Saturation of the polar cap potential: Inference from Alfvén wing arguments*

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But first: Electric potentials? What do we mean by $E$ in MHD?

- In ideal MHD (large scale, long time scales, low resistivity), $E = -u \times B$ so electric fields arise from cross-field flows.
- However, in discussions of the polar cap response, there is a long history of talking (often incorrectly) about electric potentials.
- Evidently in talking about potentials, one must assume that the system is quasi-static and a good approximation is $E = -\nabla V$.
- Using that assumption, I will discuss the problem in the standard language.
The solar wind electric field is (almost) always present

• The solar wind speed is of order 400 km/s and the magnetic field of order 10 nT (and almost never aligned with the flow). This means that there is (almost) always a solar wind electric field in Earth’s rest frame and it is of order a few mV/m.
Background assumed - The solar wind reconnects with the polar ionosphere: thus the two plasmas are linked.
How does the electric field in the polar cap vary with solar wind parameters.

- It is widely accepted that in the range of nominal solar wind parameters the cross polar cap potential, $\Delta V_{pc}$, increases linearly with $E_{sw}$.

$$\Delta V_{sw}^{RC} = E_{sw} D^{RC}$$

$D^{RC} = \text{reconnected width across flow}$ and

$$E_{sw} \propto u_{sw} B_{sw}.$$ 

If there is no potential drop along the field, isn't it obvious that $\Delta V_{pc} = \Delta V_{sw}^{RC}$?
For typical solar wind conditions the polar cap potential drop is indeed proportional to $E_{sw}$ plus a contribution from viscous effects at the magnetopause.

- Viscous contributions are small.
- For example, Boyle et al. (1997), using a data set limited to cases with the solar wind electric fields of the order of 8 mV/m and less, found linear response

$$\Delta V_{pc} (\text{kV}) = 10^{-4} \ u_{sw} (\text{km/s})^2 + 11.7 \ B_{sw} (\text{nT}) \sin^3(\theta/2)$$

- The first term (viscous interaction) is independent of the reconnection electric field.
- The second term is proportional to $E_{sw}$.

For nominal conditions $u_{sw} = 400 \ \text{km/s}$, $B_{sw} (\sim 10 \ \text{nT})$, the viscous term is 16 kV and the second term is $\sim 100 \ \text{kV}$. 
But later work shows that when $E_{sw}$ increases above nominal levels, the cross polar cap potential appears to saturate at a level near 170 kV (viscous $\sim$30 kV).

HAIRSTON ET AL.: POTENTIAL SATURATION DURING OCT–NOV 2003

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diff. $\sim$140 kV

Another example from Russell et al., 2001

Figure 8. One hour averages of the potential drop across the polar cap and the Joule heating versus the IEF with arctan fit for the October 18-20, 1995, storm.

note linear response for positive electric field up to about ~4 mV/m
Different explanations have been offered

• Hill et al. [1976] and Hill [1984] predicted saturation for large values of the electric field imposed on the magnetosphere. With $V_{mc}$ the imposed potential, $V_{pc}$ the polar cap potential and $V_S$ the saturation potential (not really defined) he suggested $V_{pc} = \frac{V_{mc}V_S}{(V_{mc}+V_S)}$. By design this implies $V_{pc} \leq V_{mc}$ and for large $V_{mc}$, $V_{pc} \rightarrow V_S$.

- In an attempt to account for the development of saturation, Hill proposed that the for large ionospheric conductance, Region 1 currents driven in the magnetosphere generate a magnetic field that opposes the Earth’s dipole field, thereby reducing the amount of magnetic flux available for reconnection on the dayside magnetopause and reducing the imposed potential drop.
More explanations

- Siscoe et al. [2002a] interpreted saturation found in MHD simulations in terms of the “Hill model” and found them to be qualitatively consistent.
- Siscoe et al. [2002b] reinterpreted the results saying:
  - “the transpolar potential saturation results not from the region 1 current system limiting the rate of reconnection at the magnetopause but instead from ram pressure . . . limiting the total amount of current that can flow in the region 1 current system.”
Still more

• Ridley [2005] argued that by taking into account the size of the magnetosphere and the fact that the post-shock magnetic field strength is strongly dependent upon the solar wind Mach number he could obtain rather good predictions of the cross polar cap potential.

• Here we offer an interpretation of saturation that gives an explicit expression similar to Hill’s but derived from very different (and fundamental) arguments.
Inspiration from studies at Io.
Saturation of the cross-moon potential is predicted by the Alfvén wing model

- The cross-moon potential imposed by the electric field arising from the flow of Jovian plasma cannot exceed a limiting value, which is smaller than the potential drop across 1 Io diameter.
- The limit is reached when the Alfvén conductance of the unperturbed flowing plasma \( (1/\mu_0 v_A) \) is small compared with the Pedersen conductance of the moon.
- LET'S APPLY THE INSIGHT FROM THIS PROBLEM.
If use MHD and circuit analysis

- For MHD, magnetic flux tubes are equipotentials. Current continuity then leads to a relation

\[ \Delta V_{pc} = \left( \frac{\Sigma_{sw}}{\Sigma_P} \right) \Delta V_{sw} \]

- This linear response is not what is observed!
Circuit theory - no

- Saturation would not arise on the basis of circuit theory, **but circuits are not the right approach**.
- **More relevant is wave reflection theory** because the interaction (both at Io and in the reconnected magnetosphere) sends Alfvén waves along the field lines from the flowing plasma (Jupiter’s plasma or the solar wind) to the ionosphere. **Alfvén waves carry} \( j_{||} \).
- While the waves are propagating from one end to another, the flux tube is moving anti-sunward. The “circuit” itself is changing. Ohm’s law applies in the plasma rest frame, but that frame does not exist if one end is moving relative to the other and the flux tube keeps changing shape.
Polar cap flux tube move, change shape

(c) shows that the flux tubes bulge. Those bulges or bends are imposed by the $j_{\parallel}$ carried by Alfvén waves.
Alfvén wave reflection

- MHD theory tells us that an Alfvén wave encountering a change of impedance $\Sigma_A^{-1}$ to $\Sigma_P^{-1}$ in a length small compared with its wavelength is reflected with signal amplitude:

$$E_r = E_i (\Sigma_P^{-1} - \Sigma_A^{-1}) / (\Sigma_P^{-1} + \Sigma_A^{-1})$$

- Note for high conductance, the electric field of the reflected wave has a changed sign... as in reflection from a mirror

- The transmitted electric field is

$$E_t = E_i + E_r = 2E_i \Sigma_P^{-1} / (\Sigma_P^{-1} + \Sigma_A^{-1})$$

or equivalently

$$\Delta V_t = \Delta V_i \frac{2 \Sigma_A}{\Sigma_P + \Sigma_A}$$
Form of solution

Cross polar cap potential
for Pedersen conductance of 10S

Vpc (kV) vs B (nT)
Apply to the magnetosphere

\[ \Delta V_{PC} = \left| \mathbf{u}_{sw} \times \mathbf{B}_{sw} \right| \sin^2 (\theta / 2) D^{RC} \]

\[ \Delta V_{sw}^{RC} = \frac{2 \Sigma_A}{\Sigma_P + \Sigma_A} \]

where \( \mathbf{u}_{sw} \times \mathbf{B}_{sw} \) is the solar wind reconnection electric field with \( \mathbf{u}_{sw} \) the solar wind velocity - along the Sun-Earth direction - in the GSM coordinate system. \( \theta \) is the clock angle of the IMF, \( D^{RC} \) is reconnected width \( \propto (p_{dyn})^{1/6} \), \( \Sigma_A = 1/(\mu_0 v_A) \) = Alfvén conductance of the solar wind and \( \Sigma_P \) is the Pedersen conductance of the relevant (northern or southern) polar cap ionosphere.

Limit when \( \Sigma_P = \Sigma_A \), gives \( \Delta V_{PC} = \Delta V_{sw}^{RC} \)

(confirming the factor of 2).
Why, to lowest order, ignore reflections as the signal crosses the bow shock and the magnetopause?

- **MAGNETOPAUSE**: the open part of the magnetopause surface is a rotational discontinuity across which $v_A$ is constant
  - so there is no reflection at the magnetopause
- **BOW SHOCK**: most of the signal enters across the downstream shock which is weak and changes $v_A$ little:

\[
\frac{(\Sigma_A - \Sigma_{A'})}{(\Sigma_{A'} + \Sigma_A)} \approx 0
\]

Almost no reflection across bow shock.
Limiting case for small $\Sigma_A \rightarrow \text{large} \quad V_A = B/(\mu_o \rho)^{1/2}$

- Saturation appears when $\Sigma_P >> \Sigma_A = (\mu_o V_A)^{-1}$, with the limiting value

$$\Delta V_{PC} = 2 \Delta V_{RC} \Sigma_A / \Sigma_P$$

$$= 2(\rho u_{sw}^2 / \mu_o)^{1/2} \sin \lambda \sin^2 (\theta / 2) D^{RC} / \Sigma_P$$

with $\lambda$ the cone angle of the IMF.

Assume $\Sigma_P = 10S$. NO MORE FREE PARAMETERS.

For nominal SW dynamic pressure (no remaining dependence on $B_{sw}$):

$$\Delta V_{PC}(kV) = 160 \sin \lambda \sin^2 (\theta / 2)$$

With the product of angles $< 1$, saturation is expected at $\sim 100$ kV (to which the viscous contribution must be added).
Does this simple expression really work?

- We compare with data from 13 storm periods previously analyzed by Ridley (2005).
- The polar cap potential is inferred from the AMIE technique. Compare the value (minus viscous) with predictions from the linear response form of Boyle et al. [2005] and from our (KR) model

\[ \Delta V_{PC} = E^{RC} D^{RC} \frac{2\Sigma_A}{\Sigma_P + \Sigma_A} \]

- Take \( D^{RC} = 0.1 \pi R_{mp} \) (roughly 3 \( R_E \) but varying as the 1/6 power of the solar wind dynamic pressure).
Data

• I shall show plots for all 8 events in which the solar wind electric field

\[ |E_{RC}| = |u_{sw,x} \times B_{sw}| \sin^2(\theta/2) \]

is greater than 15 mV/m

• For these events and fixed \( \Sigma_P = 10 \text{ S} \) we get quite good agreement whereas the linear relation overestimates all high values.

• For all 13 cases, the trend is consistent without any change of Pedersen conductance.
Sept. 17-19, 2000

Oct. 4 to 6, 2000

[Graphs showing data for the specified dates]
Mar. 30 to Apr. 1, 2001

Mar. 30 to Apr. 1, 2001 Universal Time

Apr. 11 to 13, 2001

Apr. 11 to 13, 2001 Universal Time
Onset of saturation

- The predictions follow a linear trend when the measured cross polar cap potential is < 80 kV (solar wind electric field $\approx 4$ mV/m) but diverge at 100 kV and higher.
- Consistent with conclusions of Russell et al. [2001], who give the critical electric field as 4 mV/m.
- Our equation gives 4.6 mV/m for nominal solar wind speed of 400 km/s.
- Reiff et al. [1981] estimate the critical field as $\sim 8$ nT, giving a slightly smaller critical electric field for the same solar wind speed.
Model and data compared.  
Left panels - all data.  
Right panels - Bin medians and rms deviation

Upper panels: Amie – viscous vs. modified relation of the present work.

Lower panels: Amie - viscous vs. Boyle et al. linear relation

Onset of saturation effects
Non-linear even at relatively small $E_{RC}^{\text{RC}}$

- The departure from linear predictions even for relatively small solar wind electric field can be seen in the event plotted in the event of Aug 11-13, 2000 where remains between 3 and 6 mV/m for the first 30 minutes, yet equation (13) improves the prediction of the cross polar cap potential relative to the linear prediction.
High dynamic pressure events

• In a few events $2E_{RC}^{RC}D_{RC}^{RC} > 350$ kV, and the AMIE-viscous potential exceeds 200 kV.
• This can be attributed to the effect of solar wind dynamic pressure in excess of nominal values.
• The response is proportional to the cube root of the dynamic pressure and in these events, the peak levels range from under 20 nPa to almost 80 nPa, introducing factors of 2.1 to 3.4 that can readily account for the higher values of the observed potential in the most extreme events.

\[
\Delta V_{PC} \rightarrow E_{RC}^{RC}D_{RC}^{RC} \frac{2\Sigma_A}{\Sigma_P + \Sigma_A}
\]

\[
=(2)^{1/6} \left( B_o \rho u_{sw}^2 / \mu_o^2 \right)^{1/3} \\
\times 0.2R_E \pi \sin^2(\theta/2) \sin \lambda / \Sigma_P
\]
Sources of scatter are many

- Only $\Sigma_p$ enters as an internal magnetospheric parameter. We assume it is constant, but it may change with season and surely with activity.
- Everything else depends on solar wind properties using standard assumptions for the length of the reconnection line and the reconnection efficiency.
- Some scatter may link to use of Amie method to estimate the polar cap potential.
- Still, the agreement between model and data seems to suggest that the model has some validity.
Some interpretations have been based on analysis of simulations

- Simulations make assumptions that greatly modify the results.
- Examples of important elements
  - Boundary conditions
  - Resistivity
- Selection of grid sizes affects the forms in which currents develop.
- What you put in determines what you get out.
- It can be dangerous to infer from simulations what happens in a real system.
- Some examples for identical upstream solar wind conditions follow.
Southward Bz = -5 nT

BATSRUS: Ideal MHD

OpenGGCM: Current-dependent resistive MHD

01/01/2000 Time = 04:20:00 \( y = 0.00R_E \)

01/01/2000 Time = 08:00:00 \( y = 0.00R_E \)
BATSRUS: Ideal MHD

OpenGGCM: Current-dependent resistive MHD
In conclusion: our formulation is similar to that used previously by Hill, Siscoe, and others

**What’s new?**
New is an interpretation based on fundamental principles.

- The driving field is that of the solar wind on reconnected flux tubes. The polar cap potential is a *response* to externally imposed wave signals.

- Saturation of the polar cap potential has nothing to do with changes of reconnection efficiency or changes of magnetospheric geometry or the jump in plasma parameters between the solar wind and the magnetosheath.

- It arises merely because contrasting impedances reflect incoming waves and, in the limit of large solar wind $v_A$, the reflection cancels much of the incident signal.
Suggests additional studies such as this initial input from R. L. McPherron

for nominal sw dynamic pressure, non-linear response expected at ~ 4 mV/m
thank you
\[ \Delta V_{PC} = \frac{2u_{sw} \left| B_{sw,yz} \right| D_{reconn} \left( \rho u_{sw}^2 / \mu_o \right)^{1/2}}{u_{sw} B \Sigma P + \left( \rho u_{sw}^2 / \mu_o \right)^{1/2}} \]

\[ \Delta V_{PC} = \frac{2\Delta V_{reconn}}{1 + B_{sw} \Sigma P / (\rho / \mu_o)^{1/2}} \]
Limits

- Viscous response underestimated.
- At values $100 \text{kV} < 2E^{RC}D^{RC} < 350 \text{kV}$ data trend towards an asymptotic value of the (non-viscous) polar cap potential between 100 and 150 kV.
- Compare with the limit for large $v_A$ of our relation.
- For a nominal solar wind dynamic pressure of 2 nPa, neglecting the dependence on angles, $\Delta V_{\text{max}} \approx 160 \text{kV}$ times angular factors ($< 1$), so the saturation level is consistent with our expression.

$$\Delta V_{PC} \rightarrow E^{RC}D^{RC} \frac{2\Sigma_A}{\Sigma_P + \Sigma_A}$$

$$= (2)^{1/6} \left( B_o \rho u_{sw}^2 / \mu_o^2 \right)^{1/3} \times 0.2R_E \pi \sin^2(\theta/2) \sin \lambda / \Sigma_P$$