Kelvin-Helmholtz Instability at the Magnetopause: Energy Flux Into the Magnetosphere

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A magnetohydrodynamic approach is used to investigate how compressional linearly unstable Kelvin-Helmholtz surface waves on the magnetopause extract energy from the velocity shear and radiate that energy away from the boundary in the separate rest frames of the plasmas on the two sides. On the magnetospheric side, the energy transport velocity normal to the magnetopause may become comparable with the Alfvén speed. The energy flux, for typical conditions on the dayside magnetopause, is found to be \( \sim 10^{-7} \) ergs/cm\(^2\) s, and the total energy flux over the unstable region on the dayside magnetopause is estimated to be \( \sim 10^{17} \) ergs/s in magnetic quiet times, or \( \sim 10^{18} \) ergs/s in disturbed times. The discussion of energy input into the magnetosphere illuminates how surface waves on the magnetopause provide the power to drive resonant regions within the magnetosphere at large distances from the boundary.

I. INTRODUCTION

In the past twenty years considerable attention has been given to energy transfer processes at the earth's magnetopause. Although the solar wind can provide energy and momentum to the magnetosphere by particle transport across the magnetopause, it is also possible to transport energy and momentum by magnetohydrodynamic (MHD) wave processes [Sonnerup, 1980; Hill, 1979].

There are two ways in which MHD waves can transfer energy into the magnetosphere. First, the magnetoacoustic waves in the turbulent magnetosheath refract into the magnetosphere carrying wave energy across the magnetopause [McKenzie, 1970; Verzariu, 1973; Wolfe and Kaufman, 1975]. This energy input may contribute, to a modest degree, to the energy budget of the magnetosphere. However, the average transmission coefficient of incident waves is found to be only 1--2\%, which appears to cast doubt on the 'refraction model' as an important transfer agent [Hill, 1979].

Second, the magnetopause itself may develop the Kelvin-Helmholtz (K-H) instability under certain conditions and the associated surface waves can provide energy transfer through the boundary [Southwood, 1979; Tajima and Leboeuf, 1980].

The K-H instability at the magnetopause has long been of interest to space physicists and has been studied quite extensively [Dungey, 1955; Fejer, 1964; Sen, 1964, 1965; Lerche, 1966; Southwood, 1968; Ong and Roderick, 1972; Pu and Kivelson, this issue]. Theoretical and observational work has strongly suggested that the coupling between the compressional surface waves generated at the magnetopause by the K-H instability and the resonant field lines inside the magnetosphere can be the source of the magnetic pulsations observed both in the magnetosphere and on the ground [Dungey and Southwood, 1970; Southwood, 1974; Chen and Hasegawa, 1974; Kokubun et al., 1977; Samson et al., 1971; Singer et al., 1977, 1979]. Detailed calculations of the energy flux associated with the compressional surface waves and estimates of their contribution to the transport of energy into the earth's magnetosphere have not been made. Recently, Southwood [1979] pointed out that, as the K-H instability grows, nonlinear effects can result in a net momentum and a net energy transfer from the magnetosheath to the magnetosphere. Momentum and energy transfer by K-H instability are also important in driving magnetospheric convection [Miura, 1981].

Here we discuss how the K-H instability provides a flux of energy into the magnetosphere and thus seek to illuminate how surface waves on the magnetopause drive resonances of lines of force inside the magnetosphere. In our previous paper [Pu and Kivelson, this issue] we pointed out that in incompressible plasmas, for both stable and unstable surface waves, the real part of the normal component of the wave vector \( k_z \) vanishes identically, indicating that the phase velocity and the group velocity of K-H waves are always parallel to the interface. But in a compressible plasma, the normal component of the wave vector \( k_n = k_z + ik_o \), for unstable surface waves always has nonvanishing real and imaginary parts. Nonzero \( k_o \) means the normal components of the phase velocity are finite and the waves may propagate into the bulk plasmas on the two sides of the magnetopause.

Numerical calculations, as we will confirm in this paper, show that in the separate plasma rest frames on the two sides of the boundary the group velocities, or, more exactly, the energy transport velocities, are also directed away from the boundary. Thus, the compressional linear unstable K-H waves on the magnetopause can radiate energy away from the boundary into the magnetosphere. On the magnetospheric side, the energy transport velocity normal to the magnetopause can be as large as of the order of the Alfvén velocity. This implies that the energy transfer may be quite efficient.

The paper is organized as follows. In section 2 we review the properties of the K-H instability in compressible plasmas. In section 3, we review the wave energy densities in both the magnetosphere and the magnetosheath adjacent to the magnetopause are obtained. In section 4, we present calculations of normal components of group velocities and energy flux and show that compressional unstable surface waves may transfer energy efficiently into the magnetosphere.
5, we discuss our results and compare them with other energy inputs into the magnetosphere.

2. The Kelvin-Helmholtz Instability in Compressible Plasmas

The Kelvin-Helmholtz instability in compressible plasmas has been studied by Fejer [1964], Sen [1964], Southwood [1968], Ong and Roderick [1972], and Pu and Kivelson [this issue] in the MHD approximation. To simplify the problem, it is assumed that the magnetopause is a one-dimensional tangential discontinuity located at the plane \( z = 0 \) with the magnetosheath on side 2 (\( z < 0 \)) moving with velocity \( \mathbf{v} \) relative to the magnetosphere on side 1 (\( z > 0 \)). In this paper, we rely on results derived by Pu and Kivelson [this issue] (referred to as PK-1). Assuming all perturbations to be proportional to \( \exp \left[ i(\mathbf{k} \cdot \mathbf{r} + k_n z - \omega t) \right] \) and starting from the MHD equations in the rest frame of the plasma, we obtain the following linearized equations on each side

\[
-\omega \delta p + \rho (\mathbf{k}_1 \cdot \mathbf{v} + k_n \mathbf{z} - \omega \mathbf{t}) = 0
\]  

(1)
\[- \rho \omega \delta v + C^2(k_t + k_n) \delta \rho \]

\[+ \frac{B \cdot \delta B}{4\pi} (k_t + k_n) - (k_t \cdot B) \delta B / 4\pi = 0 \quad (2)\]

\[- \omega \delta B + [(k_t + k_n) \cdot \delta v]B - (k_t \cdot B) \delta v = 0 \quad (3)\]

where \(k_t\) and \(k_n\) are the components of wave vector tangential and normal to the interface, respectively, and

\[k_n = (k_z + i k_t) \xi \quad (4)\]

\(\omega\) is the angular frequency, \(\rho, B,\) and \(C\) are the mass density, the magnetic field vector, and the sound speed in the unperturbed state, respectively, and \(\delta \rho, \delta v,\) and \(\delta B\) are the perturbations in the density, velocity, and the magnetic field, respectively. Equations (1)-(3) lead to the basic magneto-acoustic dispersion relation [Southwood, 1968; PK-1]

\[\omega^4 - \omega^2(k_t^2 + kr^2)(C^2 + A^2) + A^2C^2 \cos 2\theta(k_t^2 + kr^2) = 0 \quad (5)\]

or

\[\omega^4 - \omega^2(1 + \tau^2)(C^2 + A^2) + A^2C^2 \cos 2\theta(1 + \tau^2) = 0 \quad (5')\]

where \(A\) is the Alfvén speed, \(\theta\) represents the angle between \(k_t\) and \(B,\) \(w = \omega / k_t,\) and \(\tau = k_n / k_t.\) The surface waves occurring on the interface must satisfy the following boundary conditions: (1) the amplitudes vanish at \(z = \pm \infty\) and (2) the normal displacement and the normal total stress are continuous across the magnetopause. The combinations of (5) and the boundary conditions above then yield the dispersion equation for K-H waves

\[\frac{\rho_1(w_1^2 - A_1^2 \cos^2 \theta_1)}{\tau_1} = - \frac{\rho_2(w_2^2 - A_2^2 \cos^2 \theta_2)}{\tau_2} \quad (6)\]

where subscripts 1 and 2 represent the magnetosphere and the magnetosheath, respectively, and

\[w_1 = w - \nu \cos \alpha \quad (8)\]

In (6), the imaginary part of \(k_n\) is required to be positive on both sides of the interface and \(w_1\) and \(w_2\) are related by the Doppler shift

where \(\alpha\) is the angle between \(v\) and \(k_t.\)

PK-1 have shown that in the case of the terrestrial magnetopause, surface waves can be classified according to their phase velocities along the boundary on the magnetospheric side adjacent to the interface as fast waves (\(F\) waves), or slow waves (\(S\) waves). The normal component of the phase velocity of the \(F\) waves in the magnetosphere, as we will show in this paper is also larger than that of the \(S\) wave. The fact that unstable compressional surface waves always require a complex value of \(k_n\) with nonzero real and imaginary parts follows from (7). If \(w = w_1 + i \epsilon\) with \(\epsilon \neq 0,\)
then, except at particular values of \( w_r \), \( \gamma \) cannot be real. Similarly, \( k_z = 0 \) with \( k_i \neq 0 \) appears only at particular values of \( w_r \). Numerical solutions confirm this point. Therefore, \( v_p \), the phase velocity of unstable surface waves, must have a nonzero normal component. This is a salient feature distinguishing the compressible and incompressible situations, since in incompressible fluids, the phase velocity can never deviate from the interface. From the definition of phase velocity

\[
v_p = \frac{\omega (k_i + k_z)}{\sqrt{(k_i^2 + k_z^2)}} = \frac{w_r \tau_c \epsilon}{1 + \tau_c^2} + \frac{w_f}{1 + \tau_c^2} \tag{9}
\]

where \( \epsilon \) is the unit vector normal to the surface, \( i = k_i/k_n \), and \( \tau_c = k_z/k_i \). The phase velocity can be evaluated for the \( F \) wave \( (v_{pf}) \) or the \( S \) wave \( (v_{ps}) \). Figures 1a and 1b show the calculated \( \tau_c \) of fast waves versus \( U = v \cos \alpha /A_i \) in the case of \( \theta_1 = 90^\circ \), \( \theta_2 = \alpha = 0^\circ \) and \( \theta_1 = \theta_2 = \alpha = 0^\circ \), respectively, and \( \tau_c \) as a function of \( U \) for slow waves is shown in Figure 1c. Here only the geometry of \( \theta_1 = \theta_2 = \alpha = 0 \) is used for \( S \) waves since when \( \theta_1 = 90^\circ \), \( S \) waves vanish (PK-1). Figures 2a, 2b, and 2c, depict the corresponding normal components of the real part of the phase velocities normalized by \( A \) for situations corresponding to those of Figure 1 in the plasma rest frames on each side of the boundary. The geometry of \( \theta_1 \approx 90^\circ \), \( \theta_2 \approx \alpha = 0^\circ \) may appear near the dayside equator, while that of \( \theta_1 = \theta_2 = \alpha = 0 \) represents the nightside flanks and the high-latitude boundary near noon on the dayside.

The parameters used are those used in PK-1's previous paper [Pu and Kivelson, this issue], e.g., \( \beta_1 = 0.64, \beta_2 = 2.5 \), \( n_2/n_1 = 10 \), \( B_i/B = 1.5 \), where \( \beta \) is the ratio of thermal to magnetic pressure. The parameters above are consistent with the observed conditions in a broad region on the magnetopause (see, for example, Russell and Elphic [1979], Elphic and Russell [1979], Eastman [1979], and Paschmann et al. [1979]). We can see that in all of these cases of \( F \) waves, \( (v_{pF}) \approx (v_{pS})/A_i \) is larger than \( (v_{pS})- (v_{pF})/A_i \) in magnitude. For \( F \) waves, the normal components of the phase velocities are directed toward the bulk plasmas and increase monotonically with increasing \( U \), except over a very small range of \( U \) just above \( U_{cr} \) for \( (v_{pS}) \). On the other hand, the phase velocities of \( S \) waves on side 1 are directed inward toward the boundary. Since \( W_{rl} > 0 \) and \( W_{r2} < 0 \) (PK-1), Figures 1 and 2 are consistent with each other in sign.

### 3. THE ENERGY DENSITY ON EACH SIDE OF INTERFACE

Before proceeding with the calculations of energy transport via the K-H instability, we will obtain the energy densities associated with the surface waves. The surface waves couple magnetoacoustic waves in the plasmas on two sides of the interface with amplitudes decaying away from the interface, so we must calculate the wave energy in both bounding plasmas.

Numerical results confirm that the condition \( |\omega| \ll |\omega| (\epsilon \ll |\omega|) \) is satisfied for most of the specific cases discussed in this paper; the exception to the inequality occurs for a limited range of reduced flow velocity, \( 0.8 \ll U \ll 1.2 \), for side 2 (the magnetosheath) in the equatorial dayside case with \( \theta_1 = 90^\circ, \theta_2 = \alpha = 0^\circ \). Thus the time average density of a small amplitude magnetoacoustic wave takes the form [Anderson, 1963]

\[
\langle \delta E \rangle = \langle \delta E_i \rangle + \langle \delta E_p \rangle + \langle \delta E_k \rangle \tag{10}
\]

where, approximately,

\[
\langle \delta E_i \rangle = C^2(\delta \rho \delta \rho^*)/4\rho \tag{11}
\]

\[
\langle \delta E_p \rangle = (\delta B \cdot \delta B^*)/16\pi \tag{12}
\]

\[
\langle \delta E_k \rangle = \rho (\delta v \cdot \delta v^*)/4 \tag{13}
\]

In (10)–(12), the pointed brackets denote an average over a wave period, \( \delta E_i, \delta E_p, \) and \( \delta E_k \) represent the internal energy, the magnetic energy and the kinetic energy, respectively, and the asterisk indicates the complex conjugate. The linear terms in (10), (11), and (12) have been dropped because, if \( \omega_i \ll |\omega| \), the time averages of these terms are negligible [Stix, 1962]. Since \( \tilde{b} \cdot \tilde{z} = 0 \), where \( \tilde{b} = B/B \), it follows, for compressional surface waves, that in both bounding plasmas,

\[
\delta v = \frac{\omega}{w} \left( \frac{\omega^2 (k_i + k_n) - (k_i^2 + k_n^2)A^2 (k_i \cdot \tilde{b}) \delta \rho}{(k_i^2 + k_n^2) \omega^2 - (k_i \cdot A)^2} \right) \tag{14}
\]

and

\[
\delta B = \frac{\omega^2}{w^2} \left( \frac{(k_i^2 + k_n^2) \tilde{b} - (k_i \cdot \tilde{b}) (k_i + k_n) A^2 (k_i \cdot \tilde{b}) \delta \rho}{(k_i^2 + k_n^2) \omega^2 - (k_i \cdot A)^2} \right) \tag{15}
\]

and the expressions apply to the plasmas on sides 1 and 2 in their respective rest frames.

The time average magnetic energy density is then expressed as [Stix, 1962]

\[
\langle \delta E_B \rangle = C_B^2 |\delta \rho|^2 e^{2 \omega \tau_e}/4 \rho \tag{16}
\]

where

\[
C_B^2 = C_{Bx}^2 + C_{By}^2 + C_{Bz}^2 \tag{17}
\]

\[
C_{Bx}^2 = \left( \frac{w^2 (1 + \tau^2) - \cos^2 \theta}{(1 + \tau^2)w^2 - A^2 \cos^2 \theta} \right) (cc) A^2 \tag{18}
\]

\[
C_{By}^2 = \left( \frac{w^2 \cos \theta \sin \theta}{(1 + \tau^2)w^2 - A^2 \cos^2 \theta} \right) (cc) A^2 \tag{19}
\]

\[
C_{Bz}^2 = \left( \frac{w^2 \tau \cos \theta}{(1 + \tau^2)w^2 - A^2 \cos^2 \theta} \right) (cc) A^2 \tag{20}
\]

The time average kinetic energy can be obtained as

\[
\langle \delta E_k \rangle = C_K^2 |\delta \rho|^2 e^{2 \omega \tau_e}/4 \rho \tag{21}
\]

where

\[
C_K^2 = C_{Kx}^2 + C_{Ky}^2 + C_{Kz}^2 \tag{22}
\]

\[
C_{Kx}^2 = \left( \frac{w \cos \theta w^2 - (1 + \tau^2)A^2}{(1 + \tau^2)w^2 - A^2 \cos^2 \theta} \right) (cc) A^2 \tag{23}
\]
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Fig. 3. The ratios of \(\langle \delta E_{w} \rangle_{2}/\langle \delta E_{w} \rangle_{1}\) in geometries \(\theta_{1} = 90^{\circ}, \theta_{2} = \alpha = 0^{\circ}\), and \(\theta_{1} = \theta_{2} = \alpha = 0^{\circ}\) as functions of \(U\), where \(\langle \delta E_{w} \rangle\) is the time average wave energy density. The solid line and dashed line represent \(F\) waves, while the dot-dashed line represents \(S\) waves. The parameters are the same as in previous figures.

Thus, the ratio of wave energy on sides 2 and 1 adjacent to the interface can be expressed as

\[
\langle \delta E_{w} \rangle_{2}/\langle \delta E_{w} \rangle_{1} = R(C_{w}^{2})_{2}/(C_{w}^{2})_{1}
\]

where

\[
(C_{w}^{2})_{j} = C_{j}^{2} + (C_{B_{j}}^{2})_{j} + (C_{K_{j}}^{2})
\]

Figure 3 illustrates how \(\langle \delta E_{w} \rangle_{2}/\langle \delta E_{w} \rangle_{1}\) varies with \(U\) in the case of \(\theta_{1} = 90^{\circ}, \theta_{2} = \alpha = 0^{\circ}\), and \(\theta_{1} = \theta_{2} = \alpha = 0^{\circ}\), respectively. Note that for \(F\) waves, \(\langle \delta E_{w} \rangle_{2}/\langle \delta E_{w} \rangle_{1}\) in the most unstable region. When \(\theta_{1} = 90^{\circ}, \theta_{2} = \alpha = 0^{\circ}\), and \(0.8 < U < 1.2\), \(w_{j} = e\) and \(\langle \delta E_{w} \rangle_{2}\) must be recalculated including linear terms.

4. ENERGY FLUX AND THE ENERGY TRANSPORT VELOCITY

In dealing with energy transport problems, it is often possible to assume that in the plasma rest frame

\[
\langle \delta F \rangle = v_{E} \langle \delta E_{w} \rangle
\]

\(\langle \delta F \rangle\) and \(\langle \delta E_{w} \rangle\) being the energy flux and the energy density averaged over a wave period and \(v_{E}\) being the group velocity of waves. The group velocity by definition refers to the velocity of a wave packet and is given by

\[
v_{E} = \partial \omega/\partial k\]

If wave damping is sufficiently weak, \(v_{E}\) is also the rate at which energy is transported through the system [Ginzburg, 1970], e.g., \(v_{E} = v_{E}\), where \(v_{E}\) is the energy transport velocity defined by

\[
v_{E} = \langle \delta F \rangle/\langle \delta E_{w} \rangle
\]

For magnetoacoustic waves in the frame at rest relative to the bulk plasma, Anderson [1963] showed that

\[
v_{E} = \omega k + k^{2}C^{2}(k \cdot A) = \omega k^{2}C^{2}(2w_{0}^{2} - (A^{2} + C^{2})k^{2}) = \nu_{E}^{2}
\]

However, in the case of surface waves, strong damping (i.e., decay of amplitude normal to the boundary) is present, \(k\) and \(\partial \omega/\partial k\) are complex, and \(v_{E}\) and \(v_{E}\) are not directly related [Ginzburg, 1970]. Equation (33) fails and we must calculate \(\delta E_{w}\) in another way.

The energy flux on each side of the interface, in the frame at rest relative to the plasma on side \(j\) (\(j = 1, 2\)), takes the form [Landau and Lifshitz, 1960; Anderson, 1963]

\[
\langle \delta F \rangle_{j} = \rho_{j} \left( h_{j} + \delta h_{j} \right) \left[ \delta h_{j} - \frac{\delta \nu_{j} \times B_{j}}{4\pi} \times B_{j} \right]_{j = 1, 2}
\]

where

\[
\rho_{j} = \rho_{j} + \delta \rho_{j}, \quad B_{j} = B_{j} + \delta B_{j}, \quad h_{j} = h_{j} + \delta h_{j}, \quad j = 1, 2
\]

\(h\) being the specific enthalpy. Expanding (37) up to the second order, we obtain approximately [Anderson, 1963]
\[ (\delta F_j) = \rho [(C_j^2 \delta p/\rho) \delta v_j + (B_j \cdot \delta B_j/2 \pi p_0) \delta v_j] \]
\[ -(B_j \cdot \delta v_j/4 \pi p_0) \delta B_j = (A_j^2 \delta p/\rho) \delta v_j + (A_j \cdot \delta v_j \delta p/\rho A_j)]_{rr}, \quad j = 1, 2 \quad (38) \]

In equation (38), the subscript \( rr \) indicates that only products of real parts, such as \( \text{Re}(\delta p) \cdot \text{Re}(\delta v_j) \), \( \text{Re}(\delta B_j) \cdot \text{Re}(\delta v_j) \), or \( \text{Re}(\delta v_j) \cdot \text{Re}(\delta B_j) \) are to be retained. Taking the time average of (38) over a single period, we obtain from (14') and (15')
\[ \langle \delta F_j \rangle = C_j^2 A \delta p^2 \varepsilon_{\text{ext}}^2 \rho / 4 \rho \quad (39) \]
provided \( \omega \gg \omega_0 \), where
\[ C_j^2 = G Q^* + Q G^* + S T^* + T S^* \quad (40) \]
\[ G = C^2 - A^2 + \frac{2 A^2 \omega^2 \sin^2 \theta + \tau^2}{(1 + \tau^2)(\omega^2 - A^2 \cos^2 \theta)} \quad (41) \]

Accordingly, \( \nu_{Ej} \), the normal component of \( \nu_E \) on two sides of the interface, can be written as
\[ (\nu_{Ej})_L = \langle \delta E_j \rangle / (\delta E_{nu}) \quad (35a) \]
where \( (\delta E_{nu}) \) is given by (10), (16), (21), and (26). Finally, we obtain
\[ (\nu_{Ej})_L = (C_j^2 A / (C_{nuj}^2 \rho) \quad (45) \]

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Fig. 4a. \((U_{Ej})_L\), the normalized normal component of the energy transport velocity of surface waves (solid line) on the magnetospheric side, as a function of \( \Upsilon \) in the geometry of \( \theta_1 = 90^\circ \), \( \theta_2 = \alpha = 0^\circ \). \((U_{Ej})_L = (\partial \omega_j / \partial k_j)_L A_1 \) (dashed line) is also presented for comparison. The parameters are the same as in previous figures.

Fig. 5a. \((U_{Ej})_L\), the normalized normal component of the energy transport velocity of surface waves (solid lines) on the magnetospheric side, as a function of \( \Upsilon \) in the geometry of \( \theta_1 = \theta_2 = \alpha = 0^\circ \). \((U_{Ej})_L = (\partial \omega_j / \partial k_j)_L A_1 \) is also presented for comparison. \( S \) waves are on the left, and \( F \) waves are on the right. The parameters are the same as in previous figures.

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Fig. 4b. \((U_{Ej})_L\), the normalized normal component of the energy transport velocity of surface waves (solid line) on the magnetosheath side, as a function of \( \Upsilon \) in the geometry of \( \theta_1 = 90^\circ \), \( \theta_2 = \alpha = 0^\circ \). \((U_{Ej})_L = (\partial \omega_j / \partial k_j)_L A_1 \) (dashed line) is also presented for comparison. Here again a positive ordinate corresponds to negative velocities. \( S \) waves are on the left, and \( F \) waves are on the right. For the \( F \) wave, the dashed curve to the left of the arrow at the upper border has been multiplied by \((-1)\). The parameters are the same as in previous figures.

Fig. 5b. \((U_{Ej})_L\), the normalized component of the energy transport velocity of surface waves on the magnetosheath side, as a function of \( \Upsilon \) in the geometry of \( \theta_1 = \theta_2 = \alpha = 0^\circ \). \((U_{Ej})_L = (\partial \omega_j / \partial k_j)_L A_1 \) is also presented for comparison. Here again a positive ordinate corresponds to negative velocities. \( S \) waves are on the left, and \( F \) waves are on the right. For the \( F \) wave, the dashed curve to the left of the arrow at the upper border has been multiplied by \((-1)\). The parameters are the same as in previous figures.
We calculated \( v_E \), by using the same configurations as in previous figures. Figures 4a and 4b display \( U_E = v_E / A_1 \) for the dayside geometry for side 1 and side 2, respectively. It can be seen that within the magnetosphere, \( (U_E)_{1,2} \) is directed inward away from the magnetopause and increases with increasing \( U \). In the most unstable range of \( U \), the normalized energy transport velocity of \( F \) waves, \( (U_E)_{1,2} \), is \( 0.1-0.6 \). As to the magnetosheath, \( (U_E)_{1,2} \) also is directed out of the magnetopause. For \( U \geq U_{cf}, U_E \) on both sides 1 and 2 becomes equal to \( U_{cf} = (\delta \omega / \delta \tau) A_1 = (\delta \omega / \delta \tau) A_1 \), the normal component of the group velocities of ordinary MHD waves in the bulk plasmas. Since the unstable \( F \) wave changes into two stable fast modes of magnetoacoustic waves when \( U \geq U_{cf} \), there are two branches of \( U_E \) depicted on the plot. \( U_{cf} = (\delta \omega / \delta \tau) A_1 \) as functions of \( U \) are also shown on Figures 4a and 4b for comparison; the differences between \( U_E \) and \( U_{cf} \) are quite apparent. We will return to this point later in this section. Figures 5a and 5b show \( U_E \) in a typical nightside geometry for both \( F \) waves and \( S \) waves on side 1 and side 2, respectively. Again, \( (U_E)_{1,2} > 0 \) and \( (U_E)_{1,2} < 0 \), indicating that the directions of the energy transport velocities of compressional unstable surface waves are out of the interface. For \( S \) waves, if \( U < U_{cs} \) or \( U > U_{cs} \), \( (U_E)_{1,2} = 0 \), and for \( F \) waves, \( (U_E)_{1,2} = 0 \) when \( U < U_{cf} \), and \( (U_E)_{1,2} \neq 0 \) when \( U > U_{cf} \). Also, if \( U > U_{sf} \), \( (U_E)_{1,2} = (U_{cf}, G = 1, 2) \), and each of them breaks down into two branches corresponding to two fast modes of magnetoacoustic waves. In the most unstable range of \( U, U_{cf} \), 0.5-1.0. For both plasmas, group velocities, \( (U_E)_{1,2} \), are also represented on the plots. Again, we note that all quantities in the figures are calculated for the local plasma rest frame. Since the wave packet moves in the \( z \) direction approximately as a whole at the group velocity \( v_E = \delta \omega / \delta k \), with amplitude varying as exp \( (\omega t - k z) \), evidently, one may expect \( v_E \sim v_F \). On the other hand, if \( (40) \) is violated, as in other cases in Figures 4 and 5, the wave packet experiences distortion as it propagates \( v_E \neq v_F \). In addition, \( U_E \) goes to infinity at values of \( U \) corresponding to minimum or maximum values of \( \tau \), in Figures 1a, 1b, and 1c.

Finally, we should emphasize that the waves we have described originate in the velocity shear. Even the stable plasma waves present for \( U > U_{sf} \) originate on the interface and propagate into the plasmas on both sides \( (\omega \theta > 0, k_z > 0; \omega \theta < 0, k_z < 0) \). These bulk plasma waves differ from those studied by McKenzie [1970], that originate in the magnetosheath and propagate through the magnetopause with \( \omega \theta_1, \omega \theta_2 \), and \( k_z \) all > 0.

We have stressed that our calculations for energy transport in the magnetosheath were carried out in the rest frame of the magnetosheath plasma. Since \( (\delta F) \) is frame-dependent, it is evident that \( (\delta F)_{1,2} \) and \( (\delta E)_{1,2} \) will be different when calculated in the rest frame of the magnetosphere. Further discussion of this point will be provided elsewhere.

5. DISCUSSION

We have shown some calculated phase velocities, energy transport velocities and energy fluxes of both \( F \) waves and \( S \) waves in two representative geometries. In the frames at rest with respect to plasma on each side, the energy fluxes and energy transport velocities are directed out of the magnetopause, and the phase velocities of \( F \) waves on the magnetospheric side are also directed inward toward the bulk plasma. The results may be used to determine how the surface waves on the magnetopause couple to resonant field lines in the magnetosphere, a widely accepted model for the excitation of low-frequency magnetic pulsations [Chen and Hasegawa, 1974; Southwood, 1974].

| TABLE 2. Calculated Flux and Density of Wave Energy on the Magnetospheric Side of the Magnetopause |
|-------|-------|-------|-------|-------|-------|-------|
|       |       |       |       |       |       |       |
|       | \((\delta F)_{1,2}\) | \((\delta E)_{1,2}\) | \((\delta F)_{1,2}\) | \((\delta E)_{1,2}\) |
|       | ergs/cm² s | ergs/cm² | ergs/cm² s | ergs/cm² |
|       |       |       |       |       |
| Dayside (low latitude) | \(1 \times 10^{-3}\) | \(1 \times 10^{-10}\) | \(2 \times 10^{-6}\) | \(1 \times 10^{-11}\) |
| Dayside (high latitude) | \(5 \times 10^{-3}\) | \(2 \times 10^{-10}\) | \(4 \times 10^{-4}\) | \(5 \times 10^{-11}\) |
| Nightside (low latitude) | \(5 \times 10^{-3}\) | \(3 \times 10^{-10}\) | \(7 \times 10^{-5}\) | \(2 \times 10^{-11}\) |

Since for fixed \( k_x, \omega \), \( \omega _1 \), and \( k_x \) may all be regarded as functions of \( k_z \), we may expand them as Taylor's series in energy transport velocities and energy fluxes of both \( F \) waves and \( S \) waves in two representative geometries. In the frames at rest with respect to plasma on each side, the energy fluxes and energy transport velocities are directed out of the magnetopause, and the phase velocities of \( F \) waves on the magnetospheric side are also directed inward toward the bulk plasma. The results may be used to determine how the surface waves on the magnetopause couple to resonant field lines in the magnetosphere, a widely accepted model for the excitation of low-frequency magnetic pulsations [Chen and Hasegawa, 1974; Southwood, 1974].

TABLE 2. Parameters Assumed for Energy Flux Estimates

<table>
<thead>
<tr>
<th>F Wave</th>
<th>S Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_1, \text{ km/s})</td>
<td>(\gamma)</td>
</tr>
<tr>
<td>(U)</td>
<td>(U)</td>
</tr>
<tr>
<td>Dayside (low latitude)</td>
<td>400</td>
</tr>
<tr>
<td>Dayside (high latitude)</td>
<td>500</td>
</tr>
<tr>
<td>Nightside (low latitude)</td>
<td>200</td>
</tr>
</tbody>
</table>

TABLE 3. Estimates of Power Supplied to the Magnetosphere by Different Physical Processes

<table>
<thead>
<tr>
<th>Model</th>
<th>Total power, ergs/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unstable K-H waves*</td>
<td>(3 \times 10^7)</td>
</tr>
<tr>
<td>Quiet time</td>
<td>(1 \times 10^7)</td>
</tr>
<tr>
<td>Disturbed time</td>
<td>(1 \times 10^7)</td>
</tr>
<tr>
<td>Magnetic connection†</td>
<td>(4 \times 10^9)</td>
</tr>
<tr>
<td>Diffusive injection†</td>
<td>(\leq 8 \times 10^7)</td>
</tr>
<tr>
<td>Drift injection†</td>
<td>(\leq 7 \times 10^{16})</td>
</tr>
<tr>
<td>MHD wave transmission†</td>
<td>(3 \times 10^{16})</td>
</tr>
</tbody>
</table>

*This paper.
†Hill (1979)
that the excitation of resonant field lines requires a flux of energy inward from the boundary.

Our calculations reveal that unstable compressional surface waves themselves propagate into the bulk plasmas and create a finite energy flux into the magnetosphere.

To obtain quantitative estimates of $\langle F \rangle_F$ in different cases, we use representative numbers for $A_i$, $U$, and $\langle B \rangle_B$ listed in Table 1. Here $\langle B \rangle_B$, the amplitude of magnetic perturbations in the magnetosheath near the magnetopause, is chosen for $F$ waves to correspond to power levels in waves with periods $150-1200$ s reported by Saito et al. [1979], for example. Amplitudes for $S$ waves are reduced in these estimates because of their smaller growth rates. The reduced flow velocity, $U$, is taken to be order 1, with minor variations to reflect the kind of change with location on the magnetopause expected for field strength and flow velocity. The exceptional case of $S$ waves at dayside low latitudes ($S$ waves are stable at $U \approx 1$ for $\theta_1 \approx 90^\circ$) is calculated by using $U = 0.34$, $\theta_1 = 80^\circ$, $\theta_2 = \alpha = 0^\circ$, although for $\theta_1 < 80^\circ$, the contribution of $S$ waves to energy transport (see below) may be slightly larger. Using $\langle B \rangle_B$, we obtain $\langle B \rangle_B^2$ from (12). Then $\langle B \rangle_B^2$ can be related to $\langle B \rangle_B$ by (10) and (16) through (26).

Finally, $\langle B \rangle_B^2/\langle E \rangle_F$ may be obtained from Figure 3 for the assumed $U$. The energy fluxes are found from $\langle F \rangle_F = \langle U \rangle_F A_i \langle B \rangle_B^2$ and are multiplied by the assumed area of the magnetopause unstable to K-H waves [Boller and Stolov, 1970].

The calculated results are summarized in Table 2. If the unstable regions on the magnetopause extend from local times 1030 and 1330 in disturbed times, or from 0840 and 1540 in quiet times to the terminators [Boller and Stolov, 1970], and if the average distance between the dayside magnetopause and the earth is $14 R_E$ ($R_E$ is the earth's radius), then we find $\langle F \rangle_F$, the total energy flux of $F$ waves flowing from the dayside magnetopause into the magnetosphere, can be as large as $10^{17}$ ergs/s to $10^{18}$ ergs/s (see Table 3). These estimates can be regarded as rough lower bounds because they do not include contributions from the tail. Recently, by using data from the STARE (Scandinavian twin auroral radar experiment), Greenwald and Walker [1980] determined both the maximum Joule heating in the ionosphere by the pulsations and magnetospheric energy storage in the resonant waves. In both cases, they found power levels of approximately $6 \times 10^{16}$ ergs/s. Thus, the energy flux resulting from the compressional K-H instability on the dayside magnetopause seems to be sufficient to drive observed magnetic pulsations.

It is also interesting to compare the energy flux of unstable surface waves with other energy input into the magnetosphere. Verzariu [1973] calculated the energy flux across the magnetopause by MHD wave transmission from the magnetosheath into the magnetosphere. By assuming a typical power spectrum in the magnetosheath given by Mariani [1970], he obtained the energy density of incident waves ranging from $\sim 10^{-11}$ to $10^{-10}$ ergs/cm$^2$ and an average flux of order $10^{-9}$ ergs/cm$^2$ s, which is much lower than the values of $\langle B \rangle_B^2$ produced by unstable compressional surface waves. The comparisons of the total power supplied to the dayside magnetopause with other inputs are listed in Table 3. The numbers are taken from the review paper of Hill [1979].

From Table 3 we observe that the unstable K-H waves provide a source of energy transfer into the magnetosphere that is not insignificant and that for northward interplanetary fields may be comparable with the largest energy input from other mechanisms.

All calculations in this paper hold only in the linear unstable stage. Since in most cases surface waves exist at saturation levels, to discuss the energy transport problem completely, the nonlinear development of the waves should be considered. In addition, the entire analysis is based on the assumption of weak instability, which, as previously noted, is invalid in the dayside equatorial magnetosheath where $|\omega'| < 0.8 < U < 1.2$. The correct calculation for this case should properly include linear terms. Also, the boundary layer and kinetic effects should be taken into account in the future.

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