Frequency doubling in ultralow frequency wave signals

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Abstract. We report a new theory to explain the long-observed frequency-doubling phenomenon detected in the compressional component of "high-m" ultralow-frequency signals seen in the Earth's magnetosphere. We show that the effect is a nonlinear response directly associated with resonant particles. The phase relationship observed between the linear signal and the nonlinear frequency-doubled signal is predicted by the theory. Our explanation and the fit with observation mean that the occurrence of frequency doubling provides critical evidence of the fact that ring current particles drive high-m waves.

1. Introduction

Signals in the ultralow frequency (ULF) band which can be detected on the Earth's surface or in space in the magnetosphere are due to magnetohydrodynamic eigenmodes of the terrestrial magnetosphere. An important subclass are modes that are localized in longitudinal extent with short azimuthal wavelength. These modes are commonly referred to as high-m modes, where m is the azimuthal wave number.

In 1968, Southwood et al. [1968, 1969] proposed that high-m waves would be excited by a bounce-drift resonant interaction with the energetic particles trapped in the magnetosphere. Subsequent theoretical work [Southwood, 1973; Mikhailovskii and Pokhodtloev, 1975; Southwood, 1976; Kochanov et al., 1976; Southwood, 1977a; Lin and Parks, 1978; Pokhodtloev et al., 1986; Chen and Hanggawa, 1988; Cheng, 1991] developed the idea further. The high-m waves are a modified form of Alfvén wave standing on the local field lines (poloidal field line resonances that can have compressional components). The waves grow by tapping into free energy in the ring current population. Finite pressure and inhomogeneity introduce a coupled compressional magnetic signal. Our paper here shows that an observational feature of some equatorial high-m modes, namely, a frequency-doubled compressional magnetic component, is directly attributable to the bounce-resonant particles.

Southwood et al. [1969] were inspired by the purely transverse waves described in the earliest reports from the ATS 1 spacecraft [Cummings et al., 1969]. Like the compressional signals reported later [Coleman, 1970; Barfield et al., 1972], these were largely meridionally polarized. Southwood et al. [1969] recognized that this fact meant that the events were very likely to have a high m (as a direct consequence of \( \text{div } b = 0 \), where b is the wave perturbation field). Subsequently, m has been measured directly in situ and confirmed to be large (\( m \approx 60 \)) [Takahashi et al., 1990].

High m modes are found in the Pc5 band (0.17-6.7 mHz) and are often narrow-banded, nearly monochromatic. However, in some cases, signals appear at frequency \( \omega \) in the transverse component and at the frequency (\( 2\omega \)) in the compressional component [Coleman, 1970]. This paper is concerned with explaining the frequency-doubling effect; in fact, we show that its presence is strong evidence that the signals themselves are generated by bounce-resonant interactions with ring current particles.

Using magnetometer data from the ATS 1 geostationary spacecraft, Coleman [1970] described the compressional signal as "rectified," by which he meant that the compressional component resembles a rectified transverse component. Barfield et al. [1972] also mention the effect, and their time series plots show events similar to Coleman's. Unfortunately, Barfield et al.'s use of a principal axis coordinate system for their plotted power spectra obscures the compressional polarization of the frequency-doubled component, but its appearance primarily in the intermediate axis power is consistent.

Higuchi et al. [1986] and Takahashi et al. [1990] produced important surveys of the effect. Higuchi et al. [1986] analyzed statistical information from two later geostationary spacecraft (GOES 2 and GOES 3). In addition to the rectified events (illustrated in Figure 1a), Higuchi et al. [1986] reported a new category of event, the "transitional" event (see Figure 1b), wherein there is still phase locking but the compressional signal seems to contain power at both the frequency of the transverse signal and at the doubled frequency. The effect is as if Coleman's [1970] rectification sets in at a particular transverse amplitude threshold. Subsequently, Takahashi et al. [1990] used AMPTE CCE magnetometer and charged particle data in a survey and confirmed that the signals were associated with flux enhancements of the ring current particle population. They also determined that the azimuthal wave number of the events was high (\( m \approx 60 \)) as had been proposed in nearly all of the theoretical explanations.

Higuchi et al. [1986] described the Coleman [1990] effect as "harmonic structure" but distinguished it from the more familiar harmonic structure attributable to the simultaneous excitation of several Alfvén eigenmodes standing along the magnetic field direction as is seen in azimuthally polarized (low-m) events [Takahashi and McPherron, 1982; Engbreitung et al., 1986]. The critical distinction is that the "second harmonic" appears only in the compressional component in the Coleman effect and is systematically phase locked to the lower-frequency transverse signal. To avoid confusion, we use the term "frequency doubling" to describe the Coleman effect.

Theoretical explanations of the frequency-doubling phenomenon have not accounted for the symmetry with respect to the field equator. Takahashi et al. [1987] demonstrated that the transverse magnetic perturbations are symmetric about the equator and, consequently, that the displacement is antisymmetric. The theoretical analysis of Higuchi et al. [1986], which we discuss further below, was based on an assumed symmetric displacement now known to be inconsistent with observations. Cheng and Lin [1987] and Takahashi et al. [1990] invoked the drift mirror instability to account for the transverse
wave properties but did not succeed in explaining the compressional wave symmetry. Thus the mechanism that generates the frequency-doubled component of the signal and accounts for the unusual symmetry properties of these waves has remained undefined.

Here we present a new explanation which fits well with the observed polarization and the localization of the signals to the equator and which shows that the higher-frequency compressional perturbation arises through a wave particle bounce-drift resonance. In this way it confirms the long-held belief that high-\(m\) modes are resonantly generated. Higuchi et al.'s [1986] theory can be seen as an antecedent of ours. Hence we shall critique Higuchi et al.'s interpretation which is based on fluid theory, explain why it is inadequate, and then explain how the introduction of nonfluid concepts resolves matters. First, we provide an empirical background by reviewing more of the details of the major observational facts.

2. Properties of Signals Exhibiting Frequency Doubling

Higuchi et al. [1986] and Takahashi et al. [1990] have characterized the properties of frequency-doubling events. The former paper reports that they are a subset of storm time Pc5 low-frequency waves that occur at and beyond geostationary orbit. They are confined near the equator, are associated with the presence of ring current particles, and are most commonly observed in the afternoon-dusk sector between 1400 and 1800 UT. Higuchi et al. [1986] also distinguish the two categories, transitional and harmonic events (T and H). H events (Figure 1a) make up 4% of their total sample of storm time Pc5 wave events; T events (Figure 1b) form 11%.

Takahashi et al. [1987] proved an important symmetry property of one particular frequency-doubled event. As illustrated in Figure 1c, the transverse field amplitude is symmetric about the field line equator (whereas the field displacement is antisymmetric). Cummings et al. [1969] suspected that the purely transverse events detected very close to the geomagnetic equator had this symmetry. Thus, if the signals were standing Alfvén waves, they could not be oscillating in the field line fundamental. We illustrate this in Figure 2 which shows the field line displacements for the fundamental (field line length equal to one half of the wavelength) and the next harmonic (field line length equal to one wavelength) eigenmodes. In the former the displacement is symmetric with respect to the field line equator and so has an antinode there; it follows that there would be no equatorial transverse magnetic perturbation. In contrast, the antisymmetric second harmonic has a displacement node at the equator, the field line is tilted most there, and thus the transverse field perturbation has a local maximum.

Takahashi et al. [1990] surveyed events exhibiting frequency-doubling using data from the AMPTE CCE spacecraft in an elliptical orbit about Earth with apogee at 8.8 \(R_E\). In 700 days of data they discovered 23 events, 17 in the 1630-2000 LT sector and 6 in the 0300-0700 LT sector. They found the events predominantly near apogee at 8-9 \(R_E\) radial distance but noted a bias in their coverage favoring that distance. They questioned the direct storm time association but did not dispute the association of the events with enhanced ring current particle fluxes. The CCE spacecraft covered a range of dipole latitude \(\pm 16^\circ\), and the observations substantiated Higuchi et al.'s [1986] indication that the frequency-doubling events are equatorially confined; more specifically, the phenomenon is associated with the point on the field line where the magnetic field strength is a minimum. They showed that dusk events have right-hand polarization and dawnside events have left-hand polarization with respect to the field. By analysis of the spin phase variation of particle flux in the detectors they deduced that the angular wave number of the signal, \(m\), was large, of the order of 60. They deduced that waves were moving across the field, eastward in the morning

Figure 1. Figures 1a and 1b show examples of frequency-doubled events from Higuchi et al. [1986]. Transitional (T) and Harmonic (H) events are shown as described in the text. (a) H events correspond to Coleman's [1970] rectified events and make up 4% of the sample of storm time Pc5 wave events. (b) T events form 11%. Figure 1c illustrates the symmetry with respect to the equator. It is from Takahashi et al. [1987] and uses data from GOES 2 and 3, SCATHA, and GEOS 2 spacecraft. There is an antinode in the transverse fields at the equator. The compressional signal (at frequency \(\omega\)) has a minimum there, while the second harmonic or frequency-doubled (at 2\(\omega\)) signals are detected only close to the equator.
In addition, as the tube moves earthward, the volume decreases, and there must be a corresponding adiabatic compression of the gas. The maximum change in pressure from this effect occurs with maximum displacement earthward. Using a gas law \( pV^n = \text{const.} \), with \( V \) the volume of a flux tube and \( n \) the polytropic index, one has

\[
\Delta V < 0
\]

Figure 2. Sketch of symmetric and antisymmetric displacements of field lines. Figure 2a corresponds to the fundamental eigenmode and has a displacement antinode and, correspondingly, a transverse field node at the equator. The eigenmode in Figure 2b, the field line second harmonic, has a displacement node and a local transverse field maximum at the equator.

sector and westward at dusk. They pointed out that the high-\( m \) value means that the sense of propagation with respect to the spacecraft may be strongly affected by the sense of the externally imposed \( E \times B \) drift that is eastward at dawn and westward at dusk. It is believed that the waves propagate westward in the plasma reference frame.

Most of the observed features (high \( m \), westward propagation, low frequency, predominantly meridional polarization, equatorial symmetry, and ring current association) reported by Higuchi et al. [1986] and Takahashi et al. [1987, 1990] are entirely consistent with the theory papers attributing the waves to ring current driven bounce-drift resonance instability [Southwood, 1973; Mikhailovskii and Pokhotelov, 1975; Southwood, 1976; Kozhevnikov et al., 1976; Southwood, 1977a; Lin and Parks, 1978; Pokhotelov et al., 1986; Chen and Hasegawa, 1988; Cheng, 1991]. Many (from Southwood [1976] on) predict the occurrence of a compressional component at the same frequency as the transverse. However, none of the above mentioned theories predict the frequency doubling in the compressional component or that the effect is localized near the equator.

2.1. The Higuchi Theory

Higuchi et al. [1986] do present a simple mechanism to explain the frequency doubling. They argue that as a flux tube oscillates in the radial direction, a particle pressure change is produced as a nonlinear effect at 20. A magnetic pressure change then arises. Figure 3a shows two extrema of a transverse displacement of the equatorial part of a flux tube. The change in particle pressure between the extrema arises from two sources: advection of a gradient (producing a pressure change \( \delta p_0 \)) and compression due to flux tube volume change (producing a pressure change \( \delta p_v \)). In a preexisting gradient in pressure the advective change in pressure with displacement \( \xi \) is given, in linear theory, by

\[
\delta p_0 = -\xi \cdot \nabla p
\]
and so the second harmonic or frequency-doubled compressional magnetic component is given by

\[ \delta p_2 = -\eta \frac{\partial V}{\partial t} \]

For a plasma in a background dipole field, or indeed any field in which \( V \times B = 0 \), the linear change in local flux tube volume induced by a meridional displacement in ideal MHD is proportional to the field curvature (and also the field gradient)

\[ \frac{dV}{V} = -\frac{\xi_n \cdot \mathbf{R}}{R^2} = -\frac{\xi_n}{R} \frac{\partial \ln B}{\partial n} \]  

(3)

where \( \mathbf{R} \) is the vector radius of curvature which is in the \( n \) direction (positive inward) and \( \xi_n \) is the meridional displacement measured positive toward the Earth. Thus (5) can be rewritten

\[ \delta p_2 = -\eta \frac{\partial V}{\partial t} = \eta \xi_n \frac{\partial \ln B}{\partial n} \]  

(4)

Let us assume that the displacement \( \xi \) oscillates with frequency \( \omega \) and consider the resulting total gas pressure changes \( \delta p_t + \delta p_w \). The compressional change in pressure, \( \delta p_t \), will oscillate in precise phase with the meridional displacement \( \xi \) toward the Earth at the same frequency. An inward pressure gradient will give rise to an advective change \( \delta p_w \) precisely in antiphase, which will thus tend to cancel the effect. (In an adiabatically injected distribution the effects precisely cancel [see, e.g., Southwood, 1977b, equation (8)].)

\[ \delta p_a = -\frac{n^\prime 2}{2} \left( 1 + \cos^2 \omega \right) \]  

(6)

and one sees that the pressure contains a variation at twice the frequency.

The magnetic compressional signal and, in particular, the second harmonic magnetic signal is found by requiring overall pressure balance in the East-West direction, a natural consequence of large \( m \) [Southwood, 1976, 1977a]. Thus

\[ \delta p + \frac{Bh}{\mu_0} = 0 \]  

(7)

and so the second harmonic or frequency-doubled compressional magnetic component is given by

\[ b_2 = -\frac{n^\prime 2}{4 B} \frac{\partial^2 p}{\partial n^2} \cos 2\omega \]  

(8)

2.2. What is Wrong With the Higuchi Theory?

The explanation of frequency doubling provided by Higuchi et al. [1986], though appealing, does not fit with the observed symmetry of resonantly generated events. As pointed out above, there has long been indirect evidence that high-\( m \) modes have a field displacement that is antisymmetric with respect to the equator. Arguments that invoke flux tube compression in association with flux tube displacements are strongly dependent on the symmetry of the displacement.

Let us consider the changes in local and global flux tube volume in the light of the symmetry consideration. With an antisymmetric displacement, near the equator, the local volume change is slight, whilst the global change (i.e., the change of the entire flux tube volume) is strictly zero by symmetry. Whether one believes that the adiabatic compression is in response to the local or global volume (a point we return to), it is clear that any equatorial change in pressure due to adiabatic changes in flux tube volume is likely to be very small. Moreover, because the field displacement of an antisymmetric eigenmode is zero at the equator, the advective change in pressure is also likely to be negligible even if plasma gradients are present. As long as the field line is believed to be oscillating in an antisymmetric mode, the appealingly simple explanation of Higuchi et al. [1986] seems wrong as it stands. However, we shall argue that it has the germ of the right answer.

3. Generation of High-\( m \) Modes

The ring current plasma in the outer and middle magnetosphere is believed to be injected by the magnetospheric convection system [Southwood, 1977b]. Although the convection may be unsteady, the particles are carried toward the Earth, conserving the two adiabatic invariants \( \mu \) (magnetic moment) and \( J \) (longitudinal invariant). They gain energy as they move in [Kivelson and Southwood, 1975].

High-\( m \) modes can be generated by adiabatically injected charged particles through a bounce-drift resonant interaction [Southwood et al., 1969; Southwood, 1976; Cheng, 1991]. The resonance condition is

\[ \omega - m \Omega_l = N \alpha_b \]  

(9)

where \( \omega \) is the wave frequency, \( \Omega_l \) and \( \alpha_b \) are the bounce-averaged angular drift frequency and the bounce frequency, respectively, and \( N \) is an integer.

Southwood et al. [1969] also pointed that the most easily excited waves should have a field displacement that is antisymmetric about the equator as has been reiterated in later papers [Southwood, 1973, 1976; Chen and Hasegawa, 1988, 1991; Cheng, 1991]. The population of particles feeding energy into the wave is likely to be ions that satisfy the \( N = -1 \) resonance in the limit \( \omega < \mu_b \Omega_l \), i.e., they must satisfy

\[ m \Omega_l = \omega_b \]  

(10)

although other odd negative integer values of \( N \) can make small contributions. We shall show that these particles have a natural response at twice the wave frequency, and furthermore, we explain why such a response is visible only near the equator. Thus we explain the frequency-doubling effect and, at the same time, reaffirm ring current bounce-drift resonant instability as the source of high-\( m \) modes.
4. The Bounce Resonance Theory

In a collision-free plasma the pressure of a group of particles which is thermally isolated is still linked to the volume it occupies. However, the accessible volume is not the same for all particles. The accessible volume for any group of particles near the equator of a field line is a strong function of their pitch angle. Small pitch angle particles have large bounce amplitudes and access to a much greater fraction of the flux tube than do large pitch angle particles that mirror locally. The volume changes detected by a particle also are a function of the timescale on which the particle moves through the wave, hence of the particle's energy and perhaps the phase of its motion with respect to the wave. At one extreme a particle whose bounce period is much longer than a wave period responds to the local change in volume. At the other, a particle bouncing many times in one wave period responds to a global volume change, namely, the volume averaged over its bounce orbit. As we noted above, for an antisymmetric eigenmode neither of these extreme cases leads to net compression or rarefaction near the equator. There is no volume change at the equator, and the bounce-averaged volume change is zero by symmetry.

In fact, we shall show that the only particles that experience secular changes of the flux tube volume are the resonant particles. Resonant particle behavior is reviewed in general by Southwood and Kivelson [1982]. Here we are interested only in the antisymmetric mode case.

As the northern half of the field line tilts outward, the volume of the flux tube increases north of the equator and decreases south of the equator. The field line is in such a configuration for half a wave period, and a direct consequence is that with the right phasing a resonant particle whose bounce frequency matches the wave frequency (Doppler shifted to account for the particle azimuthal drift) can remain continually in the region where the volume decreases. Such a particle must be at its northern mirror point precisely as the field line meridian displacement changes sign, must cross the equator at the time the field displacement is maximum, and will arrive at its southern mirror point just as the displacement next changes sign. At the precise opposite bounce phase the particles start at the southern mirror and remain continually in a region where the volume is increasing as they move northward. The particles in this latter set cross the equator at the same time as the former group but are moving in the opposite direction along the field.

The two groups of resonant particles also move systematically inward or outward. The group that is always in increasing volume is moving outward because it always sees a field tilted outward. The particles at the opposite bounce phase always see a field tilted inward and execute a secular inward motion.

Now, since the displacement changes sign at all points along the field at the time that the particles mirror, the two groups continue to move respectively outward and inward through the

![Figure 4. The sketches illustrate the behavior of two groups of resonant particles during a half cycle of the wave which starts at a node in time of the displacement (i.e., at a time when at each point on the field the displacement is passing through zero). Group A achieves the maximum acceleration in the wave cycle as it sees a decreased volume for the entire time. Group B, initially moving southward, sees a continually increasing flux tube volume. Group B loses energy and will continue to do so as long as it remains in resonance. The dashed line shows the instantaneous field line at the time that the resonant particles cross the equator, and their motion is along that instantaneous field line.](image)

![Figure 5. The meridional projection of the trajectory of a resonant particle from Group A of Figure 4. One and a half bounce cycles are shown. The field line displacement is in the direction of the open arrows along the dashed portion of the trajectory and in the opposite sense along the solid portion of the trajectory. The bouncing resonant particle is phased just such that it always sees a field line tilted inward as it crosses the equator, and thus it moves radially in resonance.](image)
next phase of the bounce. Figure 4 illustrates the process. Figures 4a, 4b, 4c and 4d show the time evolution of the field line. The resonant group A that starts in Figure 4a at the southern mirror point, achieves the maximum acceleration in the half cycle. It sees a decreased local volume for the entire time. For example, as it crosses the equator in Figure 4b, it moves inward along the dashed field line onto compressed flux tubes. As it reaches the northern mirror point in Figure 4c, the sense of the displacement reverses, and it spends the next half cycle in a region where the flux tube volume continues to decrease. Again, this is apparent in Figure 4d, where it is moving onto compressed field lines south of the equator. In contrast, group B, initially at its northern mirror point, sees a continually increasing flux tube volume, and it loses energy and will continue to do so as long as it remains in resonance.

As the energy changes, so does the particle L shell. This effect is easily visualized. Group A, on a field line that is always tilted inward, always moves inward with respect to the undisturbed field for as long as it resonates with the wave. The net motion of a particle in resonance from group A is sketched in Figure 5. Correspondingly, group B sees a field always tilted outward and always moves outward. The resonant particles are not "frozen in" to the magnetic field lines. The field line motion is purely oscillatory. The resonant particles can be seen as "surfing" across the field either inward or outward on the wave.

4.1. Qualitative Description of the Frequency-Doubling Effect

The resonant behavior is so distinct that as early as 1966, Dungey [1966] suggested the observational test of using detectors back-to-back along the field to distinguish groups A and B. The rate of acceleration/deceleration of group A/B peaks twice per bounce. The peak occurs as the particles move through the equator. This fact suggests that resonant particles can excite a doubled-frequency signature in their flux and hence in their contribution to the plasma pressure. One expects a compensating compressional field change to maintain pressure balance as indicated by (7), much like the Higuchi et al. [1986] theory.

A subtlety is that, just like the process invoked by Higuchi et al. [1986], the effect is nonlinear as we now explain. The particles most affected by the wave are the groups A and B identified in the previous section that pass through the equator simultaneously. Because group A is being accelerated and group B is being decelerated, in the linear approximation their combined contribution to the pressure perturbation is zero. (Hence the need for back-to-back detectors invoked by Dungey [1966].) Only when one allows for nonlinear effects does the resonant process become manifest. To see this, we need to look a little closer at the resonant behavior.

4.2. Resonant Cross-L Motion

Higuchi et al.'s [1986] theory relies on the change in pressure being due to both advection and change of energy (or equivalently flux tube volume). Similarly, in the collision-free case, resonant particles change both energy and L shell, and both effects must be accounted for. Southwood et al. [1969] showed that the changes in energy and L in L shell are proportional to each other

$$\frac{dW}{dL_{br}} = \alpha q_{br} L R \frac{dL}{dL_{br}}$$

(11)

where the subscript br indicates "bounce resonance." The form of (11) is important as one can directly compare it with the changes in W and L that occur in adiabatic injection when conserving two adiabatic invariants μ (magnetic moment) and J (longitudinal invariant). Then [Southwood and Kivelson, 1981]

$$\frac{dW}{dL_{ad}} = \frac{dW}{dL_{br}}$$

$$\quad = -q_{br} L R \frac{dL}{dL_{br}}$$

(12)

Evidently, the ratios for the resonant process (non-J-conserving) and the adiabatic process (both μ- and J-conserving) are linked by

$$\frac{dW}{dL_{br}} = \frac{dW}{dL_{ad}} = \frac{\mu}{m}$$

(13)

Using an estimate of the wave frequency $\omega/2\pi$ of $3 \times 10^{-3}$ Hz, and noting that $m$ is of the order of 60 and the drift frequency $m \omega/2\pi$ for a ring current proton of energy 30 keV at synchronous orbit is of the order of $10^{-3}$ Hz, one finds the ratio of the energy change to the L change in resonance is much smaller than the adiabatic value.

$$\frac{dW}{dL_{br}} = 5 \times 10^{-2} \frac{dW}{dL_{ad}}$$

(14)

As the ring current is adiabatically injected, its distribution in energy and L will scale with the adiabatic ratio, and it follows that the resonant process is close to energy conserving, i.e., $\delta W = 0$.

The linear change in the local phase space density, $\delta f$, is found by applying the Liouville theorem

$$\delta f = -\delta W \frac{\partial}{\partial W} - \delta L \frac{\partial}{\partial L} = -\delta L \frac{\partial}{\partial L}$$

(15)

Now consider what happens near the equator. Here groups of particles at opposite phases of bounce motion cross each other (moving in opposite directions along the field). Groups A and B are examples distinguished by the fact that they represent particles which experience maximum inward displacement $\delta L_A$ and maximum outward displacement $\delta L_B$, respectively.

The linear contributions to the pressure perturbation integral

$$\delta p_{lin} = -2\pi \int \frac{dududv}{2} \frac{M a^2}{2} \delta f_{lin}$$

made by groups A and B precisely cancel as by symmetry

$$\delta L_A = -\delta L_B$$

(17)

so contributions to $\delta f_{lin}$ simply cancel as the slope of f is the same for group A as for group B.

To second order a difference emerges. Here groups A is moving in and group B is moving out needs to be allowed for in the assessment of the flux perturbation associated with each group. Group B has moved a total distance $\delta L_B$ purely outward. Its distribution function slope reflects conditions at slightly smaller L. The slope for group B that should be introduced is not the local value but the mean value between $L$ and $L-\delta L_B$ which we denote by a subscript (-). The second order contribution from group B is

$$\delta f_{B}^{(2)} = -\delta L_B \frac{\partial}{\partial L} \left[ \frac{\partial f}{\partial L} \right]$$

$$\quad = \frac{1}{2} \delta L_B \times \delta L_B \frac{\partial^2 f}{\partial L^2} = \frac{1}{2} (\delta L_B) \frac{\partial^2 f}{\partial L^2}$$

(18)

An equivalent expression can be derived for group A. There is no cancellation at this order and one can simply extrapolate to the expression covering all phases

$$\delta f_{A}^{(2)} = \frac{1}{2} (\delta L) \frac{\partial^2 f_{A}}{\partial L^2}$$

(19)

The derivative is taken at (close to) constant energy. It follows that the pressure perturbation can be expressed as

$$\delta p_{lin}^{(2)} = \frac{1}{2} (\delta L) \frac{\partial^2 p_{lin}}{\partial L^2}$$

(20)
4.4. H and T Events

It is shown in (5) that the square term \((\delta E)^2\) introduces the second harmonic term just as it did in the Higuchi et al. theory. As explained earlier, the plasma pressure perturbation will be balanced by an equal and opposite compressional magnetic field oscillation also at the doubled frequency. There is a definite phase prediction implied by our model. If the waves are being driven by particles, the particles that lose energy must dominate the resonant response. There must be more resonant particles moving outward and losing energy to the waves than moving inward and gaining energy (i.e., more in group B than in group A). The sense of the associated particle pressure fluctuation will be determined by group B. It follows that twice per period the pressure will reach a minimum when the field line is tilted maximally at the equator, \(\pm \delta B\), component (see Figure 1a) is at a maximum or a minimum. The field magnitude perturbation must be out of phase with the particle pressure (see equation (7)), so the field magnitude and the parallel component must be maximum at these times. This corresponds precisely to the variation observed in the Higuchi et al. [1986] events that we have considered.

4.3. Diffusion

Resonant particle motion is commonly described as diffusion. Nothing in the above is inconsistent with the idea that the long-time effect of the waves on the particles is a net diffusion in \(L\). However, despite the apparent similarity of (20) to terms in a diffusion equation [Kac, 1947], the process described here is a purely deterministic process and occurs independent of whatever long-term processes (linear or nonlinear) come into play. More work is planned to elucidate the relationship.

4.4. H and T Events

H events are seen clearly near the equator, whereas T events are found somewhat off the equator. Our argument above applies to the equatorial region and explains how an H-type compressional signal is generated at the doubled frequency \(2\omega\). T-type compressional signals contain contributions at frequency \(\omega\) as well as \(2\omega\). The difference arises because of the field-aligned symmetry of the wave displacement. In an antisymmetric signal the displacement is zero at the equator, and so nonresonant particles do not experience advection. Off the equator this is no longer true. Equation (1) shows that there is an advected change in pressure

\[
\delta p_\theta = -\mathbf{\nabla} P_{\text{loc-rad}} \tag{21}
\]

that is proportional to the displacement and the local background gradient in the nonresonant particle pressure. The T events which contain a compressional signal at \(\omega\) and \(2\omega\) result from a combination of the advected compressional change at frequency \(\omega\) with the \(2\omega\) signal which will persist near but off the equator.

The precise phasing observed is a function of whether the spacecraft is above or below the equator and the sense of the background pressure gradients in both nonresonant and resonant particles. Throughout the bulk of the ring current region the normal pressure gradient is directed toward the Earth. In this case the nonresonant plasma pressure decreases above/below the equator when the field tilts in/out. Thus the compressional magnetic field at frequency \(\omega\) rises above/below the equator when the field tilts in/out. The resonant particle fluxes oscillate at \(2\omega\) and are phased such that the peaks in field strength occur as group A/B passes through the equator, i.e., at times of maximum inward/outward field displacement. The observed phase locking shown in Figure 1b naturally follows.

5. Equatorial Localization: Coherence

There is a further reason for the equatorial confinement of both H and T events. The mechanism for frequency doubling depends on the coherent addition of the pressure contributions from all resonant particles independent of their pitch angle. If resonant particles with widely separated mirror points (i.e., very different equatorial pitch angles) cross the equator together, then off the equator they move differently, thus destroying the coherence of their wave pressure contributions elsewhere.

The dependence on equatorial pitch angle is implicit in the general bounce resonance condition given in (9)

\[
\omega - m\Omega_p = N\omega, \quad N = \pm \text{integer} \tag{22}
\]

Here the bounce-averaged drift frequency is proportional to particle energy, and the bounce frequency is proportional to particle velocity. Both vary with equatorial pitch angle. The bounce-averaged drift frequency (in a dipole field) is \(-50\%\) faster for particles with \(90^\circ\) equatorial pitch angles than for particles with \(0^\circ\) equatorial pitch angles [Hamlin et al., 1961]. The bounce frequency of particles with the same energy and different equatorial pitch angles also varies by \(-50\%\) in a dipole field with the equatorially confined particles bouncing fastest. The similarity of the dependence on equatorial pitch angle means that for given \(m\) the resonant energy depends little on pitch angle for particles in the \(m\Omega_p = \omega_0\) dominant resonance (equation (2)). However, resonant particles of different pitch angles do not move coherently as they leave the equator. Near the equator the instantaneous angular drift velocity \(\omega_0(s)\) of particles with \(0^\circ\) pitch angles is twice that of \(90^\circ\) particles even though their bounce-averaged drift velocity is \(33\%\) slower. Clearly, for particles mirroring off the equator the motion across the field through the disturbance is a strong function of latitude. The net effect of the pitch angle dependence of the instantaneous drift rate is sketched in Figure 6. The loss of coherence between particles with different mirror points reduces the resonant pressure perturbations far off the equator. Although a particular group of resonant particles of different equatorial pitch angles will spread out in longitude as they move along the field, the symmetry of the resonance process brings the particles that receive the largest radial displacements together each time they approach the equator, and this imposes coherence there.

![Figure 6](image-url)

Figure 6. A sketch illustrating how particles of different equatorial pitch angles move azimuthally with respect to wave phase fronts as they bounce. Position in the bounce orbit is shown on the ordinate, while the abscissa is the azimuthal phase in a frame rotating with the wave. Both particles cross the equator at the same point, which is required by symmetry. The near-equatorially mirroring particle has a much smaller variation in angular speed and is initially overtaken in its longitudinal motion by the off-equatorial particle which then travels much more slowly near its mirror point.
A secondary effect that works to reduce the size of any compressional disturbance away from the equator is that (3) indicates that the associated magnetic perturbation is inversely proportional to the local magnetic field strength:

$$b_n = -\frac{\mu_0 \delta p}{B}$$  \hspace{1cm} (23)

Together these effects imply that the magnetic perturbations produced by resonant particles are significant near the equator and become negligible off the equator. Nonresonant particles respond to the wave at the frequency $\omega$ as discussed in the analysis of Higuchi et al.'s [1986] model, and their contribution disappears at the equator. The sum of the contributions of resonant and nonresonant particles provides a superposition of compressional perturbations at $\omega$ and $2\omega$ off the equator, and when the amplitudes are comparable, transitional events may appear.

6. Conclusions

We have presented a new theory to explain the long-observed frequency-doubling phenomenon detected in the compressional component of high-$m$ ULF signals seen in the Earth's magnetosphere. We propose that the effect is caused by a nonlinear response of bounce-resonant ring current particles to the compressional perturbations of a transverse radially polarized wave. The resonant response accounts for the observed frequency-doubling effect and for the localization of that signature near the magnetic equator. The particular phase relationship detected in the reported wave events indicates not only that resonant particles are responsible for the frequency-doubled signal but that the waves are receiving energy from those particles. In other words, the waves are unambiguously being resonantly generated.

The nonlinear process that we have described has significant implications for the physics of the ring current which drives the waves. The coupling to the waves produces efficient cross-field transport for resonant particles. In this way the ULF waves act to limit ring current growth. We have also pointed out that the process can lead to particle acceleration if the ring current distribution gains energy through inward transport by adiabatic processes and, subsequently, resonant particles interacting with ULF waves are transported outward with little energy change. Such a process, if repeated in a cyclical manner, could result in significant particle acceleration beyond the limits posed by purely adiabatic injection from the solar wind or the magnetotail.

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