Kelvin-Helmholtz Instability at the Magnetopause: Solution for Compressible Plasmas

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The Kelvin-Helmholtz (K-H) instability of a tangential discontinuity in a compressible plasma is reexamined in the linear magnetohydrodynamic (MHD) approximation. For fixed plasma conditions, two different kinds of surface waves (labeled F for fast and S for slow) may exist simultaneously with different tangential wave vectors \( k_t \). The surface waves can be excited only for a limited range of \( U \), the relative flow speed of the plasma on the two sides of the interface. Thus the instability requires \( U_{sw} < U < U_{ess} \), where \( U_{sw} \) and \( U_{ess} \) are the lower and upper critical velocities, respectively, and the subscript \( m \) distinguishes the fast and slow surface waves, with \( U_{sw} < U_{ss} \). In the incompressible limit, there is only one surface wave with lower critical velocity, \( U_{ss} \), and no upper critical velocity. It has frequently been remarked that compressibility reduces the lower critical velocity. We show that the effect is small for the F wave but that \( U_{ss} \) is considerably smaller than either \( U_{sw} \) or \( U_{sw} \). However, for a given \( k_t \), the S wave is unstable only for a very small range of \( U \). Growth rates, \( \omega_p \), of the surface waves are calculated and \( \omega_p \) exceeds slightly the growth rate for the incompressible plasma \( \omega_c \) when \( U \approx U_{sw} \), but the slow wave growth rate is small in comparison with both \( \omega_p \) and \( \omega_c \). Consequently, plasma compressibility is relatively ineffective in reducing the critical velocity for surface wave growth below that for an incompressible plasma. In particular, for the nominal conditions of the dayside equatorial magnetopause, S mode waves do not occur near the minimum \( U_s \), with \( k_t \) perpendicular to the geomagnetic field. On the terrestrial magnetopause the F(S) surface waves can couple a quasi-fast (quasi-slow) MHD mode in the magnetosheath. These waves decay as they propagate into the bounding plasmas. Consideration of the limit \( U \rightarrow U_{sw} \) reveals the significance of the upper critical velocity. In this limit, the phase velocity of the fast surface wave approaches the MHD fast mode speed in both bounding plasmas. The imaginary part of normal component of \( k \) vanishes and the unstable surface waves change to stable MHD waves that propagate away from the boundary without damping. The above points are discussed in relation to previous work on the K-H instability. Possible applications to observations in the terrestrial magnetosphere are mentioned.

1. Introduction

The interface between two fluids in relative motion under certain conditions develops an instability known as the Kelvin-Helmholtz (K-H) instability [Dungey, 1955; Chandrasekhar, 1961; Northrop, 1956]. Since Dungey first suggested the possibility of K-H instability at the boundary of the outer magnetosphere [Dungey, 1955], the theory has been discussed by a number of authors [Fejer, 1964; Sen, 1963, 1964, 1965; Gerwin, 1968; Southwood, 1968; Ong and Roderick, 1972; Boller and Stolov, 1970, 1973; Lee et al., 1981; Melander and Parks, 1981]. It is believed that the K-H instability on the magnetopause can play an important role in many magnetospheric processes, such as the generation of geomagnetic pulsations [Atkinson and Watanabe, 1966; Southwood, 1968; Dungey and Southwood, 1970; Southwood, 1974a, b; Chen and Hasegawa, 1974; Lee and Olson, 1980], and momentum transfer across the magnetopause [McKenzie, 1970; Southwood, 1979; Miura, 1981].

Much work in this area was based upon the assumption that the plasma is incompressible [Sen, 1963; Boller and Stolov, 1970, 1973; Lee et al., 1981], an assumption valid only when the plasma \( \beta \) (ratio of thermal to magnetic pressure) is much larger than 1. Satellite observations have shown that this assumption may not apply since at low latitude, both in the magnetosheath and in the magnetosphere, \( \beta \) values can be near unity [Fairfield, 1979]. During the encounter with Saturn in November 1980, Voyager 1 also found compressional surface waves occurring on Saturn's dayside magnetopause and the associated MHD waves in the magnetosheath were slow mode [Lepping et al., 1981a]. Such waves recently have been identified in the earth's magnetosheath by use of magnetic field and plasma measurements from the Explorer 12 spacecraft by Kaufmann et al. [1970] and more recently in data from the ISEE spacecraft [Lepping et al., 1981b]. These facts are obviously incompatible with the simple 'incompressible' assumption. Thus the consequences of plasma compressibility must be fully evaluated so that the conditions at the boundary of a planetary magnetosphere can be understood. Previous work on the problem has dealt with specific aspects of the onset of the K-H instability, but the properties of the unstable system have not been fully characterized.

In this paper, we reexamine the K-H instability of a tangential discontinuity in compressible plasmas within the framework of a linear MHD treatment. The objective is to understand the properties of the unstable waves and conditions for their growth.

We begin by reviewing the previous work of Fejer [1964], Sen [1964], Southwood [1968], and Ong and Roderick [1972]. Using the MHD approximation, these authors showed that unstable surface waves can be generated on the interface when the relative speed between two fluids exceeds a threshold critical velocity. Fejer, Sen, and Southwood all
found that compressibility reduces stability by lowering the critical velocity relative to that in an incompressible fluid. Fejer and Sen, though, found that compressibility led to only a small reduction of the critical velocity, whereas Southwood found instability at streaming velocities well below the incompressible critical velocity. Ong and Roderick [1972] calculated growth rates for shear layers with both zero and finite thickness and found that compressibility reduces the growth rate and thus stabilizes the surface waves.

For the case in which the magnetic fields on both sides of the interface are parallel to the flow, Fejer found that the waves propagating along the flow can be unstable for any value of \( \beta \) if the flow speed is large enough. On the other hand, found that these modes are always stable if the sound speed goes to zero (\( \beta = 0 \)). In spite of their differences, all of these authors agree that compressibility has a stabilizing effect when the relative flow speed sufficiently exceeds the magnetoacoustic speed, which seems somewhat paradoxical.

Fejer [1964], Sen [1964], and Ong and Roderick [1972] all considered special cases that made the mathematics more tractable. They were thus limited to particular plasma parameters and geometries for which they calculated either critical velocities or growth rates. Southwood [1968] did not impose simplifying assumptions on plasma conditions but sought only to obtain the most unstable root of the dispersion equation.

In this paper we characterize the properties of the surface modes for general plasma conditions. We provide a full description of the wave properties, including the critical velocities, linear growth rates, wave vectors, and phase velocities. In section 2, we solve the dispersion relation and point out that there are two kinds of unstable surface waves that we call the fast wave (F wave) and the slow wave (S wave). We will argue that the waves discussed by Fejer [1964] and Sen [1964] are different modes than those described by Southwood [1968]. The source of the apparent disagreement of their results is then evident.

In section 3 we show that both fast and slow surface waves are K-H unstable over limited ranges of relative flow speeds between the lower and upper critical velocities, \( U_c \) and \( U_u \). Both critical velocities and growth rates are evaluated numerically for some typical parameters. The lower critical velocity of the S wave, \( U_{cs} \), is usually much lower than those of the F wave, \( U_{cf} \), or of the surface waves in an incompressible plasma, \( U_o \). For the fast wave, the growth rate \( \varepsilon_f \) is very close to but slightly larger than that in an incompressible plasma, \( \varepsilon_i \), for a range of \( U \) values just above \( U_{cf} \), although it becomes smaller than \( \varepsilon_i \) as \( U \) continues to increase. In section 4 it is shown that if the flow speed exceeds \( U_{uf} \), the wave vector of the F wave changes from complex to real, reflecting a change of the unstable fast surface wave into stable MHD body waves. This happens when the phase velocity of the unstable F wave equals the phase velocity of the fast magnetoacoustic wave in the bounding plasmas. The stabilization mechanism that occurs when \( U \geq U_{uf} \) is thus apparent. We also point out in section 4 that the normal component of the group velocity on the magnetosphere side is always directed toward the earth, a result that may be important in some magnetospheric processes.

Since we use the linear MHD approximation and neglect both plasma kinetic effects [Lerche, 1966; Melander and Parks, 1981] and boundary layer effects [Miura, 1981], we are restricted to dealing with waves whose wave lengths are much larger than the thicknesses of the magnetopause, the boundary layer and the ion gyroradius. Recent satellite observations show that such surface fluctuations do exist on earth's magnetopause [Lepping and Burlaga, 1979].

In our summary, we return to a consideration of previous calculations and point out where each study fits in to the comprehensive description we have provided.

2. The Dispersion Equation and Surface Waves

To simplify the problem, we assume the magnetopause to be a one-dimensional tangential discontinuity located in the plane \( z = 0 \) with the magnetosheath on side 2 (\( z < 0 \)) moving at a constant velocity \( \mathbf{v} \) relative to the magnetosphere on side 1 (\( z > 0 \)). Both the magnetosphere and the magnetosheath are assumed to be semi-ininitely extended homogeneous nondissipative plasmas with uniform magnetic fields \( \mathbf{B}_1 \) and \( \mathbf{B}_2 \) tangential to the interface (see Figure 1). Starting from MHD equations

\[
\begin{align*}
\frac{dv}{dt} &= -\nabla p + \frac{\nabla \times \mathbf{B}}{4\pi} \times \mathbf{B} \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B}) \\
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{v}) \\
\rho \mathbf{v} &= \text{constant}
\end{align*}
\]

Fig. 1. A schematic sketch of a local region on the magnetopause, which consists of a one-dimensional tangential discontinuity at \( z = 0 \) and two uniform plasma regions—the magnetosphere on side 1 (\( z > 0 \)) and the magnetosheath on side 2 (\( z < 0 \)) moving at \( \mathbf{v} \) relative to side 1. Both \( \mathbf{B}_1 \) and \( \mathbf{B}_2 \) are parallel to the plane of \( z = 0 \).
and looking for linearized perturbations, which vary as exp \([i(kz - \omega t)]\), we obtain the basic magnetoacoustic dispersion relation on both sides 1 and 2 [Southwood, 1968]

\[\omega^4 - \omega^2(k_z^2 + k_n^2)(C^2 + A^2) + A^2C^2\cos^2 \theta k_n^2(k_z^2 + k_n^2) = 0\]  

(5)

Here \(\rho, p, \) and \(v\) are mass density, gas pressure, and fluid velocity, \(A\) and \(C\) are Alfven velocity and sound speed. The exponent \(\gamma\) is the usual ratio of specific heats, is taken as 2 for the magnetized plasmas; \(\omega\) is the angular frequency in the frame of reference at rest relative to the plasma, \(k_z\) is the tangential wave vector, \(\theta\) is the angle between \(k\) and \(B\), and \(k_n\) is the normal component of \(k\), as well as the frequencies on the two sides of the interface, \(\omega_1\) and \(\omega_2\), are related by the Doppler shift

\[\omega_2 = \omega_1 - k_i \cdot v\]  

(6)

Introducing \(w = \omega k_z\), we can rewrite (5) as

\[k_n^2/k_i^2 = -1 + \frac{w^4}{w^2(A^2 + C^2) - A^2C^2 \cos^2 \theta}\]  

(7)

We are interested in surface waves that must satisfy the basic relations (5) and (6), as well as the following boundary conditions: (1) the amplitudes of waves vanish at \(z = \pm \infty\), (2) the normal displacement and the normal total stress must be continuous across the magnetopause.

Let

\[k_n = k_z + ik_i\]  

(8)

with \(k_i \geq 0\), where \(k_z\) and \(k_i\) are the real and imaginary parts of \(k_n\), respectively; then (5) together with the boundary conditions above yields the dispersion equation for K-H waves

\[\rho_1(\omega_1^2 - k_z^2A_1^2\cos^2 \theta_1) = -\rho_2(\omega_2^2 - k_z^2A_2^2\cos^2 \theta_2)\]  

\[\rho_1(\omega_1^2 - A_1^2\cos^2 \theta_1) = -\rho_2(\omega_2^2 - A_2^2\cos^2 \theta_2)\]  

or

\[\rho_1(w_1^2 - A_1^2\cos^2 \theta_1) = -\rho_2(w_2^2 - A_2^2\cos^2 \theta_2)\]  

(9')

where \(\tau = k_n/k_i\). In (9) and (9'), the identity \(k_n = k_z = k_i\) has been used and perturbations on sides 1 and 2 include factors of \(\exp(ik_{n1})\) and \(\exp(-ik_{n2})\), respectively.

Equation (9) or (9') can be rewritten as tenth degree equations that are impossible to solve analytically for arbitrary conditions. Before obtaining numerical solutions, it will be useful to describe some qualitative features of compressional surface waves.

In the case of surface waves, \(k_i \neq 0\), the amplitude of waves decreases away from the surface with a characteristic length of \(L \sim (2\pi/k_i)\). For incompressible waves, \(k_n^2\) must equal \(-k_i^2\), so \(k_z = 0\) and \(k_i = |k_i|\), but for compressional surface waves, \(k_n\) depends on the fluid parameters as well as the flow velocity \(v\).

For compressible plasmas, if the perturbation is stable, (7) indicates that \(k_n^2\) is real and less than zero; compressional surface waves propagate along the surface, and \(w\) is just the phase velocity. On the other hand, if the perturbation is unstable, \(k_n\) must be complex (\(k_i \neq 0\)); thus the phase velocity \(v_p\) must have a normal component, as well as a tangential component, e.g.,

\[v_p = \omega(k_i^2 + k_z^2)/(k_z^2 + k_n^2)\]  

\[= \omega(\tau + 1)/(1 + \tau^2)\]  

(10)

where \(\omega_1, w_1, \) and \(\tau_1\) are the real parts of \(\omega, w, \) and \(\tau\), respectively, \(i = k_i/k\), and \(\tau\) is the unit vector normal to the interface. It is worth noticing that since \(k_z = 0\) in incompressible plasmas, the phase velocity of K-H waves always parallels the interface, whether or not the perturbation is stable.

Equation (5) leads to

\[(w_1^2 - w_2^2)(w_1^2 - w_2^2) = 0\]  

(11)

Thus on both sides of the boundary compressional surface waves can be divided into two modes

\[w_1^2 - w_2^2 = 0\]  

(13a)

and

\[w_1^2 - w_2^2 = 0\]  

(13b)

For the special case that \(k_i = 0\), equations (11)-(13) can be divided by \((1 + \tau^2)\) to give equations for the phase velocity \(u_p\) (c.f. (10)). The familiar solutions for fast and slow magnetoacoustic waves in the bulk fluid are then recovered. We will refer to such waves as 'body waves.' In the surface wave case (\(k_i \neq 0\)), the analogous waves are described by (13), and we call them the quasi-fast ('f') and the quasi-slow ('s') modes, respectively.

The dispersion relations for the quasi-fast mode and the quasi-slow mode of surface waves can be understood by examining \(\tau^2 = \tau^2(w^2)\) in the stable or marginally unstable cases. Figures 2a and 2b show how \(\tau^2 = \tau^2(w^2)\) varies as a function of \(w^2\); solid lines represent the \(f\) mode and dashed lines indicate the \(s\) mode. It can be seen that the \(s\) mode has two branches and that the \(f\) mode and the \(s\) mode meet at the point where \(w^2 = w_f^2, \tau^2 = \tau_f^2\); while at \(w^2 = w_s^2, \tau^2\) becomes infinite, where

\[w_f^2 = A^2C^2 \cos \theta(A^2 + C^2)\]  

(14a)

\[w_d^2 = 2w_g^2\]  

(14b)

and

\[\tau_f^2 = -1 + 4A^2C^2 \cos^2 \theta(A^2 + C^2)\]  

Naturally, surface waves correspond to values of \(\tau^2 < 0\), or, for the \(s\) mode, \(0 \leq w^2 \leq w_s^2\) and \(0 < w^2 \leq w_f^2\), and for the \(f\) mode, \(w_f^2 \leq w^2 \leq w_s^2\). The quasi-slow mode vanishes when \(\theta \rightarrow \pi/2\). Then \(\tau^2\) becomes a linear function of \(w^2\). The \(\tau^2 > 0\) portions of the curves represent body waves; dot-dash line and double dot-dash lines refer to the \(f\) and \(s\) modes, respectively. The existence of two modes is a consequence of the compressible assumption, for in the incompressible case only one mode can be excited and \(\tau^2 = -1\).

The \(f\) and \(s\) modes are extensions of the ordinary MHD fast and slow modes with phase velocities related by
Fig. 2. The ratio of \( r^2 = k_n^2/k_t^2 \) as a function of \( w^2/A^2 = \omega^2/(k_i^2A^2) \) in the stable or marginally unstable cases. Several characteristic values of \( w^2/A^2 \) are shown in the figure, including \( w_2^2, w_3^2, w_f^2, w_g^2 \), all identified by the subscripts, and \( A^2 \cos^2 \theta \) identified by an \( e \). The solid line and dashed line represent the \( f' \) and \( s' \) mode of surface waves, respectively, and the dot-dash line and double dot-dash line represent the fast and slow mode of magnetoacoustic waves, respectively. (a) \( \beta = 2.5, \theta = 45^\circ \). (b) \( \beta = 0.64, \theta = 45^\circ \).

Thus, for solution (15), the waves in the magnetosphere (side 1) must be the quasi-fast mode (\( w_1^2 > w_{dt}^2 \)) and those in the magnetosheath (side 2) must be the quasi-slow mode (\( w_2^2 < w_{dt}^2 \)). We call this combination the \( f' \)-\( s' \) mode. Numerical calculations (see section 3) show that \( w_{1,2}^2 \) grows with increasing flow speed. When \( v \) becomes larger than the lower critical velocity, \( w_{1,2}^2 \) becomes complex. As \( v \) continues to increase, the waves on side 2 change from \( s' \) mode to \( f' \) mode, and we call the combination the \( f' \)-\( f' \) mode. We identify both the \( f' \)-\( s' \) mode and the \( f' \)-\( f' \) mode as fast surface waves, or \( F \) waves, since on the magnetosphere side waves are the quasi-fast mode with larger phase velocities.

On the other hand, solution (16) corresponds to the stable surface wave that, with increasing \( v \), can appear first as the quasi-slow mode on side 1 and as either a quasi-slow or quasi-fast mode on side 2. We call these two combinations the \( s' \)-\( s' \) mode or \( s' \)-\( f' \) mode, and identify both as both slow surface waves, or \( S \) waves, since on the magnetosphere side they are quasi-slow mode with lower phase velocities. Numerical calculations show that \( S \) waves cannot change to \( F \) waves as \( v \) increases if the propagation direction is fixed. For some values of \( v \), it is possible to have unstable \( F \) and \( S \) waves simultaneously present on the surface, but when that occurs they will propagate in different directions. In summary, in dealing with the K-H instability on the magnetopause with \( \beta_1 < 1 \) and \( \beta_2 > 1 \), we have two unstable surface waves, the \( F \) waves and the \( S \) waves.

3. Kelvin-Helmholtz Instability: Criterion and Growth Rate

In order to understand conditions that result in the development of the K-H instability, we examine the dispersion equation directly and obtain critical velocities, growth rates, wave vectors, and phase velocities simultaneously. The square of equation (9') in conjunction with equation (7) yields
\[
\rho_i^2(w_i^2 - A_i^2 \cos^2 \theta_i)^2 - 1 + \frac{w_i^4}{w_i^4(A_i^3 + C_i^3) - A_i^2 C_i^2 \cos^2 \theta_i}
\]
\[
= \rho_s^2(w_s^2 - A_s^2 \cos^2 \theta_s)^2 - 1 + \frac{w_s^4}{w_s^4(A_s^3 + C_s^3) - A_s^2 C_s^2 \cos^2 \theta_s}
\]

which, after a lengthy procedure, can be rearranged as a tenth order algebraic equation

\[
\sum_{i=1}^{11} A_i w_i^{11-i} = 0
\]

Only those roots of (20) that satisfy equation (9') as well as the boundary condition at \( z \to \pm \infty \) represent K-H waves, and among them a complex root with a positive imaginary part corresponds to the unstable solution.

For numerical calculations we set \( \beta_1 = 0.64, \beta_2 = 2.5, n_2/n_1 = 10, B_1/B_2 = 1.5 \), values chosen to characterize situations \( B_1 = 30-20 \gamma, B_2 = 20-10 \gamma, n_1 = 1-2/cm^4, n_2 = 10-20/cm^4 \) typical of the terrestrial magnetopause [Russell and Elphic, 1979; Elphic and Russell, 1979; Eastman, 1979; Paschman et al., 1979]. The geometries used correspond to the average situation at different sections of the magnetopause. Consistent with the previous discussions, our calculations show that there are two unstable surface waves for which the phase velocity and the growth rate are quite different. Both of them have two critical flow velocities, which we call the lower critical velocity, \( U_c \), and the upper critical velocity, \( U_u \). Expressing all velocities in units of \( A_i \), we find that the instability can appear only if \( U_c < U < U_u \), with the notation \( U = v/A \). The relations \( U = U_c + \delta \) and \( U = U_u - \delta \) with \( \delta \to 0^+ \) indicate the lower and upper marginally unstable states, respectively.

Initially we take conditions typical of the (idealized) dayside equatorial magnetopause and assume \( B_1 \perp B_2 \) and \( B_2 \parallel v \), i.e., in the magnetosheath \( B \) is parallel to the flow. Figure 3 shows \( U_c \) and \( U_u \), the lower and upper critical velocities plotted against \( \theta_1 \), the angle between \( k_t \) and \( B_1 \), for each of the two unstable surface waves. Also plotted is the critical velocity in an incompressible fluid \( U_t \). The lower critical velocity of the S wave, \( U_{s,S} \), is much smaller than the \( U_t \), while the lower critical velocity of the F wave, \( U_{s,F} \), is only slightly smaller than \( U_t \). Both \( U_{s,F} \) and \( U_{s,S} \) decrease with increasing \( \theta_1 \). It can also be seen that the unstable flow band of the S wave, \( \Delta U_S = U_{s,S} - U_{s,s} \), is much narrower than that of F wave, \( \Delta U_F = U_{s,F} - U_{s,s} \). The results remain qualitatively unchanged if other parameters for the magnetopause are used.

It should be pointed out that the F waves usually start as the \( f'-s' \) mode when \( U \) increases past \( U_t \). As \( U \) continues to increase they will change from the \( f'-s' \) mode to the \( f'-f' \) mode for fixed \( \theta_1 \). For example, if \( \theta_1 = 90^\circ (60^\circ) \), the mode transfer takes place for the F waves at \( U = 0.72 \) (0.94). Meanwhile, S waves may start as either the \( s'-s' \) or the \( s'-f' \) mode. As \( U \) increases, they do not change to the \( f'-s' \) or the \( f'-f' \) mode, indicating that S waves and F waves are distinct if \( \theta_1 \) and \( \theta_2 \) are fixed.

Perhaps the most striking feature in Figure 3 is the fact that when \( \theta_1 \to 90^\circ \), the asymptotic value of the lower critical velocity \( U_{S,S} \) equals \( U_{S,F} = A_2/A_1 \), which is Southwood’s instability criterion [Southwood, 1968]. On the other hand, the range of flow velocities for which the S wave is unstable goes to zero at the same time. Consequently the S wave vanishes in the limit that \( \theta_1 \to 90^\circ \). We will return to this point later in the paper.

Figure 4 shows the effective growth rate, \( \varepsilon = \omega/(k_i A_i) \), as a function of \( \theta_1 \) for several \( \theta_1 \) values in the case of the dayside equatorial region. The geometry and parameters are the same as in Figure 3.

Fig. 3. The lower and upper critical velocities, \( U_c \) and \( U_u \), plotted as functions of \( \theta_1 \) and \( \theta_2 \) for the dayside magnetopause near the equatorial plane, \( \theta_1 \) and \( \theta_2 \) being the angles between \( k_t \) and \( B_1 \) and \( B_2 \), respectively. It is assumed that \( U \parallel B_2 \) and \( \theta_1 + \theta_2 = 90^\circ \). The subscripts \( F \) and \( S \) represent \( F \) waves and \( S \) waves, respectively. The subscript \( i \) indicates the wave in an incompressible plasma. \( U_{S,F} \) is Southwood’s instability criterion. The parameters used are shown in the figure.
Fig. 5. Phase velocities normalized by $A_1$ plotted as functions of directions of $k$, for the dayside (indicated by subscript $d$) and nightside (indicated by the subscript $n$) cases. The phase velocities in the incompressible plasma (indicated by subscript $i$) are also shown in the figure.

is also shown for comparison. Figure 4 shows that $e_F$ is slightly higher than $e_i$ for a range of $U$ just above $U_c$, then becomes lower than $e_i$, whereas $e_S$ is much smaller than both $e_i$ and $e_F$. When $\theta_1 \rightarrow 90^\circ$, $e_S \rightarrow 0$. The phase velocities normalized by $A_1$, $U_{pS}$, $U_{pF}$, and $U_{pi}$ for marginally unstable states at the lower critical velocity are plotted in Figure 5. It is clear that $U_{pF} < U_{pi}$, $U_{pS} < U_{pi}$, $U_{pF}$, and $U_{pS} \rightarrow 0$ if $\theta_1 \rightarrow 90^\circ$.

Figures 3 and 4 tell us that unstable $S$ waves can be excited at rather low flow velocity. Thus, they may appear at the magnetopause on the dayside equatorial plane near the subsolar point even when the Mach number of the solar wind is low. On the other hand, magnetic pulsations on the ground should probably be attributed to coupling with $F$ waves not $S$ waves, since the growth rate of $S$ waves is much lower than that of $F$ waves.

In Figure 6, the variation of $U_{pF}$ and $U_i$, with $B_2/B_1$ is shown for the case of $\theta_1 = 90^\circ$ and constant $B_1$. We can see that $U_{pF}$ decreases and approaches $U_i$ as $B_2$ decreases. This limiting behavior is reasonable since $B_2$ increases with decreasing $B_2$, and the incompressible approximation becomes valid. The curve $U_{pF}$ in Figure 3 is similar to the result obtained by Fejer [1964] (see curve B of his Figure 1). Fejer considered only the case of $\theta_1 = 90^\circ$ where the $S$ wave disappears; hence he discussed only the $F$ wave and calculated only the lower critical velocity. On the other hand, as has been pointed out, the ‘most unstable mode’ discussed by Southwood [1968] is the $S$ wave in the case $\theta_1 = 90^\circ$. Thus the discrepancy between Southwood and Fejer arises because each was considering a different mode. In appendix A we will prove that Southwood’s criterion

$$U_{cS} \cos \alpha = A_2 \cos \theta_2 / A_1$$

(21)

describes the limit of the $S$ wave with zero phase velocity on side 1 in the case of $\theta_1 \rightarrow 90^\circ$. Unfortunately, in this case $\Delta U_2$ and $\varepsilon_S \rightarrow 0$, and this greatly restricts the usefulness of criterion (21). In addition, since $A_1^2$ is often much larger than $A_2^2$ on the dayside magnetopause near the subsolar point, as soon as $\theta_1$ begins to depart from $90^\circ$ (say, around $80^\circ$) $A_1 \cos \theta_1$, $A_2 \cos \theta_2$ and $w_S$ may be of the same order. Then criterion (21) may not be a good approximation; it gives an estimate only within one order of magnitude (see appendix A). An alternative estimate of the lower critical velocity that may be better than (21) is derived in appendix A where we show that

$$U_{cS} \cos \alpha = A_2 \cos \theta_2 / A_1 + C_1 \cos \theta_1 / (A_1^2 + C_1 \lambda)^{1/2}$$

(22)

The second set of calculations (nightside case) corresponds to conditions typical of the flanks of the magnetosphere where we assume $B_1 \parallel B_2 \parallel v$. Note these conditions
can also apply on the noon-midnight meridian. Figure 7 shows how the critical velocities vary with the direction of $k_t$. The magnitudes of the parameters are the same as those used for the dayside case. The results remain qualitatively unchanged if different parameters are used. It can be seen that $U_{cs}$ and $U_{cf}$ change little with $\theta$ and relative to the dayside case are larger when $\theta_1$ is near 90°. Again, $S$ waves disappear if $k_t$ is perpendicular to $B$.

Figure 8 is a plot of $\xi_S$ and $\xi_F$ versus $U$ for various $\theta$. Generally, $\xi_F$ is much larger than $\xi_S$, except in the parallel or quasi-parallel propagation situations where maxima of $\xi_S$ and $\xi_F$ are near equal, indicating the relative importance of $S$ waves in these cases. It should be noticed that $\xi_F$ is slightly higher than $\xi_S$ for a range of $U$ values provided that $\theta$ is far from 0°. The same thing happens in dayside case if $\theta_1$ is not close to 0°. These can be explained as follows. Since in general $U_{cf} < U_i$, there must be a range of $U < U_i$ in which $\xi_F > \xi_S$. Nevertheless, the growth of the $F$ wave must vanish when $U \geq U_{cf}$, so as $U$ increases, $\xi_F$ must decrease, at some point becoming less than $\xi_S$. Thus we cannot conclude that compressibility always reduces the growth rate and tends to stabilize the K-H instability. It depends on the actual configuration as well as the parameters under consideration, especially the flow velocity $U$. The results of Ong and Roderick [1972] apply only if $p_1 = p_2$, $B_1 = B_2 = B$ and $U \perp B$; in this case $U_{cf} = U_i = 0$ and $\xi_F < \xi_S$ as they found.

One particular configuration investigated by previous authors has parallel magnetic fields of equal magnitude, which are also parallel to the flow, and plasma with identical acoustic properties on two sides of the interface and is a special example of the nightside case. In this case, if $k_{||}B$, the dispersion relation simplifies in a system moving with velocity $U/2$ with respect to side 1. Thus, if we set

$$w_1 = (W + U/2)A$$

$$w_2 = (W - U/2)A$$

(23)

$$W^2 = W_+^2 = (\beta + U^2/4) \pm \sqrt{\beta U^2 + \beta^2(\beta - 1)(\beta + 1)}$$

(24)

Figure 9. The normalized lower critical velocity, $U_c$, as a function of $\beta$ in the case of $\beta_1 = \beta_2 = \beta$, $B_1 = B_2$, $p_1 = p_2$, and $B = B_{||}$ $U_{fejer}$ and $U_{fejer}$ are plotted in the figure where $U_{fejer}$ is Fejer's instability criterion in the same configuration.

Equation (9') is satisfied provided

$$W^2 = \frac{W^2}{\beta} = (\beta + U^2/4) \pm \sqrt{\beta U^2 + \beta^2(\beta - 1)(\beta + 1)}$$

(24)

(see appendix B). Only the negative root leads to unstable waves. For that root it can be demonstrated that

$$(U_{fejer})^2 = 4(\beta \pm \beta \sqrt{(\beta - 1)(\beta + 1)})^2$$

(25)

provided $\beta > 1$.

For the same problem, Fejer [1964] found that $w_1 = A$ and $w_2 = w_F$ is a double real root of (19), and he stated that

$$U_{fejer} = 1 + (\beta(1 + \beta))^{1/2}$$

(25)

was the lower normalized critical velocity. Obviously, $U_{fejer} < U_i$ (see Figure 9), and we have verified that when $U_{fejer} < U < U_c$, K-H waves are stable. Since (19) is the square of the original dispersion equation (9'), its double root does not necessarily represent the marginal state, but may be a spurious solution. This is why $U_{fejer}$ does not represent the lower critical velocity. Sen [1964], on the other hand, pointed out that if $\beta \rightarrow 0$ (zero sound speed), the interface is stable for any value of $U$. Actually, as $\beta \rightarrow 0$, equation (19) simplifies to

$$w_1^2 - A^2 = w_F^2$$

Clearly this solution, discussed by Sen, represents Alfvén waves propagating along $B$ with $w_1^2 = w_F^2 = A^2$ and $U = 0$, so his stable solution does not represent a surface wave and is not relevant to the discussion of the K-H instability.

Another point to notice for the special configuration being discussed is that when $\beta \rightarrow 1$, $U_{fejer} \rightarrow 2$ (see (25)), $\Delta U = 0$, $\epsilon = 0$ and the unstable surface wave vanishes. If $\beta < 1$, both $U^2$ and $U^2_{fejer}$ are complex; there is no $U$ that can lead to instability. At the end of this section, it is worth noticing that for both $F$ wave and $S$ wave, $w_1^2/A \leq U \cos \alpha$; as a consequence, by Doppler shift (6), $w_2$ is always negative.

4. THE CHANGE FROM UNSTABLE SURFACE WAVES TO STABLE BODY WAVES

The most striking feature of K-H instability in compressible plasmas may be the existence of the upper critical velocity $U_c$. Once $U \geq U_c$, the instability disappears [Fejer,
Fig. 10. The variations in the absolute value of \( r_i = k_i/k_e \), the normalized imaginary part of the perpendicular component of wave vector, with increasing flow \( U \) for the dayside (solid lines) and nightside (dashed lines) magnetopause. The critical velocities for the two cases, \((U_{cF}^1, U_{cF}^2, U_{cB}^1, U_{cB}^2)\), are also marked in the figure. Again, the symbols \( \perp \) and \( \| \) are used to identify the dayside and nightside geometries. The parameters used are the same as in Figure 3 and Figure 7.

1964; Sen, 1964]. A new observation in this paper is the point that as soon as \( U > U_c \), unstable surface \( F \) waves change to MHD fast body waves and that just at the upper critical flow velocity, the phase velocity of the surface waves increases to a value characteristic of the fast mode MHD waves in the bulk fluid.

This property of the upper critical velocity can be demonstrated for a special simple configuration that can be studied analytically:

\[
B_1 = B_2 = B \tag{27}
\]
\[
\rho_1 = \rho_2 = \rho \tag{28}
\]

Now (19) can be written as

\[
W^4 = \left( \frac{U^2}{2} + (2 - 2\beta) \right) W^2 + \left( \frac{U^4}{16} - \frac{(1 + \beta) U^2}{2} \right) = 0 \tag{30}
\]

where \( W \) is defined by (23). We obtain

\[
W^2 = \frac{U^4}{4} + (1 + \beta) \pm \sqrt{U^2(1 + \beta) + (1 + \beta)^2} \tag{31}
\]

and \( U_{1}^2 = 0, U_{2}^2 = 8(1 + \beta) \). Hence, as the wave becomes unstable, then \( W_1 = U/2, W_2 = -U/2 \), and

\[
\varepsilon = \left[ \sqrt{U^2(1 + \beta) + (1 + \beta)^2} - \left( \frac{U^2}{4} + 1 + \beta \right) \right]^{1/2} \tag{32}
\]

Using (17), we find

\[
\tau_{1r} = \tau_{2r} = [(1 + \beta)/2]^{1/2} \tag{33}
\]
\[
\tau_{1i} = [(1 - 1/2)]^{1/2} \tag{34}
\]
\[
\tau_{2i} = -[((1 - 1/2)]^{1/2} \tag{35}
\]

where

\[
\xi = \frac{U^2}{2(1 + \beta)} - \left[ 1 + \frac{U^2(1 + \beta)}{4} \right]^{1/2} \tag{36}
\]

The phase velocities are then (see (10))

\[
(U_{p_j}^a) = \left( \frac{\omega_j^a}{A^2} \right)^2 = \frac{U^2(6 + 2\xi)}{4\pi} \quad j = 1, 2 \tag{37}
\]

Obviously, \( U_{p_j}^a \) increases and \( \tau_i^a \) decreases with increasing \( U \), and once \( U \to U_c = [8(1 + \beta)]^{1/2} \), then \( \xi \to 0 \), \( (\tau_i^a) \to 0 \), and \( (\omega_j^a) \to A^2 + C^2 = v_f^2 \). Thus the change from the unstable surface wave to the stable body wave happens when the phase velocity grows to the MHD fast wave velocity \( v_f \).

Another simple example is the special example of the nightside case studied in section 3 and appendix B with \( B_1 = B_2 = B, \|k_\|, \rho_1 = \rho_2 \). When \( U = U_a, W^2 = 0 \),

\[
\omega_j^a A_j^2 = U_a^2 (\beta + \beta \sqrt{(\beta - 1)/(\beta + 1)}) \tag{38}
\]

from (7),

\[
\omega_j^a A_j^2 = \omega_j^a (A_j^2 + C_j^2) \tag{39}
\]

\[
(\beta - 1) + \beta \sqrt{(\beta - 1)/(\beta + 1)} > 0 \tag{40}
\]

Therefore, \( (\tau_i^a) = (k_i^a) = 0, j = 1, 2 \). Thus, the surface wave becomes a body wave, and the square of the phase velocity

\[
(U_{p_j}^b)^2 = \omega_j^b (A_j^2 + C_j^2) = \frac{(A_j^2 + C_j^2) - 4A_j^2 C_j^2 \cos^2 \theta_j}{2} = v_f^2(A, C, \theta_j) \tag{41}
\]

where \( \theta_j = \pm \cos^{-1}(1/\sqrt{1 + \tau_i^a}) \) with \( 0 < \cos^2 \theta_j < 1 \). We find that \( (\omega_j^b) \) is the phase velocity of the fast mode in the bulk plasma that propagates at the angle \( \theta_j \) with respect to \( B_j \).

Numerical calculations give us more general information. Figure 10 shows the dependence of \( \tau_i \) on \( U \). The parameters are the same as in Figures 3 and 7. Once \( U \geq U_{af} \), \( \tau_i \) vanishes and \( \tau_i > 0 \) (\( j = 1, 2 \)); surface waves change to body waves. Figure 11 depicts the variations of \( (U_{p_j}^b) \) and \( (U_{p_j}^b)^2 \) as functions of \( U \) for those configurations in Figure 10. Two fast modes exist in each bulk plasma. These two modes appear only if \( U > U_{af} \). Let \( U_{p_j}^b \) be the phase velocity of the mode with higher wave speed (if \( \theta = 90^\circ \), the two fast modes have the same phase speed). It is found that \( (U_{p_j}^b) \) can never exceed \( (U_{p_j}^b) \), \( j = 1, 2 \). \( (U_{p_j}^b) \) and \( (U_{p_j}^b)^2 \) reach the minimum value of \( (U_{p_j}^b) \) and \( (U_{p_j}^b)^2 \) simultaneously when \( U = U_{af} \). Once that happens, the change from surface wave to MHD body waves occurs.

Although the analyses and calculations above are carried out only for some special configurations and particular parameters, the fact that surface waves change to body waves when \( U \geq U_{af} \) may also be a general feature of K-H waves.

It has been pointed out (section 2) that an essential difference between the K-H waves in an incompressible plasma and a compressible plasma lies in the fact that in an incompressible plasma \( \tau_i \approx -1 \) (hence \( k_i = \pm ik_e \) and \( k_i = 0 \) for both stable and unstable situations, while in a compressible plasma, \( \tau_i < 0 \) holds only for stable surface waves. Unstable surface waves are always related to complex \( k_i = \)
It is the variation of $k_z + ik_\perp$ that makes K-H waves in compressible fluids so different from those in an incompressible fluid. Furthermore, in compressible plasmas, the fastest characteristic speed for MHD waves is $U_\|$, which has a finite value. The phase velocities of surface $F$ waves on both sides appear to be required to remain less than $v_{\|1}$ and $v_{\|2}$, respectively. Once $v_{p1}$ and $v_{p2}$ reach $v_{\|1}$ and $v_{\|2}$, surface waves change to body waves. One may think of the incompressible plasma as the limit for which the acoustic speed of a compressible plasma becomes infinite. Correspondingly, $v_\| \to \infty$ and the fast mode speed cannot be attained by the surface wave at any relative flow velocity so the surface waves do not change into body waves.

The transition from surface waves to body waves with increasing $U$ is a salient feature of K-H waves in compressible plasmas, which may possibly occur in the waves generated at the earth’s magnetopause and play an important role in certain physical processes. Theoretical studies have shown that the dayside magnetopause can be unstable to K-H wave growth [Sen, 1964; Southwood, 1968; Boller and Stolov, 1973] and satellite observations indeed have found tailward propagating surface waves there [Fairfield, 1979; Lepping and Burlaga, 1979]. On the other hand, as has been noticed by some authors, in the tail region the streaming speed in the magnetosheath may be larger than the upper critical velocity $U_\|_n$ [Sen, 1965] for the waves propagating in the direction not close to perpendicular to the magnetic fields. Thus, the change from unstable surface waves to stable body waves may occur somewhere on the magnetopause away from local noon. Besides, both analytical and numerical calculations reveal that on the magnetosphere side the normal components of the energy transport velocity are always directed from interface toward the magnetosphere and can be of the order of the Alfvén speed, e.g., there is a net energy flux from the interface going to the plasma. Thus, if $U \to U_\|_n$, the penetration distance $L = 2m/k_z$ becomes very large. The energy associated with unstable surface waves can be transferred deep into the magnetosphere. We discuss this problem and present some calculations in a companion paper.

One final point may be made from the results displayed in Figure 11. In the absence of boundary conditions, the dispersion relation (19) provides only a relation between the frequencies and tangential wavelength. To determine the frequencies of K-H surface waves, the wavelength must be specified from observations or selected to satisfy boundary conditions. Observed wavelengths of surface waves on the magnetopause [Aubry et al., 1971; Lepping and Burlaga, 1979] range from the order of 0.5 $R_E$ to 10 $R_E$. Figure 11 indicates that on the magnetospheric side of the boundary for typical dayside low latitude conditions the $F$ wave becomes unstable when the phase velocity is of order 0.5 $A_1$. With $A_1 \approx 400$ km/s, wave periods in the range 0.5 $R_E$/0.5 $A_1 \approx 20$ s to 10 $R_E$/0.5 $A_1 \approx 300$ s (i.e., Pc 3-Pc 5 periods) are anticipated. On the night side, $F$ waves appear first with phase velocities of order $A_1$. With $A_1 \approx 200$ km/s (reflecting the reduced magnetic field magnitude in the tail), wave periods in the range 0.5 $R_E$/ $A_1 \approx 20$ s to 10 $R_E$/ $A_1 \approx 300$ s are again anticipated. $S$ waves have smaller phase velocities than $F$ waves, so longer periods are anticipated for a given wavelength.

5. CONCLUSIONS

We have found that in compressible plasmas:

1. The MHD surface wave is associated with a quasi fast mode or a quasi slow mode on each side of the interface, hence it can be described as $F$ wave ($f^\prime$-$f^\prime$ mode and $f^\prime$-$s^\prime$ mode) and $S$ wave ($s^\prime$-$s^\prime$ mode and $s^\prime$-$f^\prime$ mode). The $F(S)$ is always associated with an $f^\prime(s^\prime)$ mode in the plasma on side 1.

2. For both $F$ waves and $S$ waves one can find two critical velocities: $U_c$ and $U_\|_n$. The K-H instability can be excited only when $U_c < U < U_\|_n$, where $U$ is the relative velocity between two plasmas.

3. For typical conditions occurring on the earth’s magnetopause with the magnetosphere on side 1 and the magnetosheath on side 2, the lower critical velocity, $U_{c,F}$, and the phase velocity of the $F$ wave on side 1, $U_{p,F}$, are a little lower than those in an incompressible plasma, $U_{c,F}$ and $U_{p,F}$. The growth rate $\epsilon_F$ is very close to but slightly larger than $\epsilon_I$ for a
range of $U$ values just above $U_{cF}$, but $\epsilon_F$ becomes smaller than $\epsilon_i$ as $U$ increases.

4. As to the $S$ wave, in general $U_{cS}, U_{us}, \epsilon_S$, and the unstable range of flow speeds, $\Delta U_S$, are much smaller than those of $F$ waves or those of waves in an incompressible plasma. If $\theta_1 \to 90^\circ$, the $S$ wave vanishes; the asymptotic values of $U_{cS}$ and $U_{us}$ tend toward the Southwood criterion, and $\epsilon_S$ and $\Delta U_S$ equal zero.

5. As the streaming speed increases, an unstable surface wave can start from a stable surface wave and, in some special cases, can also be generated from a stable body wave as well. In any case, it is always related to the complex normal component of the wave vector, $k_n = k_z + i k_i$. Thus the phase velocity has a normal component and the decay distance varies with different $U$.

6. The phase velocity of surface $F$ waves does not exceed the MHD fast mode speed $U_f$. Once $U_{cF}$ reaches $U_f$, which will occur simultaneously for side 1 and side 2 when $U = U_{cF}$, $k_i = 0$ and the unstable surface $F$ wave ($f'-f'$ mode) will change to a stable MHD body wave. We assert without proof that, on the magnetosphere side, the normal components of the group velocities of unstable $F$ waves are directed out of the interface. There is a net energy flux going to the magnetosphere. Thus, if $U \leq U_{cF}$ the energy associated with unstable surface waves may propagate deep into the plasma.

The conclusions listed above hold if the interface is a one-dimensional tangential discontinuity and the plasmas and the magnetic fields on two sides of the interface are homogeneous. The boundary layer and kinetic effects should be taken into account if the K-H instability on the earth's magnetopause is studied in detail.

**APPENDIX A: S WAVES IN THE CASE OF $\theta_1 = 90^\circ$**

The most interesting geometry on dayside magnetopause is that of $\theta_1 = 90^\circ$, which corresponds to the surface waves propagating nearly along the equatorial plane. Southwood [1968] recognized that in this situation the minimum instability criterion is obtained. Our numerical calculations show that this 'most unstable mode' is the limit of $S$ waves when $\theta_1 \to 90^\circ$ (see section 3). In this appendix we investigate the properties of $S$ waves in the case of $\theta_1 \to 90^\circ$ in detail.

First, let us consider the behavior of unstable $S$ waves when $\theta = 90^\circ - 0^\circ$. It is easy to see that in this limited situation, the roots of the dispersion equation (9') become degenerate, yielding a pair of roots at

\[
w_1 = 0^+ \quad w_2 = -A_2 \cos \theta_2 \quad (A1)
\]

It follows that (A1) represents both the lower and upper marginally unstable $S$ waves in the limit of $\theta_1 = 90^\circ - 0^\circ$. In the marginally unstable state, $\epsilon = 0^+$. If we write equation (9') as

\[
D_1(w_1) = -D_2(w_2) \quad (A2)
\]

then $w_1$ and $w_2$ must satisfy the following marginal equation as well [Southwood, 1968]

\[
\frac{dD_1(w_1)}{dw_1} = -\frac{dD_2(w_2)}{dw_2} \quad (A3)
\]

Substituting for $D_1$ and $D_2$, we obtain

\[
\frac{dD(w_1)}{dw_j} = \frac{2w_j\rho_j}{(1 + w_j^4/G_j)^{1/2}} - \frac{\rho_j(w_j^2 - A_j^2 \cos^2 \theta_j)}{(1 + w_j^4/G_j)^{3/2}} \cdot \frac{2w_j^2}{G_j} - \frac{w_j^2(A_j^2 + C_j^2)}{G_j^2} \quad (A4)
\]

where

\[
G_j = w_j^2(A_j^2 + C_j^2) - A_j^2C_j^2 \cos^2 \theta_j \quad (A5)
\]

and $j = 1, 2$.

It has been pointed out in section 2 that for the lower marginal state of the $S$ wave, $0 \leq w < w_{gl}$. The left-hand side of (A3) equals zero if $w_1 = 0^+$. As $w_1$ increases, the left side of (A3) goes through all purely negative imaginary values reaching $-iw_1$ for $w_1 = w_{gl} - 0^+$ (see (14)). This assertion may be argued as follows. When $0^+ < w_1 < w_{gl} - 0^+$, $\tau_1^2 = -1 + w_1^4/G_1 < 0$; hence $dD_1(w_1)/dw_1$ must be purely imaginary. And at $w_1 = w_{gl} - 0^+$, $G_1 = -\tau_1^2 = -w_{gl}^4/0^+$, then $dD_1(w_1)/dw_1 = ip_1(w_1^2 - A_1^2 \cos^2 \theta_1)(A_1^2 + C_1^2)w_{gl}(-G_1)^{1/2} = -ip_1 \omega_{gl}$.

On the other hand, if $w_2 = -A_2 \cos \theta_2, G_2 = A_2^2 \cos^2 \theta_2, -dD_2(w_2)/dw_2 = -2w_2\rho_2(-1 + w_2^4/G_2)^{1/2} = -ip_2A_2 \omega_{gl}$. Therefore, there must be a value $w_1 = w_{gl}$ between $0^+$ and $w_{gl} - 0^+$ for which the left-hand side of (A3) equals $-ip_2A_2 \omega_{gl}$.

Numerical calculations show that the instability of $S$ waves terminates at $w_1 = w_{gl} < w_{gl} = \sqrt{2} w_{gl}$. Naturally, as long as $\theta_1 = 90^\circ - 0^+$, $w_{gl} = 0^+$. As a result, $\Delta U_S, \Delta w_2$, and $\Delta w_1$, which are the unstable bands of $U_S, w_2$, and $w_1$, respectively, and $\epsilon$ all approach 0 from above which is consistent with the fact that when $\theta_1 = 90^\circ$, $S$ waves should vanish. Equation (A1) gives by use of (6) that

\[
U_u = U_c = A_2 \cos \theta/\cos \alpha A_1 \quad (A6)
\]

where $\alpha$ is the angle between $k_i$ and $U$. Equation (A6) is Southwood's criterion, which has been used for years in studying the generation of micropulsations in the magnetosphere or on the ground [Wolfe and Kaufmann, 1975; Lee and Olson, 1980].

The fact that both $\Delta U_S$ and $\epsilon$ is greater than 0 greatly restrict usefulness of criterion (A6) in the limit of $\theta_1 = 90^\circ$. Although $\Delta U_S$ and $\epsilon_S$ are relatively smaller than the corresponding values for $F$ waves, $S$ waves can still appear on the magnetopause provided $\theta_1$ is not very close to 90°, say, $\leq 80^\circ$. In this case, assuming $w_2 = A_2 \cos \theta_2$, we can find that $w_{gl}$ may be closer to $w_{gl}$ than to $0^+$, especially when $\sin \theta_2 \approx 1$, (see (A4) and (A5)). Then, instead of being expressed by (21), the lower critical velocity should be approximately equal to

\[
U_{cS} \cos \alpha = A_2 \cos \theta_2 A_1 + w_{gl} A_1 \quad (A7)
\]

Since the inequalities (17) and (18) hold if $\beta_1 < 1$ and $\beta_2 > 1$, the wave must be $s'$-$s'$ mode.

To show the validity of criterion (A7) and the discussion above, let us take $\cos \theta_1 = 0.2(\theta_1 = 78.5^\circ), \cos \theta_2 = \cos \alpha = 0.98$ as an example. Using the same parameters as in Figure
3, we obtain $A_2 \cos \theta_2/A_1 = 0.212$, $|w_{z2}|/A_1 = 0.213$, $|w_{z2}|/A_1 = 0.214$, $w_{z2}/A_1 = 0.254$, $w_{z2}/A_1 = 0.125$, $w_{z2} = A_1 = 0.126$, $w_{z2}/A_1 = 0.125$, $w_{z2}/A_1 = 0.177$, $U_{c5} = 0.345$, $U_{cs} = 0.347$.

Since $U_{c5} \cos \alpha = 0.338$ and $A_2 \cos \theta_2/A_1 + W_{c1}/A_1 = 0.337$, it is apparent that (A7) gives a good estimate of the lower critical velocity. The inequalities: $w_{z1} < w_{z2}$ and $|w_{z2}| < |w_{z1}|$ show that the wave is $s'$-$s'$ mode on both sides of the boundary.

The phase velocity of $S$ waves is much smaller than that of $F$ waves. Therefore, the surface waves found by Howe and Siscoe [1972] with velocities ranging typically from 10 to 20 km/s should be attributed to the excitation of unstable $S$ waves on the magnetopause.

Similar to the limit of $\theta_1 \rightarrow 90^\circ$, the case $w_1 \rightarrow A_1 \cos \theta_1$, $w_2 \rightarrow 0$ represents the lower marginally unstable state for the $f'$-$s'$ mode if $\beta_1 < 1$ and $\beta_2 > 1$. The lower critical velocity can then be expressed as

$$U_c \cos \alpha = \cos \theta_1$$

which is the same as that given by Southwood [1968].

**APPENDIX B: A SPECIAL EXAMPLE OF THE NIGHTSIDE CASE**

A simple configuration previous authors have studied is a special example of the nightside case in which $B = B_2 = B$, $k(U_1) B$ and $p_1 = p_2 = p$. Under these conditions, (19) can be broken down as

$$w_1^2 - A^2 = 0$$

and

$$(w_1^2 - A^2)(w_2^2 - w^{2}_2 - C^2)$$

Equations (B1) represent Alfvén waves propagating along $B$ without decaying away from the interface, while equation (B2) can be rewritten as

$$W^2 = \left( \frac{U_1^2}{2} + 2\beta \right) W^2 + \left[ \frac{U_1^2}{16} \frac{\beta U_1^2}{2} + \frac{2\beta^2}{(1 + \beta)} \right] = 0$$

provided $w_1 \neq 0$. Here

$$W = w_1/A - U/2$$

$$= w_2/A + U/2$$

From (B3),

$$W^2 = (\beta + U^2/4) \sqrt{\beta^2 + \beta^2(\beta - 1)/(\beta + 1)}$$

When $\beta > 1$, $W^2$ always $> 0$, while $W^2$ becomes $< 0$ for $U$ such that

$$U_c < U < U_u$$

where

$$U_{c,s}^2 = 4(\beta \pm \beta \sqrt{(\beta - 1)/(\beta + 1)})$$

Obviously a negative value of $W^2$ means the interface must be unstable; thus $U_c$ and $U_u$ are the lower and upper critical velocities, respectively.

It is worth noting that if $U = U_c$, $w_{1,s}^2 = w_{2}^2$ and if $U = U_u$, $w_{1,s}^2 = w_{2}^2$ where

$$w_{c,u}^2 = C^2 \pm C^2[(\beta - 1)/(\beta + 1)]^{1/2}$$

Therefore, $w_{2}^2 < w_{c}^2 < w_{c}^2 = A^2$, and $w_{u}^2 > w_{c}^2 = C^2$. When $U \approx U_c$, just below the instability situation, the $K$-$H$ waves on both sides of interface are slow body waves, since $w_{2}^2 < w_{c}^2 = 2\beta/(1 + \beta)$. So, in this special case, the unstable surface waves start from slow MHD body waves as $U$ passes over $U_c$ and are $s'$-$s'$ mode. If $U$ continuously increases they will change to $f'$-$f'$ mode and finally becomes fast body waves when $U \geq U_u$.

**NOTATION LIST**

- $A$: Alfvén velocity;
- $B$: magnetic induction;
- $c$: light speed;
- $C$: sound speed;
- $F$: fast surface wave;
- $f$: fast mode of MHD waves;
- $f'$: quasi-fast mode of surface waves;
- $k$: wave vector;
- $k_r$, $k_i$: real and imaginary parts of the normal component of the wave vector;
- $L = 2\pi/k$: decay distance of surface waves away from the interface;
- $p$: pressure;
- $S$: slow surface wave;
- $s$: slow mode of MHD waves;
- $s'$: quasi-slow mode of surface waves;
- $v$: magnetosheath flow velocity relative to the magnetospheric plasma;
- $v_f$: phase velocity of the fast mode of MHD waves in bulk plasmas;
- $v_p$: phase velocity of surface waves;
- $v_{c,s}$: lower and upper critical velocities;
- $U$: normalized magnetosheath flow velocity in units of $A_1$, the Alfvén velocity in the plasma;
- $U_{c,s}$: normalized lower critical velocities for $F$ waves and for $S$ waves;
- $U_i$: normalized critical velocity for the K-H instability in an incompressible plasma;
- $U_{F\text{e}}$: Fejer's criterion for the K-H instability;
- $U_{ps}$, $U_{ps}$: normalized phase velocities of $F$ waves and of $S$ waves;
- $U_{ps}$: normalized phase velocity of K-H waves in an incompressible plasma;
- $U_0$: Southwood's criterion for driving the 'most unstable mode';
- $U_{uk}$, $U_{uk}$: upper critical velocities of $F$ waves and of $S$ waves;
- $U_{W}$: normalized phase velocity of the fast mode of MHD waves in bulk plasmas;
- $W$: normalized tangential phase velocity in the frame of reference in which the magnetosheath and magnetospheric plasmas move at $v/2$ and $-v/2$, respectively;
- $w$: tangential phase velocity;
- $w_f$, $w_s$: phase velocity of the fast mode and the slow mode of MHD waves times $(1 + \beta^2)^{1/2}$;
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