ASPECTS IN COMMON OF HIGH LATITUDE IONOSPHERIC VORTEX MOTIONS

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ABSTRACT

It is shown that travelling vortices in the ionospheric flow convey momentum in the direction of the phase motion. We use a simple approximate description of flow in the ionosphere based on an analogy with two dimensional incompressible flow. A non-linear calculation is given which shows that although to first order a travelling vortex may carry no net momentum, the net momentum in the direction of travel is proportional to the square of the flow amplitude. The result shows that transport in the ionosphere of both momentum and magnetic flux is independent of the mechanism by which the vortex pattern is generated at high altitude.

INTRODUCTION

Recently there has been much interest in large scale travelling vortices in the ionospheric flow which move tailward in the vicinity of the polar cap boundary. There now exists a large literature [1], [2], [3], [4], [5], [6], [7], [8], [9] on the occurrence of large amplitude travelling ionospheric vortices (TIVs) derived from a variety of data sources.

The motivation for such studies has very largely been the importance attributed to the identification of the ionospheric counterparts of magnetospheric boundary phenomena associated with transfer of energy and momentum to the terrestrial system from the solar wind. The major theoretical result that we report in this paper contains the fascinating implication that at ionospheric heights the sense of momentum and flux transport in a traveling vortex structure is determined from the sense of (phase) motion of the traveling structure rather than on any features of the detailed structure of the flow pattern. The result follows from the incompressible nature of large scale magnetohydrodynamic flow in the ionosphere. It implies that if structures representing vortical motion in the ionosphere are detected moving tailward then momentum transfer is taking place regardless of the precise mechanism by which the vortices are generated. Another remarkable conclusion is that if solar wind pressure changes are responsible for generating vortices traveling in the anti-solar direction then decreases in solar wind pressure are as effective in transferring momentum as increases in pressure.

SOURCES OF IONOSPHERIC VORTEX FLOW

The global high latitude flow in the ionosphere was originally proposed to be driven by coupling to the solar wind through magnetic reconnection by Dungey /10/ and there is now fairly general acceptance that this is the dominant process. The pattern of flow produced in the ionosphere on a global scale is a twin vortex with flow cells on the dawn and dusk sides of Earth. By analogy localised reconnection is expected to produce twin vortex patterns on the local scale /11/. However this is not the only potential source of localised vortex flows. In addition to localised reconnection at the magnetopause which are often attributed to flux transfer events (FTEs) /11/,/12/,/13/,/14/,/15/,/16/,/17/ potential sources for TIVs include solar wind pressure perturbations travelling along the magnetopause [ref. /18/ and references therein] or nonlinear Kelvin Helmholtz waves at the magnetopause /19/,/20/. There have been some fierce debates over the mechanism of generation of TIVs. However, a recent paper by the present authors /21/ has shown that the general form of TIVs carries important implications for the mean transport of flux and material from day to night. The simple calculation reproduced here shows that, independent of the details of the mechanism by which the signatures are excited, TIVs transport significant amounts of both momentum and magnetic flux in their direction of travel. The magnitude as well as the sense of transport is independent of the nature of the high altitude source.

APPROACH AND PURPOSE OF PAPER

In this paper we reproduce using a slightly simpler argument than that presented by Southwood and Kivelson /21/ illustrating the very powerful result that traveling vortices in horizontal ionospheric flow are carrying momentum in the direction of phase motion (i.e. in the direction of travel of the overall flow pattern).

For theoretical analysis of vortex motion, we introduce a particularly simple model of plasma behaviour which uses the close analogy between incompressible fluid flow ($\nabla \cdot \mathbf{u} = 0$) and mesoscale ionospheric flow (flow on horizontal scales $\gg 10$ km but much less than global). Because the Earth's field strength is large enough that the ionospheric flow cannot significantly compress the Earth's magnetic field, horizontal ionospheric flow resembles classical two dimensional incompressible flow in being magnetically incompressible.

The ionosphere is modelled as a flat thin collisional layer of thickness, h , threaded by a vertical uniform magnetic field $\mathbf{B} = B \hat{\mathbf{z}}$ with a vertical magnetic field. Generalisation to a tilted field is straightforward but the mathematics is much simplified if we make the vertical assumption. The ionosphere is a low β plasma with $c_s \ll V_A$, where $c_s^2 = \gamma p / \rho$ with γ , the ratio of specific heats, p the plasma pressure, and ρ the ion density) is the sound speed and V_A is the Alfvén speed ($V_A^2 = B^2 / \mu_0 \rho$).

THE VORTICITY EQUATION

Vorticity plays a very basic role in 2D incompressible hydrodynamic flow. The magnitude of the Laplacian of the streamfunction (ψ) is equal to the magnitude of the vorticity (Ω). Subject to sign convention, one has

$$\nabla^2 \psi = -\Omega \quad (1)$$

Analogy with the relation between electrical charge and electrical potential shows that the vorticity can be seen as the source of the flow field. Southwood and Kivelson

/21/ derive a vorticity equation for a simple model ionosphere threaded by a vertical magnetic field. In their model vorticity is conserved following the motion of the plasma except where it is put in from higher altitude by field aligned currents or modified by non-uniform neutral collisions.

$$\frac{1}{v} \frac{D\Omega}{Dt} + \Omega = \mathbf{u} \times \nabla(\ln v\rho) + \frac{B}{h\rho v} j_{\parallel} \quad (2)$$

where the differential operator, D/Dt , represents the (Lagrangian) derivative taken along the path of a parcel of fluid, h is the vertical thickness of the ionosphere, ρ is mass density and it has been assumed that there are no local sources of ionisation. When the left hand side is zero, the right hand terms show that vorticity is brought to zero on the time scale of the (height averaged) ion-neutral collision time, v^{-1} , a relatively short time ($\ll 1$ s) compared with high latitude ionospheric flow time scales in the terrestrial ionosphere. The time derivative in the equation is the Lagrangian derivative; thus once the vorticity of any parcel of fluid is brought to an equilibrium value the vorticity of the parcel is conserved. In equilibrium one has (3)

$$\frac{D\Omega}{Dt} = 0 \quad \text{and} \quad \Omega = \nabla \times \mathbf{u} = \mathbf{u} \times \nabla(\ln v\rho) + \frac{B}{h\rho v} j_{\parallel}$$

assuming for now that $j_{\parallel}$ does not vary with time. Note how the equilibrium vorticity and field aligned current are closely but not exactly linked. They are proportional only if (4)

$$\nabla \times (\rho v \mathbf{u}) = 0.$$

a condition that is discussed further in ref. /21/.

SOLVING FOR FLOW IN A HORIZONTALLY UNIFORM MODEL IONOSPHERE

If the ionosphere is uniform and there is no field aligned current input, the equations governing the possible flow systems are

$$\nabla \cdot \mathbf{u} = 0 \quad \text{and} \quad \nabla \times \mathbf{u} = 0 \quad (5)$$

In the nearly uniform case, the two equations represent continuity and the momentum equation, respectively, and thus should be capable of producing a unique flow pattern for a given set of boundary conditions. In a flow system governed by the equations it is well known that one may describe the flow by either a stream function ψ or a potential, ϕ . Both, ψ and ϕ satisfy Laplace's equation

$$\nabla^2 \psi = \nabla^2 \phi = 0 \quad (6)$$

It is also well known that in an infinite medium or a closed system with no boundaries (such as a thin spherical shell) the only solution of Laplace's equation is the trivial one with ψ, ϕ constant, leading in either case to the solution, $\mathbf{u} = 0$.

For a nontrivial flow solution which closes everywhere or vanishes at large distances, one must have somewhere either the Laplacian of the streamfunction nonzero or the Laplacian of the velocity potential nonzero. The former condition corresponds to the vorticity being specified somewhere, the latter to there being a source of material

somewhere. An alternative possibility is to introduce a boundary in the flow on which inhomogeneous boundary conditions are specified. It turns out that in such a case the boundary conditions are equivalent to there being a source of vorticity or a source of material at the boundary.

MOMENTUM AND FLUX TRANSPORT BY TIVS

In an incompressible medium any motion can be instantaneously described by streamlines which all close. In a stationary vortex, it follows that the streamlines do not change and any fluid element passes repeatedly through the same points. Streamlines and particle orbits are the same and the maximum displacement possible for any element in the flow is bounded by the scale size of the streamline. In such a flow there cannot be any net transport in the mean. The situation is very different when the flow

Figure 1: The flow pattern of a surface wave in a plane containing the normal to the surface. The pattern as a whole moves along with the phase motion of the wave. It follows that the pattern represents the instantaneous form of the stream lines.

pattern is not time stationary. The streamlines vary continually with time and the trajectories of fluid elements are not the same as the instantaneous streamline pattern. We next discuss these concepts further.

A surface wave can be regarded as a series of travelling vortices on the fluid surface and an early nonlinear calculation of the net transport in a surface wave of sinusoidal form was done by Stokes in 1848 /22/. In Figure 1 we show an illustration of the instantaneous flow in a surface wave. IN the case of a magnetospheric flow projected from the magnetosphere into the ionosphere, the "surface" on which the vortices are excited is formed from the magnetic shell of field lines wich map from the boundary down to the high latitude ionosphere where they mark the boundary of the polar cap.

For a very small amplitude traveling sinusoidal surface wave, elements of the fluid move in circles in planes perpendicular to the surface (i.e. horizontally in the ionosphere), the sense of rotation being a function only of the direction of wave propagation along the surface. After an integral number of periods the elements return to their original positions. To first order in wave amplitude there is no net transport. This is illustrated in Figure 2(a)

However once the variation of the flow field over the fluid element orbit is allowed for, the orbits depart from circularity. Stokes showed that when the variation was included across the orbit each element moves at a higher speed parallel to the

boundary in that part of its orbit closest to the surface of the fluid. Because it is moving in the direction of wave propagation at the "top" of its orbit (near the surface) there is a systematic motion imposed in the direction of the wave propagation. The Southwood and Kivelson /21/ calculation is similar but is not restricted to the simple

Figure 2: On the left is illustrated the motion of an element of the fluid in a small amplitude surface wave. The orbit is circular and there is no nett transport of the material in any direction as the waves pass. On the right the non-linear motion is illustrated for a wave propagating to the right. In this case the curvature of the orbit varies across the orbit and the result is net motion of the

form of wave chosen by Stokes. The linear and the non-linear motions are illustrated in Figure 2 below.

NONLINEAR CALCULATION

We do not repeat the Southwood and Kivelson /21/ calculation here but give a shorter slightly more restricted calculation. As in /21/ we use a model in which the (two dimensional) ionospheric plasma occupies a half space bounded by the line $y=0$ which might be seen as representing the polar cap boundary. The third dimension is along the magnetic field and its exclusion means we are assuming that there is good mapping along the field into the magnetosphere. The vorticity in the flow is imposed from above, i.e. from the magnetosphere. We shall assume that vorticity is imposed along the surface $y=0$ which represents the polar cap boundary. The essence of the

TIV is its travelling nature and we represent this feature by assuming that the vorticity Ω varies in x (longitude) and t (time) such as to to simulate the motion of a pattern along the direction of the boundary at a phase speed c .

$$\Omega = \Omega(x - ct) \delta(y) \quad (7)$$

where $\delta(y)$ is the Dirac delta function.

Formally solutions of the two differential equations (5) can be found by a standard method such as separation of variables, once the vorticity distribution has been specified.

Separation of variables naturally yields a solution in terms of Fourier transforms. Solutions for the x -component of flow field for $y > 0$ and $y < 0$ are

$$u_x(x, y, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dk \frac{\Omega(x - ct)}{-2} \exp(\pm ky) \exp(-ikx) \quad (8)$$

where the upper (lower) sign within the integral corresponds to $y < 0$ ($y > 0$) respectively. A similar expression can be found for u_y . [The decay of the solutions away from the source at $y = 0$ means that the boundary conditions far away are not important.]

The form of the solution shows that the flow field varies in x and t with the same phase structure as the source vorticity and so

$$u(x, y, t) = u(x - ct, y) \quad (9)$$

For calculating the nonlinear impact of the disturbance on the ionospheric plasma Southwood and Kivelson /21/ introduce the plasma displacement ξ . ξ represents the displacement that an element of the plasma at x and y at time t has received during the passage of the flow pattern. By definition ξ and u are related by the integral relation,

$$\xi(x_0, y_0, t) = \int_{\text{orbit}}^t dt' u(x(t'), y(t'), t') \quad (10)$$

where the orbit is given by

$$\mathbf{r} = (x(t), y(t)) = (x_0, y_0) + \xi(x, y, t) \quad (11)$$

and where the integral is carried out over the orbit of the fluid element. We can draw a distinction between the velocity of the fluid and the velocity of an element by noting that at each point in the fluid at any instant

$$\mathbf{u}(x, y, t) = \frac{\partial \xi}{\partial t} \quad (12)$$

Thus velocity of any element can be calculated

$$\dot{\mathbf{r}}(x, y, t) = \frac{\partial \xi}{\partial t} + \mathbf{u} \cdot \nabla \xi = \mathbf{u} + \mathbf{u} \cdot \nabla \xi \quad (13)$$

As $\mathbf{u}$ is known, our problem is to calculate $\xi(x, y, t)$ which we do by successive approximation. To first order, we have

$$\xi^{(1)}(x, y, t) = \int_{-\infty}^t \mathbf{u}(x, y, t) dt. \quad (14)$$

(i.e. ignoring the displacement in the orbit in the integral). Note that $\xi^{(1)}$ shares the phase variation of the flow field $\mathbf{u}$, thus

$$\frac{\partial \xi^{(1)}}{\partial t} = c \frac{\partial \xi^{(1)}}{\partial x} \quad (15)$$

and also

$$\nabla \times \xi^{(1)} = 0 \quad (16)$$

We then substitute in (11) to obtain the second order correction

$$\dot{r}^{(2)}(x, y, t) = u \cdot \nabla \xi^{(1)} \quad (17)$$

We now can write down the expression for the second order momentum of the element

$$\rho \Delta V \dot{r}^{(2)} = \rho \Delta V u \cdot \nabla \xi^{(1)} \quad (18)$$

where ΔV is the elemental volume and ρ is the mass density.

Now in any steady vortex flow pattern in which all streamlines close $\langle u \rangle = 0$ where the brackets indicate the average (over t). Once the flow pattern is allowed to be time dependent one finds that to first order also there is no net displacement. However, to second order there is the contribution of the term given in (18) to the displacement and so there is a net second order displacement and accordingly velocity and momentum. The second order momentum is found by differentiation and has the form

$$\rho \left[u_x \cdot \frac{\partial \xi_x^{(1)}}{\partial x} + u_y \cdot \frac{\partial \xi_x^{(1)}}{\partial y} \right] \quad (19)$$

Using the relations (14), (15) and (16), we can rewrite the terms in the following way

$$\frac{\partial \xi_x^{(1)}}{\partial x} = \frac{1}{c} \frac{\partial \xi_x^{(1)}}{\partial t} = \frac{u_x}{c} \quad \text{and} \quad \frac{\partial \xi_x^{(1)}}{\partial y} = \frac{\partial \xi_y^{(1)}}{\partial x} = \frac{1}{c} \frac{\partial \xi_y^{(1)}}{\partial t} = \frac{u_y}{c} \quad (20)$$

An exactly similar calculation can be done for the mean momentum in the y-direction. In this case the terms cancel precisely and thus the only nonlinear contribution to the momentum is in the x-direction i.e. in the direction of propagation of the travelling disturbance.

Thus the total second order momentum density, P , in the x direction is given at any point by

$$P = \rho(x, y, t) ([u_x(x, y, t)]^2 + u_y(x, y, t)^2] / c \quad \hat{x} \quad (21)$$

IMPLICATIONS

The important insight afforded by the above calculation and the result (21) that there is necessarily a net transport of magnetic flux as well as of momentum in the direction of phase motion of *any* propagating ionospheric disturbance. This is true even in the absence of actual net flow in the instantaneous flow pattern. Propagating disturbances, whether quasi-isolated vortical flows or wave-like motions, transport magnetic flux and momentum in the direction of phase motion independent of the nature of the

boundary disturbance driving the ionosphere. In other words, if travelling vortex motions are detected in the ionospheric flow, one may immediately draw conclusions in regard to overall transport without needing to determine the precise mechanism (for example, sporadic reconnection or magnetopause Kelvin Helmholtz instability) generating the vortices high in the magnetosphere. Conversely, the ionospheric signatures of magnetospheric processes cannot necessarily be used for distinguishing between the mechanisms operating at high altitude (In essence the problem lies in the fact that the flow in the ionosphere is constrained to be incompressible; the compressional element of the source flow which is suppressed in the ionosphere may be a critical feature of the flow near the source (cf. the excitation of vortex flows by solar wind compressions /8/)).

The amount of flux transported increases as the square of the ratio of the mean eddy speed to the phase speed of the disturbance, a quantity that must be small for the approximations that we have made to be strictly valid.

For a typical TIV with phase speed of order 5 km/s and rms eddy speed of 1000 m/s, the magnetic flux transported is ~20% of the total flux within the area spanned by the eddy. If a typical eddy has a spatial scale of ~1000 km and the polar cap diameter is taken to be of order 5000 km, a single eddy can transport 1/25 of the polar cap flux. Individual eddies make relatively minor contributions on a global scale, but, as for flux transfer events (FTEs), with frequent repetition their effect can become significant.

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