THE EFFECT OF MASS LOADING ON THE TEMPERATURE OF A FLOWING PLASMA

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Abstract. We have investigated how the addition of ions at rest (mass loading) affects the temperature of a flowing plasma in a magnetohydrodynamic (MHD) approximation, using analytic theory and time dependent, three-dimensional MHD simulations of plasma flow past Io. The MHD equations show that the temperature can increase or decrease relative to the background, depending on the local sonic Mach number ($M_S$) of the flow. For flows with $M_S > \sqrt{9/5}$ (when $\gamma = 5/3$), mass loading increases the plasma temperature. However, the simulations show a non-linear response to the addition of mass. If the mass loading rate is large enough, the temperature increase may be smaller than expected, or the temperature may actually decrease, because a large mass loading rate slows the flow and decreases the thermal energy of the newly created plasma.

Introduction

The term "mass loading" refers to the addition of mass to a flowing plasma by ionization of neutrals. Io, Jupiter's innermost Galilean satellite, is embedded in a flowing plasma (the Io torus) and is also surrounded by a cloud of neutral atoms and molecules [Brown et al., 1983, and references therein]. In this paper we present results from three-dimensional MHD simulations of plasma flow past Io, including the effect of ionization of neutrals in the equations. Ionization may modify the plasma temperature significantly. We demonstrate these effects by examining the MHD equation for temperature and the results of simulation runs for three different values of $M_S$.

Mass loading plays an important role not only in Io's interaction with the plasma torus, but also in the solar wind interaction with comets and with Venus, in plasma flow past Saturn's satellite Titan, and possibly in other plasma-satellite interactions. Although the simulation parameters we use in this paper correspond most closely to the Io torus parameter regime, our remarks regarding the plasma temperature (in an MHD approximation) apply to the plasma in these other systems as well.

The MHD Equations and the Plasma Temperature

We use the time-dependent MHD equations, including the effect of neutrals ionized near Io. When Jupiter's gravity is accounted for, the escape velocity for neutrals from the surface of Io is about 2.3 km/s [Linker et al., 1985] and the neutral density in Io's exosphere is dominated by lower velocity neutrals on non-escape trajectories [Watson, 1981; Sieveka and Johnson, 1984; Linker 1987; McGrath and Johnson 1987]. The torus plasma flows at 57 km/s relative to Io, so on average, the neutrals can be assumed to be at rest relative to Io. With that assumption, the normalized MHD equations can be written in the form:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{1}
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \nabla \frac{\mathbf{p}}{\rho} = \frac{\mathbf{J} \times \mathbf{B}}{\rho} \tag{2}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = - \nabla \times (\mathbf{v} \times \mathbf{B}) \tag{3}
\]

where \(\rho\) is the density, \(\mathbf{v}\) the velocity, \(P\) the plasma pressure, \(\mathbf{B}\) the magnetic field, \(\eta\) the resistivity, and \(\mathbf{J} = \nabla \times \mathbf{B}\) the current density. \(\rho_s\) is the mass added per unit volume per unit time. We take \(\gamma = 5/3\). (Viscous terms are added to (2) and (3) in the simulation runs for numerical stability purposes, but they are irrelevant to the present discussion.) Equation (3) can be derived in a straightforward manner, using (1) and an equation for the conservation of energy (note that \(d/dt\) is a convective derivative) [Linker, 1987]. Because we consider that ionization of neutrals results from electron-impact, there should also be a heat loss term in (3) representing the energy given up by the electrons. However, the flow energy gained by an oxygen ion created in the background flow (280 eV) is large compared to the energy lost by the electron (10-12 eV). The inclusion of the electron energy loss term did not alter the simulation results significantly; therefore, we have ignored the term.

From (1-3) we can also derive equations for the plasma velocity and plasma pressure similar to those used by Schmidt and Wegmann [1982] and Ogino et al. [1988]:

\[
\frac{d\mathbf{v}}{dt} = - \nabla \cdot \left( \left( P + \frac{B^2}{2} \right) I - \mathbf{B} \mathbf{B} \right) - \rho_s \mathbf{v} \tag{5}
\]

\[
\frac{\partial P}{\partial t} + \nabla \cdot (P \mathbf{v}) = (\gamma - 1) \left( - P (\nabla \cdot \mathbf{v}) + \frac{\eta J^2}{\rho_s} + \frac{1}{2} \rho_s v^2 \right) \tag{6}
\]

We see from (1), (5), and (6) that mass loading causes increases in the plasma density and pressure, and decreases in the plasma velocity. In the context of the Io torus, it has been noted that mass loading can increase the plasma temperature [e.g., Goertz, 1980; Brown et al., 1983; Shemansky, 1988]. However, if we assume an ideal gas law ($P = \rho RT$), (6) implies that the plasma temperature, \(T\), satisfies

\[
\frac{dT}{dt} = (\gamma - 1) \left( \frac{3 \rho_s v^2}{\rho R} - T (\nabla \cdot \mathbf{v}) + \eta J^2 \right) \frac{1}{\rho R} - \frac{\rho_s T}{\rho} \tag{7}
\]
If we neglect effects on the plasma velocity, then, relative to cases with no mass loading, ionization adds to (7) one term that decreases the temperature \(\left(\rho_b T_b / \rho_0\right)\) and one term that increases it \(\left((1/2)\rho_b u^2 / \rho_0\right)\). The former term arises because when mass is added to a fluid element, the thermal energy initially present must be shared by a larger number of ions, and this decreases the local temperature. The latter term arises because when an ion is added to the fluid element and is accelerated up to the background flow speed, some flow energy of the plasma is converted to thermal energy. This increases the thermal energy in the fluid element. If the flow energy of the plasma is greater than the thermal energy, the plasma will be heated by mass loading. Conversely, if the flow energy is less than the thermal energy, the plasma cools.

The ratio of the heating term to the cooling term is \(\frac{\rho_b (1-\beta) (M_s^2)}{\rho_0} = \frac{1}{2} (M_s^2)^3\) where \(M_s\) is the sonic Mach number. One would expect heating by mass loading to occur only if this ratio is greater than 1, or \(M_s > \sqrt{9}/5\). As we have assumed minimal effects of mass loading on the flow velocity, the scaling for thermal effects that we have derived here is only approximate. We note that the thermal effects are not directly dependent on \(\beta = 2P/B^2\). \(\beta\) describes the relative importance of thermal energy and magnetic energy, whereas the effects of mass loading on the plasma temperature depend on the relative importance of thermal and flow energies locally in the plasma.

The \(M_s\) in the Io torus is uncertain; \(M_s = 1.8\) for oxygen ions and 2.5 for sulfur ions in the thermal plasma, assuming a background temperature of 100 eV [Bagenal et al., 1985]. However, the flux of ions with energies > 6 keV has not been measured, and their contribution to the pressure is probably significant. Thus, \(M_s\) is probably smaller than the above values. For example, assuming an average mass number of 20, if all of the ions > 6 keV gave the same contribution to the plasma pressure as a 1% population of 30 keV ions, \(M_s\) would be 1. Even a 1% population of 10 keV ions would make \(M_s = 1.4\). In this paper we consider results for simulations with \(M_s = 0.55, 1,\) and 1.5. These values allow us to examine the effects of mass loading on plasma temperature for parameter regimes ranging from subsonic to supersonic, and the values are close enough to the torus parameter regime to permit useful comparison. Simulations at higher \(\beta\) presented fewer numerical difficulties and were less expensive to perform, so we used values of the Alfvén Mach number \(M_A\) of 0.5 and 0.7, higher than those that actually occur in the Io torus.

The MHD equations obviously do not describe all of the important phenomena associated with the creation of new ions in a plasma. However, they do give a good description of the global effects of ionization on the flow. We note that by using the MHD equations, we make the implicit assumption that when ionization occurs, the ion is introduced into a volume element in thermal equilibrium with the fluid contained in it. The assumption seems reasonable for the Io torus, where the velocity of the plasma is nearly perpendicular to the magnetic field, and the \(\nabla \times \mathbf{B}\) electric field accelerates newly created ions to the local flow velocity. Even for flow parallel to the magnetic field, Omidi and Winske [1987] have shown that instabilities driven in the plasma can cause the ions to be picked up by the background flow (although the process is not so efficient as in the case of flow perpendicular to the magnetic field). Plasma instabilities can also help to bring about thermal equilibrium, but in general the plasma distribution is likely to be more complicated than can be described by a single temperature for all plasma species. Nevertheless, a nominal plasma temperature is useful for describing the net thermal energy gained or lost as a result of ionization. We note that assumptions about the thermal effects of ionization are an important component of studies of the energy balance of the Io torus [e.g., Barbosa et al., 1983; Smith and Strobel, 1985; Semeshnky, 1988].

**Simulation Results**

From our analysis of (7), we would expect that when the flow is subsonic or transonic, mass loading would cool the plasma, whereas when \(M_s > \sqrt{9}/5\), mass loading would heat the plasma. In this section we examine the results for simulations run in these different parameter regimes. Our simulation code solves equations (1-4) as an initial value problem in spherical coordinates using a two-step Lax-Wendroff finite difference scheme [Linker et al. 1988]. We note that for the results shown in this paper, the fluid Reynolds number \(R_e = 50\) and \(R_m\), the magnetic Reynolds number \((V_A L/\eta, \text{where } V_A \text{ is the Alfvén speed}\) and the length scale \(L\) is an Io diameter), is 100. All results are shown in Io’s rest frame, after the simulation has reached a quasi-steady state.

For electron impact ionization and a constant electron temperature, the mass loading rate \(\rho_0\) is proportional to the neutral density. We assume that the neutral density is spherically symmetric about Io, and we consider two types of radial variations for the neutral density. For most of the runs presented, the neutral density falls off like \(1/r^2\). We also consider a radial variation based on a model of Io’s exosphere [Linker, 1987]. We parameterize the mass loading models in terms of the total number of ionizations per second occurring within 6 Io radii \((R_{Io})\) of Io (roughly the dimension of Io’s exosphere). Voyager observations limit the ionization rate of sulfur and oxygen atoms near Io to \(< 10^{27} \text{s}^{-1}\) [Shemansky, 1980], but there is no limit on the ionization rate of molecular species. Accordingly we consider models with ionization rates both less than and greater than \(10^{27} \text{s}^{-1}\). We note that because the electron impact ionization process reduces the electron temperature, it is not clear that the torus plasma could actually support the highest ionization rate that we consider here \((6 \times 10^{27} \text{s}^{-1})\). A drop in the electron temperature near Io that would reduce ionization efficiency has been reported [Sittler and Strobel, 1987].

Figure 1 shows the plasma temperature in the xz plane, represented as a surface, for two different simulation runs.

**Fig. 1.** (a) A surface representation of the plasma temperature in the xz plane for a simulation with \(M_A = 0.5, M_S = 0.35\) and no mass loading. The xz plane is perpendicular to the initial magnetic field. The flow direction is from left to right. Io is in the center of the plot and has a radius of 2 in the units shown. (b) The same as (a) but for a simulation with \(6 \times 10^{26}\) ionizations \(\text{s}^{-1}\) occurring within 6 \(R_{Io}\) of Io, the ionization falling off as \(1/r^2\). Mass loading decreases the plasma temperature in the wake.
The $xz$ plane cuts through Io's equatorial plane, perpendicular to the initial $y$-aligned magnetic field and parallel to the initial $z$-aligned flow in the simulation (Figure 1 of Linker et al., [1988] depicts the simulation coordinate system). A temperature value of 1 in Figures 1, 2, and 4 corresponds to 670 eV (an average mass number of 20 is assumed). Both runs shown in Figure 1 initially had $M_A = 0.5$, and $M_S = 0.55$ and $\beta = 1$. Figure 1a is for a case with no mass loading, and 1b is for a case with a total of $6 \times 10^{24} \text{s}^{-1}$ (ionizations per second) and a $1/r^2$ fall off of the ionization rate. The plasma temperature increases slightly in Io's wake for the case with no mass loading; this results from the slow mode compression described by Linker et al. [1988]. Figure 1b shows the behavior anticipated from our examination of (7). When a small mass loading rate is introduced into a subsonic flow, the plasma temperature decreases slightly in the wake relative to the case with no mass loading. Figure 2a shows the results for a simulation with the same parameters as the cases shown in Figure 1, but with a total of $6 \times 10^{27}$ ionizations $\text{s}^{-1}$. For this case the decrease of plasma temperature in the wake is much larger. Figure 2b shows the plasma temperature for a simulation with the same mass loading rate as for the case in Figure 2a, but with $M_A = 0.5$, $M_S = 1$, and $\beta = 0.3$. The parameter choice for the plasma is probably closer to the actual torus plasma than the parameters used in Figures 1a, 1b, and 2a. We find that as in the case of subsonic flow, mass loading in a transonic flow decreases the plasma temperature in the wake.

The simulations results shown in Figures 1 and 2 are qualitatively consistent with our expectations based on (7). However, the decrease in the plasma temperature becomes especially dramatic as the mass loading rate increases. We can interpret the dramatic decrease by examining the effect of mass loading on the plasma velocity. Figure 3a shows the plasma velocity in the $xz$ plane for the same simulation as Figure 1a (no mass loading). The flow velocity is represented with vectors, for a portion of the total simulation region. Figure 3b shows the same data as Figure 3a, but for the parameters of Figure 2a (high mass loading rate). Comparison of the two figures shows that the plasma velocity in Io's wake diminishes in the presence of a large mass loading rate. The result suggests that the decrease of plasma temperature can be a non-linear process. Initially mass is added to the plasma which slows the flow and reduces $M_S$ locally. Slowly flowing fluid elements remain longer in regions of high mass loading rates, becoming nearly stagnant. As mass is added in the stagnated (low $M_S$) regions of the flow, the plasma cools even further. Eventually the plasma temperature reaches a steady state, because reduction of the temperature reduces the local value of the sound speed, and this acts to increase $M_S$.

Thus far we have examined cases where mass loading led to cooling of the plasma. We now examine what happens when we run cases with $M_S > \sqrt{\beta/5}$. Figure 4 shows the plasma temperature in the same format as Figures 1 and 2, but for simulations with $M_A = 0.7$ and $M_S = 1.5$; $\beta = 0.26$. For Figure 4a, $6 \times 10^{26}$ ionizations $\text{s}^{-1}$ was assumed. In this simulation, mild heating of the plasma occurs in Io's wake region, as we would expect for $M_S > \sqrt{\beta/5}$. Figure 4b shows results from a simulation with $5 \times 10^{27}$ ionizations $\text{s}^{-1}$ and other parameters the same as in Figure 4a. Even though the background $M_S$ is large enough for mass loading to heat the plasma, the plasma has cooled in the wake region. Once again, the explanation of the cooling is apparent from examination of the flow velocity (similar to the results shown in Figure 3b). The high ionization rate greatly reduces the plasma flow velocity in the wake, so that the plasma flow energy in the wake drops below the thermal energy. As more mass is added to the regions of reduced flow (primarily in the wake region), the plasma cools.
Discussion

Equation (7) predicts that the value of $M_S$ determines whether mass loading heats or cools the plasma, and our simulations have confirmed this prediction qualitatively. However, Figure 4 shows that the plasma temperature depends on the local value of $M_S$. The process is non-linear because mass loading affects the plasma velocity as well as the temperature. If the mass loading rate is high enough, cooling of the plasma can occur even when the initial value of $M_S > \sqrt{9/5}$.

As we noted earlier, values of $M_S$ in the torus could be larger than the values we have used. It is possible that if the background flow were more strongly supersonic, the plasma in the wake would still be heated even though the flow speed was reduced in the wake. However, our results suggest that even if $M_S$ in the torus is $> 1.5$, it is difficult to heat the torus plasma strongly by heavy mass loading near Io. Increasing $\rho_s$ adds more mass and thermal energy to plasma in the wake, but because the velocity of the plasma decreases as $\rho_s$ increases, the thermal energy of the pickup ions also decreases. In the Io torus at locations remote from Io, the mass loading rate is expected to be small enough that the non-linear effects we have described will not be important.

Our simulations assume ionization is produced by electron impact. Large rates of charge exchange near Io have also been invoked as a means of providing thermal energy to the torus plasma (Shemansky, 1988). We have not yet investigated the effects of charge exchange near Io in our simulation, but we note that charge exchange can affect the plasma velocity in much the same way as impact ionization. Therefore, strong heating of the torus plasma by charge exchange near Io may have difficulties similar to what we have shown for ionization.

Values of $M_S$ pertinent to other plasma regimes, such as the solar wind near Venus and comets, are much higher than the values we have considered here, so far away from these bodies mass loading will only increase the plasma temperature. However, close to the coma of a comet, the plasma velocity decreases considerably, so the pickup ions could have cooler temperatures than the background plasma. Cooling of the plasma near comets by charge exchange has been noted previously (e.g., Galeev et al., 1985).

Conclusions

We have used three-dimensional MHD simulations of background plasma flow past Io to investigate the effects of mass loading on the plasma temperature. A straightforward analysis of equation (7) reveals that mass loading can increase or decrease the plasma temperature, depending on the value of $M_S$. When $\gamma = 5/3$, $M_S > \sqrt{9/5}$ is required for heating to occur. Our simulations show this analysis to be qualitatively correct when the mass loading rate is relatively small. For large mass loading rates, if $M_S$ is only marginally greater than $\sqrt{9/5}$, heating is less significant than expected, and in some regions cooling may occur because the addition of mass also decreases the plasma flow velocity. The cooling of the plasma by mass loading when the flow is initially supersonic is most likely to be of interest for the Io torus parameter regime where $M_S$ is probably in the range of 1-2. Our simulations show that the effects of mass loading on plasma temperature and velocity are most pronounced in Io's wake region.

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References


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