/Fe solar wind abundance examined by Aellig et al. [1999] using solar wind CELIAS composition data from SoHO. Our results for the solar wind reproduce those of Sorriso-Valvo et al. [1999] as seen in Figure 1. For the fast solar wind, Figure 2 shows that our results follow the trend of the Sorriso-Valvo et al. study but differ in magnitude. This difference is most likely due to the number of samples in the PDFs. The Sorriso-Valvo et al. study combines many periods of slow and fast solar wind flow to obtain many samples, whereas our study examines only a continuous nine day interval of slow solar wind and a five day interval of fast solar wind. By using a single period we eliminate the possibility that the characteristics of the turbulent fluctuations are different.
in each slow solar wind intervals. It is likely that $\lambda^2$ will increase if such changes are present.

Figure 3, which has a format similar to that of Figures 1 and 2, demonstrates a decrease in $\lambda^2$ (blue circles) for slow solar wind periods for increasing spatial separations. This plot used several slow solar wind periods: November 23-26, 1990 (Galileo Earth fly-by with blue text); May 17-26 (red text), 1998, and June 12-21 (black text), 1998. The two periods from 1998 include solar wind magnetic field measurements from 4 different spacecraft (ACE, Geotail, IMP-8, and Wind). In this plot we have used several different intervals and we have assumed that the characteristics of the turbulent fluctuations in each interval are similar. Otherwise we would not have a wide enough range of spatial separations to examine in each individual interval. Again, we have plotted the results from the Sorriso-Valvo et al. [1999] study (black squares) to demonstrate the similarity in the magnitude (despite the different intervals used) and trend in $\lambda^2$. Equivalent spatial separation values for the Sorriso-Valvo et al. [1999] study were obtained using a solar wind speed of 420 km/s, which is approximately the mean solar wind speed for a year’s worth of data from 1998, and adopting the Taylor hypothesis.

Figure 4 presents results for the fast solar wind and also indicates that the $\lambda^2$ of the log-normal variance (blue circles) decreases with increasing spatial separation. Again, the black squares are the results from Sorriso-Valvo et al. [1999] for the fast solar wind where we have assumed a solar wind speed of 550 km/s to identify the equivalent spatial separation. Whereas the general trend in the data is for $\lambda^2$ to decrease with increasing $\Delta x$, the first two values of $\lambda^2$ do not support the trend. We believe that our estimates of $\lambda^2$ based on the Geotail IMP-8 spacecraft pair in Figures 3 and 4 are low. The magnetic field measured on Geotail has an uncertainty of 0.1 nT [Kokubun et al., 1994]. The magnetic field is measured on IMP with a quantization uncertainty of 0.3 nT. [See http://sd-www.jhuapl.edu/IMP/imp_merge_user_info.html.] The uncertainty in the Geotail magnetic field data is similar to that of data from most other solar wind spacecraft, but the uncertainty for IMP-8 is higher than average (however, remarkable for a spacecraft that has supplied magnetic field data for about 27 years). Large amplitude noise may dominate the ambient fluctuations and lead to a PDF that is more Gaussian than if the noise level were smaller. In Figure 5 we illustrates how adding a known
amount of noise affects the PDFs and decreases the magnitude of the kurtosis. Shown in the top left panel of Figure 5 are the fluctuations of the $B_x$ component, which have a mean amplitude of 0.5 nT, from the ACE measurements for a small temporal separation in the data. On the upper right is the normalized PDF of those fluctuations whose kurtosis is 8.8. The distribution has a sharp peak and resembles a double exponential distribution. The second row is the PDF from the same data with 0.3 nT Gaussian distributed noise added to the fluctuations. The second row shows that the resulting PDF is more rounded at the peak and the kurtosis has decreased to 6.8 (i.e., a 22% decrease). The bottom two rows repeat the comparison with a large temporal separation in the data. In the third row the mean amplitude of the fluctuations is much larger and the PDF has a kurtosis of 3.7. In the bottom row the 0.3 nT noise has been added without affecting the kurtosis. From this simple experiment, we conclude that the larger amplitude noise in the IMP-8 magnetometer decreases the value of the log-normal of the variance of the PDF at small temporal or spatial scale sizes, but does not affect the fluctuations when the scale size of the separation is large. This suggests that the two estimates of $\lambda^2$ for small separations in Figures 3 and 4 are underestimates, while the spacecraft pairs that use IMP-8 at large separations are not significantly affected.

Figures 1 and 2 help demonstrate that we are able to reproduce the trends obtained in the Sorriso-Valvo et al. [1999] study and Figures 3 and 4 indicate that we obtain analogous results with two point measurements at different spatial separations over the range of spacings for which measurement limitations do not affect the analysis.

Another approach to understanding the properties of the PDFs used to obtain Figures 1 to 4 is to calculate the kurtosis of the PDFs and to establish its dependence on $\Delta x$. The kurtosis indicates if there is a high probability of large fluctuations in the wings of the PDFs. Figure 6 is a plot of the kurtosis versus spacecraft separation for the slow solar wind. The blue squares are the results for fluctuations observed between two spacecraft and the red circles are the results determined from one spacecraft and mapped from time into space as previously described. The black line joins points obtained by the two methods to allow the reader to compare values of the kurtosis. For each pair of points we have indicated which spacecraft pair was used to calculate the kurtosis; the first spacecraft in the pair is the one used for the single spacecraft measurements. A vertical
black dashed line shows the Matthaeus et al. [1986] estimate of the minimum turbulent eddy scale size. Figure 7, for the fast solar wind, has the same format as Figure 6. In both figures the kurtosis decreases from some initial large value at small spacecraft separation to a value of about 3 for large spacecraft separation of about 500 $R_\oplus$. If the kurtosis of the fluctuation is thought to become 3 at spatial separations larger than a turbulent eddy scale size, then the separation of about 500 $R_\oplus$ can be interpreted as roughly the scale size of the largest turbulent eddies. For both Figures 6 and 7 the mean difference between the single spacecraft kurtosis and the two point method kurtosis is about 12% where the largest difference is 29%.

4.2 Plasma sheet Probability Distribution Functions

Figures 8 and 9 display the dependence of $\lambda^2$ on temporal and spatial separation of the magnetic field fluctuations in the plasma sheet. These figures have the same format as Figures 1 and 3. Local peaks in $\lambda^2$ are seen for time differences of about 0.03 and 0.086 hr (Figure 8), which corresponds to frequencies of 9.1 and 3.2 mHz. It is not clear what causes these peaks. Walker [2005] has indicated that density fluctuations in the solar wind may excite characteristic frequencies of the magnetosphere at 1.3, 1.9, 2.6, and 3.3 mHz [Samson et al., 1992]. This possibility needs further analysis. If the peaks are omitted, then the trend in the data is a decrease of $\lambda^2$ with increase of $\tau$.

The dependence of $\lambda^2$ on spatial separation is provided in Figure 9 for the limited range of two-spacecraft separations available from Cluster separations. The events used for this figure are those for which the bulk plasma sheet flow is greater than 80 km/s in the $X_{GSM}$ direction. The selection criteria of a bulk plasma sheet flow of 80 km/s was motivated by the need to measure turbulent fluctuations with spacecrafts pairs that were located within the same parcel of plasma sheet plasma, but still low enough to present acceptable statistics in our study. The dashed vertical line in Figure 9 indicates the turbulent eddy scale size estimate according to Neagu et al. [2002], which was determined from the eddy turn over time and the RMS flow velocity in the same manner as Borovsky et al. [1997]. A similar decrease in the log-normal variance is also found for slower bulk plasma flow, but more scatter is present in the data.

Figure 10 displays the kurtosis versus spatial separation for the plasma sheet magnetic field fluctuations using both one and two spacecraft data as previously described for the
solar wind. Although the Taylor hypothesis most certainly does not apply under nominal plasma sheet conditions, it should apply under plasma sheet conditions when the bulk flow is large and the Alfvén Mach number is greater than or about 1. In the figure both the one spacecraft (triangles) and the two spacecraft (squares) kurtosis values show a decreasing trend over the full range of the measurements. The vertical dash-dot line is shown at the turbulent eddy scale size of about 5000 km estimated in Neagu et al. [2002] and the vertical dashed line at about 10,000 km is the estimated eddy scale size in Borovsky et al. [1997]. The mean difference between the one and two spacecraft kurtoses is about 32% where the maximum difference is about 70%.

5. Hnat et al. [2002] Analysis Method

In this section we apply methods outlined in Hnat et al. [2002] as an additional means of identifying non self-similar scaling. Hnat et al. [2002] show that the PDFs of solar wind fluctuations exhibit an approximately mono-power scaling law in the inertial range of 46 s to 26 hr using the functional form:

\[ P(\delta B^2, \tau) = \tau^{-s} P_s(\delta B^2\tau^{-s}, \tau) \]  

where \( \delta B^2 = B^2(t + \tau) - B^2(t) \), \( \tau \) is the scale size (i.e., the temporal separation over which the magnetic field fluctuation is identified), and \( s \) is the constant scaling exponent. PDFs that satisfy the scaling relation given in equation (4) can be collapsed onto a single master curve. Hnat et al. [2002] showed that such a collapse is possible for magnetic fluctuations in the solar wind. If the scaling exponential is a constant and the PDFs collapse onto a single master curve, the fluctuations are monofractal. In the event that the PDFs are multifractal, Hnat et al. [2002] demonstrate that PDFs do not collapse to a single master curve, but instead display peaks with approximately the same height and wings whose height varies systematically with the scale size as shown in their Figure 5.

As an intermediate step to rescaling the PDFs, Hnat et al. [2002] determine the scaling exponential by examining the log-log plot of the peak of the PDFs versus scale size for a range of scale sizes. Figure 3 in the Hnat et al. [2002] study demonstrates for self similar scaling the peak value of the PDF normalized to the area under the curve decreases linearly with increasing temporal scale. This figure shows that the scaling exponential is best described by a single constant. We have applied this procedure to the PDFs calculated using both the one and the two spacecraft method.
Figure 11 displays log-log plots of the peak of the PDF versus a scale parameter, $\tau$, for the PDFs calculated from a single spacecraft (Figures 11a and 11c) and that calculated from $\Delta x$ for PDFs determined from two spacecraft (Figures 11b and 11d) for the solar wind (Figures 11a and 11b) and the plasma sheet (Figure 11c and 11d).

The results differ for the one spacecraft PDFs and the two spacecraft PDFs. Several factors should be considered such as the spacecraft transverse separation, the combined intervals, and the instrumental noise. The first factor we consider that is the spacecraft may not be separated along the direction of the flow. As a result the actual temporal separation may differ from the temporal separation calculated from the mean separation and mean flow. When we restrict the solar wind spacecraft separations to $\leq 30$ RE (figure not shown) there is less scatter in Figure 11b, but the slope still does not reflect the one determined in the upper left panel. A transverse separation of $30$ RE was adopted because the correlation between the two spacecraft begins to decrease significantly beyond this separation [Richardson and Paularena, 2001]. Unfortunately, limiting the transverse separation for the plasma sheet reduces the data set to an unacceptably small number of measurements. An additional factor that may influence the points determined from spacecraft pairs is the fact that several different intervals of slow solar wind and fast solar wind have been combined. Turbulence properties may differ in each interval and the combined intervals may affect the slope of the log-log plot. However, plots of the individual solar wind intervals still do not produce the same features as the top panels. Lastly, some of the scatter in the lower panels may result from the instrumental noise. As mentioned in the previous section the use of multiple spacecraft adds noise to the fluctuations and for the IMP-8 spacecraft the uncertainty in the measurements is larger than the other spacecraft. Additional noise would act to reduce the height of the PDFs at small scale sizes, similar to the decrease in the kurtosis of the PDFs, and could change the slope, but would not significantly affect the PDFs at large scale sizes. Our best estimate is that the lack of agreement between the top and bottom plots is the result of a combination of all three factors discussed, but we require more events in order to test this hypothesis.

While Figures 11a and 11b are not similar, Figure 11a resembles the relation reported by Hnat et al. [2002], especially for the slow solar wind. The most significant difference is the range over which a monofractal scaling (this down trending linear portion of the
curve) applies. In Hnat et al. this occurs from scales of about 46 s to 26 hr, but the range in our interval extends from 1 min to about 100 min depending on the solar wind interval. Why there is a difference in the range is not clear. An additional observable difference is the fast solar wind contains two regions of self similar scaling as indicated by the break in the slope of the linear fit in the top left plot.

Like the solar wind plots of Figure 11, the plasma sheet plots of the peak of the PDFs from the single spacecraft and spacecraft pairs (Figures 11c and 11d) do not resemble one another. The reasons for this are the same as those given for the solar wind. We also note that Figure 11c plot does not have the same linear trend in the PDF peaks in the equivalent solar wind plot. This is not unexpected since Weygand et al. [2005] has demonstrated that the plasma sheet PDFs scale non self-similarly, which indicates we should not have a single linear trend, whereas Hnat et al. [2002] has shown that the solar wind scale self-similarly. Finally, we also call to the reader’s attention the peaks in Figure 11c between 100 and 1000 s. These peaks are believed to be real because the points that make up the peak do not appear to be a random scatter of points and the peaks are observed in other log-log plots (not shown), but not necessarily at the same position. More on these peaks will be discussed in section 6.3.

Figure 12 compares all individual slow and fast solar wind periods that we have examined to determine if mono-scaling in the slow solar wind and bi-scaling in the fast solar wind are consistent features. The left panel compares three different slow solar wind regions and the panel on the right shows three different fast solar wind intervals. The diamonds in each figure are the values taken from Figure 3 of Hnat et al. [2002]. In the fast solar wind are there systematically two different scaling regions. The slow solar wind in the left panel seems to systematically show a single scaling region. The slow solar wind interval of November 1990 after the scaling region between about log_{10}(τ) 3.25 to 3.59 shows two small peaks. These peaks are similar to, but smaller than, the peaks observed in Figure 11c for the plasma sheet. Table 1 compares the scaling exponential for each of these periods and indicates that the first scaling exponential for each slow solar wind interval is close to the value calculated in Hnat et al. [2002], which is given in the top row of the table. The mean scaling exponential is about -0.4, within the uncertainty given by Hnat et al. [2002]. Table 1 also shows that in the slow solar wind the
break point of the self similar scaling differs considerably for the three periods and ranges from about 0.5 hr to 5 hr. For the fast solar wind, the scaling exponentials for the two scaling ranges and the positions of the two break points differ little from one interval to another. Our hypothesis regarding the physical significance of the break points will be discussed further in the next section.

6. Discussion

By examining temporal fluctuations of the magnetic field on a single spacecraft and comparing them with the fluctuations of the field measured simultaneously at two different positions, we have been able to investigate the validity of the Taylor hypothesis for studies of the solar wind and the plasma sheet. Comparing our results with those reported by other investigators for fluctuations in time, we have in some cases reached different conclusions or extended previous analysis. Here we attempt to account for the features of our work that differ from or extend those previously reported.

6.1 Scaling Parameters in the Solar Wind and the Plasma Sheet

Following Sorriso-Valvo et al. [1999], we analyzed temporal variations of the magnetic field using data from Wind near 1 AU and formulated the scaling behavior in terms of the Castaing et al. [1990] parameter $\lambda^2$ introduced in equation (1). Sorriso-Valvo et al. used data from Helios acquired between 1 AU and 0.29 AU. We reproduced the trend of the Sorriso-Valvo et al. scaling ($\lambda^2$ vs. $\tau$ in Figures 1 and 2), but found systematically lower values for the fast solar wind. It is possible that the shift results from the different nature of the two data sets. Our work used continuous intervals of uninterrupted fast (or slow) solar wind, whereas Sorriso-Valvo et al. used a superposition of multiple fast (or slow) solar wind intervals and did not exclude localized events such as coronal mass ejections or current sheet crossings. It is also possible that solar wind conditions were more extreme in the interval that he used or that solar wind turbulence evolves as the plasma propagates from 0.29 AU to 1 AU. It is interesting to note that Sorriso-Valvo et al. found larger $\lambda^2$ values for the fast solar wind than the slow solar wind whereas we found just the opposite. Why this is the case is not clear at this time.

We have also applied the Castaing et al. [1990] formulation to the differences of the magnetic field measured simultaneously at pairs of solar wind spacecraft. The variation of $\lambda^2$ with spacecraft separation in radial distance from the sun, $\Delta x$, followed the trend
found from the Sorriso-Valvo et al. [1999] variation with $\tau$ assuming $\tau = \Delta x/v_{sw}$. The
points in the two-spacecraft study that did not correspond to the single spacecraft results could be understood as contaminated by noise in the (aging) IMP-8 magnetometer.

The plasma sheet was also investigated by using the same types of analysis. For this plasma, as for the solar wind, a non-linear decrease in the variance of the log-normal distribution was found over a large range of scale sizes (Figures 8 and 9). This non-linear decrease is consistent with intermittent turbulence of the plasma sheet and with properties of non self-similar scaling. These results are consistent with those for field fluctuations [Weygand et al., 2005] and for flow fluctuations [Borovsky et al., 1997; 2003; Angelopoulos et al., 1999; Petrukovich and Yermolaev, 2002]. Angelopoulos et al. [1999] used two spacecraft (Geotail and Wind) to demonstrate intermittency in the plasma sheet flow. However, the present study is the first to use spacecraft at a range of separations to investigate both intermittency and scaling features of the plasma sheet plasma. In comparing the present work with that of Weygand et al. [2005], we found that the results for plasma sheet properties based on the single spacecraft data and the study using spacecraft pairs corresponded well provided that the sunward flow exceeded 80 km/s in the $X_{GSM}$ direction. Noting that the properties of plasma in different parts of the magnetotail (lobe, plasma sheet boundary layer, central plasma sheet) differ considerably, the question arises in any multi-spacecraft study whether both spacecraft are probing a single continuous plasma regime. We believe that by limiting the analysis to data from intervals of relatively high speed flow, we select cases in which both spacecraft are embedded in the same plasma regime.

The form of the PDF is expected to become Gaussian at the eddy scale size. In the Angelopoulos et al. case, the PDF of the flow fluctuations remained non-Gaussian at separations as large as 4.8 $R_E$, whereas Borovsky et al., Neagu et al., Weygand et al. and the present study find turbulent eddy scale sizes that range from 5,000 to 20,000 km. Possibly the plasma sheet was particularly thick in Angelopoulos event. Wind magnetic field $B_z$ data shows a dipolarization at about 1100 UT and hence a substorm occurred and the tail was in the recovery phase for most of the interval, which means the plasma sheet was thicker than normal. However, Weygand et al. [2005] also showed that the turbulent eddies in the plasma sheet are stretched along the $x$ axis and thinner along the $z$ axis.
Furthermore, Thompson et al. [2005] has shown that the plasma sheet can be as thick as 8-10 $R_E$. Additional studies are clearly needed to test relationships between turbulence and plasma sheet thickness or other features of the plasma sheet that may dictate eddy scale size. Such studies would provide insight into the differing results in the multiple investigations.

6.2 Eddy Scale Size Inferred from the Kurtosis of the PDFs

The characteristics of solar wind fluctuations can be quantified in terms of the kurtosis of the distribution [e.g., Burlaga and Viñas, 2004] as an alternative to an analysis in terms of the log normal variance, $\lambda^2$. Our results for the variance of magnetic field fluctuations versus temporal or spatial separation support the conclusions of Burlaga and Viñas in showing that the kurtosis decreases with increasing separation, asymptoting to a value of $\sim 3$, applicable to a Gaussian distribution. However, the limiting value for the Burlaga and Viñas study is reached at about 280 h, which for a nominal solar wind speed of 400 km/s implies a spatial separation of $\sim 63,000 R_E$ whereas we find that the kurtosis approaches 3 at $\sim 500 R_E$ (see Figures 6 and 7). Our result is within a factor of 3 of the eddy scale size of 190 $R_E$ determined by Matthaeus et al. [2005] and within the range of 190 to 4400 $R_E$ reported by Matthaeus et al. [1986].

It is unclear why the Burlaga and Viñas study identifies Gaussian distributions only at extremely large spatial scales. Their work does not separate fast and slow solar wind, but we find that for both portions of the solar wind, the kurtosis of 3 is attained at several hundred $R_E$. It is possible that the properties of flow and field fluctuations differ among events, but it is hard to imagine such extreme differences.

From the kurtosis of the PDFs of variations of the magnetic field in the plasma sheet, we have inferred eddy scale sizes of 0.9-1.5 $R_E$, consistent with sizes reported by Weygand et al. [2005]. Values of 0.8 $R_E$ and 1.6 $R_E$ have been found by Neagu et al. [2002] and Borovsky et al. [1997], respectively. The range in the size of the turbulent eddies suggest that external conditions may control the scale size of plasma sheet eddies. However, more work needs to be done to confirm this hypothesis.

6.3 Turbulence Analysis in Terms of Peak Values of PDFs
Another approach to turbulence analysis follows the method of Hnat et al. [2002] who characterized the fluctuations of various solar wind properties versus $\tau$ or $\Delta x$ by looking for trends in the peak values of the PDFs of the fluctuations. A linear trend implies self-similar scaling. Our analysis of two intervals of slow solar wind in Figure 12 agrees with the Hnat et al. conclusion that self-similar scaling is present over a range of temporal or spatial separations and demonstrates that the scaling exponential changes little from one to another interval of slow solar wind. In one particular interval, though, there appears to be several peaks similar to those observed in the plasma sheet (Figure 11c). The nature and meaning of these peaks is unclear at this time. In the slow solar wind intervals we also found variations in the upper cut-off of the range of temporal or spatial separations for which self-similar scaling applies. Our cut-offs varied between 0.5 hours and 5 hours, and were all smaller than the 26 hour cut-off reported by Hnat et al. We believe that the differences between our results relate to selection criteria. We treated intervals of slow and fast solar wind separately and avoided structures such as CMEs, whereas Hnat et al. used several years of solar wind data including all solar wind conditions.

For nominal solar wind speed, our cut-offs imply eddy scale sizes of 370, 110, and 1100 $R_E$ for the three solar wind intervals that we investigated. These can be compared with the range of eddy scale sizes (190 to 4400 $R_E$) reported by Matthaeus et al. [1986] and with the 190 $R_E$ reported more recently by Matthaeus et al. [2005] who combined more than 100 solar wind intervals for their study. These variations of the cut-off scale suggest the possibility that the eddy scale size may change depending on properties of the coronal source of the solar wind, a proposal worthy of further investigation.

In Figure 11 we compared the results for PDFs obtained from single spacecraft and pairs of spacecraft and observed almost no agreement between the plots. We attributed this lack of agreement to a combination of factors that include differences in spacecraft alignment, noise in the data, and differences in the turbulent properties in each of the intervals.


There is a long history of using power spectral density as a tool for identifying the properties of turbulent systems, the idea being that in regions of self-similar fluctuations, the power varies linearly with the frequency. The analyses described in the previous
sections provided a cut-off in time whose inverse is expected to correspond to the low frequency end of the linear trend in the power spectrum. For the May 1998 slow solar wind intervals of our study (see Figure 13), we found the expected linear trend (corresponding to a slope of $-5/3$) over a range of frequencies starting slightly above the expected minimum value. For a second interval, November 1990, the cut-off in time at about 30 min did not correspond to a power density spectrum break at about 20 µHz (power spectrum not shown). Above 20 µHz the spectral index is about -1.55, which is near the Kraichnan [1965] estimated inertial range spectral index and a spectral index commonly observed in solar wind turbulence. The power spectra of the third interval did not reveal the same behavior due to the shortness of the interval and the low resolution of the data.

For the fast solar wind (Figure 14), a break in the power spectral slope in the February 2000 interval did correspond to a frequency at which a break occurs within the log-log plot at about 10 min (about 1.67 mHz). The two break points (at about 1200 s and 4300 s) observed in the right plot of Figure 12 for the February 2000 fast solar wind event are plotted in Figure 14 at 1.67 and 0.16 mHz. At frequencies higher than the break point at 1.6 mHz the spectral index in the figure is about -1.5. In between the break point at 0.167 mHz and 1.67 mHz the spectral index is about -1.0, which is the spectral index commonly associated with the driving range of a turbulent power density spectrum [Goldstein et al. 1995]. The February 2000 interval was the only one of the three examined intervals in which the power spectra displayed breaks at the same periods as the log-log plots of Figure 12. The other two did not have enough high resolution data.

As in the other approaches, the critical frequencies (or time scales) appear to vary by as much as an order of magnitude from one interval to another. The critical frequencies bound regions in which the power spectral density displays a linear decrease with frequency. This behavior is compatible with the behavior identified with other analysis techniques and suggests to us that the solar wind behaves in a self-similar manner over one or more ranges of temporal or spatial fluctuations but that the critical scale sizes can change from one portion of solar wind to another.

6.5 Isolated Spectral Peaks
Several of our plots of properties of the magnetospheric plasma sheet (note especially Figures 8 and 11c) contain isolated points that depart from the trend. We have remarked that these anomalous points could be related to the “magic frequencies” of Samson et al. [1992]; see also Walker, [2005]. In Figure 8 only one of the peaks at 3.3 mHz corresponds to one of the Samson et al. [1992] frequencies. The other peak at about 13.9 mHz is much larger than any of the Samson et al. frequencies but falls in the Pc 4 band that corresponds to magnetospheric field line resonances. In Figure 11c one of the peaks at about 2.1 mHz is close to the magic frequency of about 1.9 mHz, but none of the other peaks are similar to any of the other frequencies. The true nature of the peaks in both Figures 8 and 11c is not clear. Additional intervals should be analyzed to compare peaks with the Samson et al. frequencies. It is clear in both figures that the plasma of the plasma sheet is non self-similar over a large range of scale sizes.

6.6 The Taylor Hypothesis

It is widely accepted that the Taylor hypothesis applies in the solar wind. The use of both one and two spacecraft techniques in this study presents us with the opportunity to check this assumption. A comparison of Figures 1 and 2 with Figures 3 and 4 and the one and two spacecraft values of the kurtosis in Figure 6 and 7 show that similar values are obtained from spatial and temporal analysis. The results are consistent with the Taylor hypothesis. Differences between the two methods can be explained by either instrument noise or by the assumption that spacecraft did not probe a homogeneous regime. The mean difference between the kurtosis determined with a single spacecraft and the kurtosis determined with the spacecraft pairs is about 12 % with a maximum difference of 29%.

In the plasma sheet, the Taylor hypothesis is most certainly not true under nominal conditions. However, under some conditions the Taylor hypothesis could apply. By selecting intervals in which the bulk speed of the plasma is high, as we have done, the results of Figure 10 indicate that the kurtoses calculated using the two different methods do not differ greatly. The mean difference is about 32% and the largest difference is 70%. We recommend that additional studies be done in order to demonstrate under what conditions the Taylor hypothesis begins to fail.

6.7 Is Solar Wind Turbulence Self-Similar?
The analysis of this and other papers lead to conflicting conclusions on the question of whether turbulence in the solar wind is self-similar. Using the parameter $\lambda^2$ introduced by Castaing [1990], we concluded that the solar wind has non self-similar scaling properties over a large range of scale sizes. Using the approach of Hnat et al. [2002], we were led to similar conclusions for the fast solar wind and the plasma sheet but we obtained ambiguous results for the slow solar wind. Possibly the differences arise because of the analysis methods, but there may be underlying physical reasons for the different results obtained at this time. For example, the concepts that we apply to the description of turbulence were developed for the analysis of homogeneous systems characterized by fixed boundaries (such as the radius of an obstacle in a flow) but long intervals of the solar wind include portions with different scale sizes. If the characteristic eddy sizes differ not only between fast and slow solar wind but also among different intervals of slow or fast solar wind, the properties of the superposed PDFs may be ambiguous. Many of the properties of the solar wind are imposed in the solar corona. Particularly because of our evidence that the cut-off scales differ from one to another interval of slow solar wind, it seems quite plausible that features of the turbulence are imposed at the coronal source and that properties inferred from long intervals of solar wind data produce ambiguous results because they are inferred from data that superimposes parameters from distinct plasma regimes.

7. Summary and Conclusions

We have applied several different methods of identifying self-similarity in the solar wind and plasma sheet. Both the Hnat et al. [2002] method and Sorriso-Valvo et al. [1999] method demonstrate that the fast solar wind and plasma sheet have non self-similar scaling properties. In the slow solar wind our results are conflicting. The Sorriso-Valvo et al. [1999] method suggests that the slow solar wind has non self-similar scaling properties, while the Hnat et al. [2002] method suggests that it has self-similar scaling properties.

In addition to obtaining results similar to those of Sorriso-Valvo et al. [1999] and Hnat et al. [2002] studies, we used spacecraft pairs in both the solar wind and the plasma sheet to demonstrate that we are able to obtain results with spatially separated spacecraft that are similar to those obtained from a single spacecraft. The fact that we are able to obtain
analogue PDFs and kurtoses with both methods in the solar wind indicates that the Taylor hypothesis is a valid assumption in the solar wind and most likely applicable under specific conditions of high speed flow in the plasma sheet.

Furthermore, the PDFs of magnetic field fluctuations in both the solar wind and plasma sheet are non-Gaussian at scales smaller than about 500 $R_E$ and 1.5 $R_E$ in the solar wind and plasma sheet, respectively. The highly peaked shape with high probability wings (i.e., a distribution with a kurtosis $>3$) generally indicates that intermittent turbulence is present within the plasma.

Finally, plot of the kurtosis versus spatial separation suggests that the turbulent eddy scale size for the slow and fast solar wind is about 500 $R_E$. This is consistent with the previous work of Matthaeus et al. [1986], but a factor of about 3 higher than their more recent work [Matthaeus et al. [2005]. In the plasma sheet we found that the turbulent eddy scale size was about 0.9 $R_E$ for the spacecraft pairs, but 1.5 $R_E$ for the single spacecraft measurements. Both these values are consistent with the previous work of Weygand et al. [2005], but additional values from 2006 are required to refine the estimate.

Acknowledgments

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References


Table 1. Table of the scaling exponentials determined from Figure 12. Given in the left column is the type of solar wind (slow solar wind: SSW and fast solar wind: FSW) and the date of the intervals. In then second column is the value for the first scaling region plus the range over which the scaling applies. The far right column is the value for the second scaling exponential if it is applicable. The top row shows the value given in Hnat et al. [2002].

<table>
<thead>
<tr>
<th></th>
<th>1st Scale Exp.</th>
<th>2nd Scale Exp.</th>
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<td>Hnat et al. [2002]</td>
<td>-0.42±0.02</td>
<td></td>
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<tr>
<td></td>
<td>(46s to 26 hr)</td>
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<td>(1 min to 300 min)</td>
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<tr>
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<tr>
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<td>(1 min to 102 min)</td>
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<tr>
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<td>(10 min to 102 min)</td>
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<td>(1 min to 20 min)</td>
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Figure 1. Scaling behavior of PDFs of magnetic field fluctuations in the slow solar wind. The grey circles are the log-normal variances obtained from the Castaing et al. [1990] model. The error bars represent $2\sigma$ standard deviations. The magnetic field data was recorded by the Wind spacecraft from May 17 to May 26, 1998. The black squares were obtained from the Sorriso-Valvo et al. [1999] study.
Figure 2. Scaling behavior of fast solar wind PDFs of magnetic field fluctuations. This figure has the same format as Figure 1. These data were obtained by the Wind spacecraft from February 24 to February 29, 2000.
Figure 3. Scaling behavior of slow solar wind PDFs of magnetic field fluctuations. This figure has the same format as Figure 1 except that the plot is of $\lambda^2$ versus spacecraft separation and the blue circles have been obtained using pairs of solar wind spacecraft: A is for ACE, C is for Cassini, Ga is for Galileo, Ge is for Geotail, I is for IMP-8, and W is for Wind.
Figure 4. Scaling behavior of fast solar wind PDFs of magnetic field fluctuations. This figure has the same format as Figure 3.
Figure 5. The effect of instrumental noise on the PDF shape and the kurtosis. These data were obtained from the ACE spacecraft. The column of panels on the left side has arbitrary units of time along the x axis. The title above each panel on the left gives the amplitude of the Gaussian distributed noise. The column of panels on the right side has units of counts on the y axis and the kurtosis is given in the upper right corner for each distribution.
Figure 6. Kurtosis versus spatial separation determined from one (red circles) and two (blue squares) spacecraft PDFs of magnetic field fluctuations for slow solar wind. As in Figure 3, the letters identify the spacecraft pairs with the spacecraft used for the single spacecraft PDF given first. The vertical solid lines link the pairs of points determined from the two different methods. The vertical dashed line is placed at the minimum scale size of a turbulent solar wind eddy as determined by Matthaeus et al. [1986].
Figure 7. Kurtosis versus spatial separation determined from both one and two spacecraft PDFs of magnetic field fluctuations for fast solar wind. This figure has the same format as Figure 6.
Figure 8. Scaling of PDFs observed in the plasma sheet for a range of temporal separation. This figure has the same format as Figure 1. These magnetic field fluctuations for this figure were recorded by Cluster on August 15, 2001 from 1013 UT to 1330 UT.
Figure 9. Scaling of PDFs for the plasma sheet over a range of spatial separations. This figure has the same format as Figure 3 and is a composite of several different events from 2001, 2002, and 2003. The vertical dashed line indicates the turbulent eddy scale size estimate from Neagu et al. [2002].
Figure 10. Variation of kurtosis versus spatial separation in the plasma sheet. This figure has the same format as Figure 6. The vertical dash-dot line is the turbulent eddy scale size in the plasma sheet estimated by Neagu et al. [2002] and the vertical dashed line is the turbulent eddy scale size estimated by Borovsky et al. [1997].
Figure 11. Log-log plots of the peak of the PDFs of the solar wind magnetic field fluctuations versus the scale size using the same method as in Hnat et al. [2002]. The points in the top row were determined using solar wind measurements. The points in the bottom row were calculated using plasma sheet spacecraft. The points in the left column are from single spacecraft PDFs and the plots in the right column are determined with pairs of spacecraft PDFs. The black points in Figure 11a are slow solar wind PDFs and the grey points are from fast solar wind PDFs.
Figure 12. This figure has the same format as Figure 11. In the left panel are the points from three different slow solar wind periods. The circles apply to an interval in May 1998, the *s for November 1990, and the squares for June 1998. In the right panel is three different fast solar wind periods. The circles are for February 2000, the squares are for November 2000, and the *s are for August 1999. The slow solar wind periods show only one interval of self similar scaling while the right panel shows two regions of self similar scaling. In both panels the diamonds are the points taken from Figure 3 of Hnat et al. [2002].
Figure 13. Power density spectrum for the slow solar wind period in May 1998. The black vertical dashed line marks the break observed in left panel of Figure 12 at about 6000 s. The solid black line is a linear fit to the data at approximately the break in the power density spectrum. The slope of this line is $-5/3$. 

\[ \text{May 1998 Power Density Spectrum} \]
Figure 14. This figure has the same format as Figure 13, but for the fast solar wind interval in February 2000. At approximately 1.67 mHz there appears to be breaks in the power density spectrum. This break is close to the first break observed in the right panel of Figure 12 at 600s, which is marked with the right black vertical dashed line. The left vertical dashed line marks the second break observed in Figure 12 at about 6000 s.