Lower-Hybrid-Drift Instability and Its Associated Anomalous Resistivity in the Neutral Sheet of Earth's Magnetotail

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It has been suggested previously that at the center of the earth's plasma sheet there exists a thin non-adiabatic layer, the neutral sheet, whose thickness is of the order of the ion gyroradius. The ion distribution in such a thin sheet may have some non-Maxwellian form. The purpose of this paper is to construct a one-dimensional 'non-Maxwellian' model for the steady neutral sheet and to examine its instability properties with respect to the lower-hybrid-drift (LHD) wave. A combination of a Boltzmann distribution and a modified Alpers' distribution for ions is taken. We show that in the limit of a thin neutral sheet, this distribution is a more powerful source of free energy than the usual drifting Maxwellian. It is found that most regions within the neutral sheet are unstable. The frequency spectrum of the unstable waves is nearly the same as in the drifting Maxwellian case. As shown previously by Huba et al. (1978), the frequency spectrum is in good agreement with the observations. The growth rate, assuming a modified Alpers' distribution function, is found to be enhanced compared with that due to a drifting Maxwellian model in a portion of the sheet at some distance from the null of the field. The saturation level of the fluctuating field and the anomalous resistivity are calculated in the entire sheet by using current relaxation as the stabilizing mechanism and are found to be 3-4 times larger than those in a Maxwellian sheet.

1. INTRODUCTION

A problem of continuing interest to magnetospheric plasma physicists is the study of anomalous transport in collisionless plasmas. In the absence of Coulomb collisionality, transport coefficients such as resistivity, viscosity, and thermal conductivity will be strongly affected by the presence of short wavelength microturbulence. In turn, these transport coefficients will regulate the flux of mass, momentum, and energy across current layers, such as the magnetopause, and could modify the reconnection rate at the nose of the magnetopause and in the tail. Thus an understanding of microturbulence is essential to accurate large-scale modeling of the magnetospheric convection pattern.

In this study we wish to consider the linear and quasilinear evolution of lower-hybrid-drift (LHD) waves [Krall and Lieber, 1971] and to assess their importance to plasma dynamics in the earth's plasma sheet. As has been shown by Huba et al. [1978], a significant level of electrostatic turbulence of LHD waves can be generated in the magnetotail when the magnetic field gradients are sufficiently large and the plasma $\beta$ (ratio of thermal to magnetic pressure) is sufficiently small. Recent satellite observations of electrostatic and magnetic noise in the distant magnetotail [Gurnett et al. 1976] have shown the presence of strong electrostatic noise in a frequency range consistent with the prediction of the LHD instability theory near the edge of the plasma sheet and sometimes in the region near the neutral sheet.

There remain a number of questions concerning the applicability of the LHD to the interior of the plasma sheet. First, to generate significant growth rates, it is necessary for the local magnetic gradient scale lengths to be of the order of $r_e$, the mean ion gyroradius, much thinner than is generally observed for the earth's plasma sheet. Second, as has been demonstrated by Drake [1981], in the absence of $\nabla B$ drift, the electron response to the fluctuating field spectrum is adiabatic until $e\phi/T_e \approx 1$, where $\phi$ is the fluctuating electrostatic potential and $T_e$ is electron temperature. Thus, if the LHD instability generates a dissipative electron response (necessary for directly generating anomalous resistivity) we require $E_i \approx (eB/2m_eT_e)\sqrt{\alpha}$, $T_i$ electron temperature. Taking $T_i = 5$ keV, $T_e = 1$ keV and $B = 2 - 20$ $\gamma$, we require $E_i \approx 100 - 1000$ MV/M. Thus, for even small magnetic field strengths, we require extremely large fluctuating electric fields. Finally, there is the difficulty that plasmas with $\beta \approx 1$ are stable to LHD growth because of $\nabla B$ drift resonances with the electrons. Thus, the instability works well near the lobe where the gradients are large and the density is low but becomes increasingly difficult to destabilize as we approach the center of the plasma sheet. It is not entirely clear, therefore, how the mode can penetrate significantly into the plasma sheet or even if it does, how this will result in true particle diffusion, given that the electron response is adiabatic.

A possible resolution to the latter difficulty is saturation of the mode by electron resonance broadening. It has been shown by Huba and Papadopoulos [1978] that over a large range of parameter space (in particular $\beta \sim 1$, $T_i > T_e$) the LHD instability will saturate due to a broadening of the effective velocity space over which electron $\nabla B$ drift damping is significant. In the earth's magnetosphere, however, $T_i \gg T_e$ is generally satisfied, so it is unlikely that resonance broadening will dominate over other mechanisms such as current relaxation [Davidson, 1978]. Nevertheless, for $\beta \sim 1$, there should still be significant electron damping so that the electron response will be dissipative in this region. It follows that in the broad central region of the plasma sheet when the $\beta$ is large enough so that drift resonances generate dissipation, but not so large as to completely stabilize the mode, we should expect true particle diffusion.

There still remains the difficulty of how the mode can penetrate close to the null line, where we should normally expect $\beta > 1$ and thus stability. Two methods have thus far been proposed. First, Huba et al. [1978] have suggested that low frequency turbulence from the tearing instability could create
Then allow LHD growth. Thus the LHD instability would be
low. If the saturation amplitudes are high enough for
proposal, by Drake [1981], assumes that initially growth is
triggered as a consequence of tearing mode growth. A second
sharp current gradients within the plasma sheet that would
confined to those regions where the gradients are sharp and
the null line. The ambient magnetic field is in the -x direction for z>0 and
in the +x direction for z<0.

Recent particle simulations [Winske, 1981] reveal that the
LHD instability grows at early times in regions of strong den-
sity gradients away from the reversal point. At late time, after
saturation, a primarily electromagnetic mode with a longer
wavelength appears at the field null, a result of the nonlinear
diffusion to occur, then magnetic flux will diffuse inward to-
ward the null region, increasing the gradients and destabiliz-
ing the LHD waves closer to the null line.

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sity gradients away from the reversal point. At late time, after
saturation, a primarily electromagnetic mode with a longer
wavelength appears at the field null, a result of the nonlinear
penetration of LHD unstable waves.

In this paper we limit ourselves to linear and quasi-linear
analysis, and consequently our calculations are applicable
only to the first part of the above scenario. We will not be con-
cerned with understanding the mechanism for the penetration
of the LHD mode into null region. We suppose that a thin (\( \geq r_i \))
current layer exists at the center of the plasma sheet, across which a sizable fraction of the total plasma sheet mag-
netic field reversal occurs. We hereinafter designate this cur-
rent layer the 'neutral sheet.' The possibility of such a neutral
sheet existing is not without precedent. Cowley [1978] has
shown that if the marginal firehose criteron is exceeded in the
plasma sheet, it is possible to find model equilibria, which
necessarily require a thin nonadiabatic layer at the center of
the sheet. Similarly, Eastwood [1972] has shown that a 'beam'
solution to the steady state tail problem requires thin current
layers. Observationally, there exist a number of cases in which
such sheets are thought to have been observed [Ness, 1969;
Speiser, 1973; Hones et al., 1973; Thomas and Hedgecock,
1975].

Ideally, we should like to construct a solution to the neutral
sheet problem including the effects of the normal component
of the magnetic field and then analyze its stability to LHD
waves. This is, unfortunately, an extremely complicated prob-
lem. Instead, we will solve the reversed-field sheet pinch prob-
lem with a self-consistent kinetic equilibrium. The distribu-
tion function chosen contains many of the salient features of
the actual current layer and all those necessary for studying
the stability of LHD waves. In particular, we choose equilibria
that have a scale length of \( O(r) \) and whose ion pressure ten-
sors are nonsymmetric. We impose the latter condition because
within the actual neutral sheet, the time history of the mirror-
trapped protons is significantly different from that of the pro-
tons that rapidly transit this region [Francfort and Pellat, 1976].
The resultant mixing of these two populations, and the
requirement that together they generate the self-consistent
field and maintain charge neutrality, should lead to a signifi-
cant 'non-Maxwellian' component within the neutral sheet.
Thus, we choose as our distribution function for ions a combi-
nation of a modified Alpers' distribution [Alpers, 1969, 1971]
and a Boltzmann distribution, giving a new one-dimensional
model of the neutral sheet. It can be seen that for the distribu-
tion chosen there is an ion flow in the -y direction generated
by the non-Maxwellian piece. As the scale length decreases,
the ion distribution becomes more non-Maxwellian, and
the magnitude of the ion flow increases. It is recognized that
the LHD instability is excited by a resonance between the mo-
tions of ions and the LHD wave. Obviously, the additional
motion of ions can enhance the resonance and gives rise to a
potential source of free energy for the LHD instability.

Recently, in his article on transport phenomena in the
earth's magnetopause, Tu [1979] dealt with the problem nu-
merically, indicating that both growth rate and anomalous re-
sistivity in a non-Maxwellian magnetopause are stronger than
those in a 'Maxwellian magnetopause.' The purpose of this
paper is to discuss a similar problem analytically for the tail
neutral sheet. In section 2 we list all the basic assumptions
used for simplifying the problem. In section 3, the distribution
functions of ions and electrons are described, and the non-
Maxwellian neutral sheet model is constructed. The disper-
sion equation based on the assumption of \( j_0/k_v \) < 1 is de-
vised in section 3. The maximum growth rate \( \gamma \) and the cor-
responding real frequency \( \omega \) of the wave are calculated for the
whole neutral sheet. The saturation level of the fluctuating
field and the anomalous resistivity are estimated by using the
quasi-linear theory of electrostatic plasma turbulence and
compared with similar analysis for a Maxwellian neutral
sheet. In the final section we summarize our theoretical re-
results, showing that it is possible that the non-Maxwellian
distribution of ions in the neutral sheet can enhance the LHD
instability and provide stronger saturation level and anomalous
resistivity than those in a Maxwellian sheet.

2. Basic Assumptions

Here we summarize the basic assumptions of the present
analysis.

1. The geometry of one-dimensional neutral sheet under
consideration is shown in Figure 1. The only spatial variation
is in the \( z \) direction with \( z = 0 \) representing the null of the
magnetic field. The ambient magnetic field is in the \(-x \) direc-
tion for \( z > 0 \) and in the \( x \) direction for \( z < 0 \). The normal
component of field, \( B_z \), is neglected since we are interested in
the regions not very near the null.

2. Both the cross-tail electric field, \( E_x \), and the resulting
\( E \times B \) drift speed \( v_{drift} = c E_x / B \) are neglected as we assume that
\( E_x \) is rather weak and \( v_{drift} \) is too low to contribute significantly
to the equilibrium conditions in the magnetotail. Thus, elec-
trons drift in the \( y \) direction with a mean fluid velocity \( v_{ey} \)

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**Fig. 1.** The geometry of one-dimensional neutral sheet model. The only spatial variation is in the \( z \) direction with \( z = 0 \) representing the null. The ambient magnetic field is in the \(-x \) direction for \( z>0 \) and in the \(+x \) direction for \( z<0 \).
+ \nu_{d\alpha}\), where \(\nu_{d\alpha} = c E_{\alpha}/B\), and \(\nu_{d\alpha}\) is the diamagnetic velocity. This one-dimensional model represents the geometry of the part of the neutral sheet that is far away from the X type neutral point.

3. Electrons are strongly magnetized, e.g., \(|\omega| \ll \Omega_{Te}\), where \(\omega\) is the wave frequency, \(\Omega_{Te} = eB/m_e c\) is the gyro-frequency of an electron, and \(r_{eL}^2 = |\partial \ln n/\partial z|, r_{eL}^2 = |\partial \ln B/\partial z|\).

4. The trajectory of an ion within a wave period can be regarded as straight because \(|\omega| \gg \Omega_{Te}, \Omega_{Li}, (k\lambda)^2 \gg 1\), where \(k\) is the wave number.

5. Wave perturbations are flute-like, e.g., \(\vec{B} = 0\), and the inhomogeneity is weak, \(k_{\perp}^2 \gg k_{\parallel}^2 \gg (\partial \ln n/\partial z)^2, (\partial \ln B/\partial z)^2\), so that local approximations can be used.

6. \(|\omega/\nu_{\perp}\lambda| < 1\), where \(\nu_{\perp}\lambda\) is average thermal velocity of the ions perpendicular to the ambient magnetic field, which means the instability is kinetic, not fluid-like.

7. The perpendicular temperature of the electrons is much lower than that of the ions.

8. The steady drift velocity \(\nu_{d}\) is constant in the whole neutral sheet, so \(E_{\alpha}/B = \) constant, too.

9. The perturbations in the field are electrostatic. This approximation is made because the wave observations in situ have shown that there are broadband electrostatic noises in the magnetotail, and the anomalous transport effects are easier to calculate in this approximation.

### 3. STEADY STATE DISTRIBUTION AND THE NEUTRAL SHEET MODEL

In constructing a new model of the neutral sheet, it is necessary to determine the steady state distribution of particles. The steady state distribution function can be written as a function of the single particle constants of the motion in the unper- turbed fields. It is easy to show that the constants of motion are

\[
\begin{align*}
n_{\perp} &= C_1 \\
m\nu_{\perp}^2/2 + q\phi &= C_2 \\
m\nu_{\parallel} + qA/C &= C_3
\end{align*}
\]

where \(m\) is the mass of particles, \(q\) is the electric charge, and \(\phi\) and \(A\) are the potentials of the electric field and the magnetic field; \(-\nabla \phi = \vec{E}, \nabla \times \vec{A} = \vec{B}\). Therefore, for ions we can take the steady state distribution function as

\[
F_{\infty} = \frac{n_0}{\pi^{3/2} e^{\nu_{\perp}^2/2}} \exp \left[-\nu_{\parallel}^2/2\right] \left\{1 - \frac{\alpha}{1 + \alpha} \text{erf} \left[\frac{w}{(1 - w^2)^{1/2}} \left(1 + \frac{w}{w^2 + A^2}\right)\right]\right\}
\]

where

\[
\begin{align*}
v_{\parallel}^2 &= \frac{2T_{\parallel}}{m_i} \\
v_{\perp}^2 &= \frac{2T_{\perp}}{m_i} \\
\xi A' &= \frac{eA}{m_e\nu_{\perp}^2}
\end{align*}
\]

both \(T_{\parallel}\) and \(T_{\perp}\) are constant, and

\[
A' = \frac{eA}{m_e\nu_{\perp}^2}
\]

\(c\) is the speed of light, \(e\) is the electron charge. In (1) there are three parameters, \(\alpha\), \(\xi\), and \(w\), for which \(\alpha > 0\), \(\xi > 0\), and \(0 < w < 1\). We have assumed that in the null of the neutral sheet \(z = 0, A = A(z = 0) = 0\). (This should be compared with the analysis of Alpers [1969] in which \(A(z \rightarrow -\infty) = 0\) was assumed.) We choose (1), since it is a simple non-Maxwellian distribution which we can treat analytically. Besides, it goes to the Maxwellian distribution far away from the null and contains more free energy, as we can see in the following. Figure 2 shows how \(F_{\infty}\) varies with \(\nu_{\perp}\).

For electrons, according to assumption (3), the steady state distribution function is a bi-Maxwellian of the form

\[
F_{\infty} = \frac{n(z)}{\pi^{3/2} e^{\nu_{\perp}^2/2}} \exp \left[-\nu_{\parallel}^2/2\right] \left\{1 - \frac{\alpha}{1 + \alpha} \text{erf} \left[\frac{w}{(1 - w^2)^{1/2}} \left(1 + \frac{w}{w^2 + A^2}\right)\right]\right\}
\]

where

\[
\begin{align*}
\nu_{\perp}^2 &= \nu_{\parallel}^2 + \nu_{\perp}^2 \\
v_{\parallel}^2 &= \frac{2T_{\parallel}(Z)}{m_i} \\
v_{\perp}^2 &= \frac{2T_{\perp}(Z)}{m_i}
\end{align*}
\]

\(Z\), the coordinate of the electron's guiding center [Davidson et al., 1977] is a constant of motion. Under the assumption (3), \(|v_{\parallel}/E_{\alpha\perp}\lambdaencer; | < 1\) and \(Z \simeq z - (\nu_{\parallel} - \nu_{\parallel})/\nu_{\perp}\lambdaencer; \), approximately, we have

\[
F_{\infty} = \frac{n(z)}{\pi^{3/2} e^{\nu_{\perp}^2/2}} \exp \left[-\nu_{\parallel}^2/2\right] \left\{1 - \frac{\alpha}{1 + \alpha} \text{erf} \left[\frac{w}{(1 - w^2)^{1/2}} \left(1 + \frac{w}{w^2 + A^2}\right)\right]\right\}
\]

Here, as well as in (1), \(\nu_{\perp}^2 = 2T_{\perp}/m_i\) may be different from \(\nu_{\parallel}^2 = 2T_{\parallel}/m_i\). Choosing \(T_{\parallel} \neq T_{\perp}\) does not complicate the problem, since, as it can be seen in section (4), the instability is only related to \(T_{\perp}\), if \(k \cdot B = 0\).

Assuming \(n_{\infty} \approx n_{\infty}\), it is found that

\[
n_{\infty} = n_{\infty} = n_{\infty} e^{-\nu_{\perp}^2/2} \left[1 - \frac{\alpha}{1 + \alpha} \text{erf} \left(wA'\right)\right]
\]

where \(n_{\infty}\) is the number density at the null.

From (1) it can be obtained that
Equations (1), (6b), (7) and (15) give a new self-consistent model for the neutral sheet or for the whole plasma sheet. Note that $A'$ is used to represent the distance from the null, which is meaningful since $A'$ varies monotonically with $z$. Actually, it can be seen that (1) is a combination of a modified Alpers’ distribution [Alpers, 1969, 1971] and a Boltzmann distribution with $\varphi/\Theta_n = \xi A'$. If $\xi = 0$, $F_0$ becomes a bi-Maxwellian when $A' \to \infty$ ($z \to \infty$). In that case, $1 + \alpha = n_0/n_\infty$ and the meaning of $\alpha$ is easy to understand. We also can see that if $\xi = 0$ and $\alpha > 1$, then as soon as $A' \approx 1/w$, $n$ becomes much smaller than $n_0$. The physical significance of $A' = 1/w$ can be understood by expressing (5) as a ratio of lengths:

$$A' = \frac{d \ln A/\Delta z}{r_i}$$

$L_A$ is scale length of the sheet with respect to the variation of $A$. So, $wA' \sim 1$ means $w^{-1} = L_A/r_i$; for example, if $w \to 1$, then $r_i \to L_A$.

Generally speaking, if none of the parameters $\xi$, $\alpha$, or $w$ vanishes, the scale lengths of the sheet are shorter than those that apply when these parameters do vanish. Figure 3 shows the variations of $n$ and $B$ with $A'$. It is clear that the larger $\alpha$, $w$, and $\xi$ are, the more quickly $n$ decreases or $B$ increases with $A'$. We can also see that

$$\xi = 2\varphi_0/v_{||}$$

while $w$ and $\alpha$ characterize the departure from a Maxwellian distribution. The larger $w$ and $\alpha$ are, the more (1) deviates from a Maxwellian, as can be seen from Figure 2. It is also clear in Figure 2 that the majority of ions have a negative $y$ component of velocity. Only when $A' \to \infty$ does the distribution become Maxwellian.

Since we will ignore effects associated with finite $\beta$, including both $\nabla B$ drift resonance of electrons and electromagnetic effects, our calculations apply only where $\beta < 1$. For typical parameters ($w = 0.6, \alpha = 4$) we have calculated $\beta$ as a function of $A'$ and find that $\beta \approx 1$ for $A' \approx 0.1$. For the same parameters, the current sheet thickness, $L$, can be identified in Figure 3 as $L = z(A' \approx 2)$. We also find $z(A' = 0.1) \approx 0.2L$. Consequently, our calculations apply between $0.2L$ and $L$ or between $A' = 0.1$ and $A' = 2$.

At the end of this section it is worth pointing out that the ion pressure and temperature are tensors

$$P_{xx} 0 0$$

$$0 0 0$$

Finally, the Maxwell equation in a steady state, $\nabla \times B = 4\pi j/c$ together with (13) leads

$$\frac{B^2}{8\pi} + n(T_{\perp} + T_{\parallel}) = \text{constant}$$

Thus, it follows from (7) and (14) that

$$B = B_\infty = \left\{ 1 - e^{\omega A'} \left[ 1 - \frac{\alpha}{1 + \alpha} \text{erf}(wA') \right] \right\}$$

where

$$B_\infty = \left[ 8\pi(T_{\perp} + T_{\parallel})n_0 \right]^{1/2}$$
Newtonian equation in the steady state, only exists in the z direction.

4. LOCAL DISPERSION RELATION AND THE LOWER-HYBRID-DRIFT INSTABILITY

Initially we consider the electrostatic wave within the context of a local analysis where the z variation of perturbation amplitudes is neglected. The general dispersion equation describing the lower-hybrid-drift instability can be found in the usual way from the relation

$$1 + X_x + X_z = 0$$  \hspace{1cm} (21)

where, for $T_{ex}/T_{iz} \ll 1$,

$$X_x = \frac{\omega_{pe}^2}{\Omega_e^2} \frac{2\omega_{pe}^2}{k_r^2\nu_{ez}^2} - \frac{k_r\nu_e}{w - k_r\nu_e}$$  \hspace{1cm} (22)

$$\nu_e = -\frac{T_{ex}}{m_i\Omega_e} \left( \frac{\partial \ln n_i}{\partial z} - \frac{\partial \ln B}{\partial z} \right)$$  \hspace{1cm} (23)

and

$$\omega_{pe}^2 = 4\pi e^2/m_e$$  \hspace{1cm} (24)

In getting (17) we have used the approximations that $|\omega/k_r\nu_{iz}| \ll 1$ and $|\omega - k_r\nu_e| \gg |k_r\nu_e|$ where $\nu_e = v_e/k_r$. The effect of $\nabla B$ orbit modification is stabilizing for small $k_r$ and destabilizing for large $k_r$ [Huba and Wu, 1976; Davidson et al., 1977] when $v_e/v_{iz} \ll 1$. With these assumptions, we see that $X_z$ is independent of $T_e$.

As to $X_x$, we find that

$$X_x = 2\omega_{pe}^2(1 + \xi Z_2 + Z_3)/(k_r^2\nu_{iz}^2)$$  \hspace{1cm} (26)

where

$$\xi = \omega/(k_r\nu_{iz})$$

$$Z_2 = \frac{1}{w^{1/2}} \int_{-\infty}^{+\infty} (1 - R \text{erf} [S(\lambda + A')] \exp (-\lambda^2)) \, d\lambda$$

$$Z_3 = \frac{RS}{2w} \int_{-\infty}^{+\infty} \exp (-S^2(\lambda + A')^2 - \lambda^2) \, d\lambda$$

$$R = \alpha/(1 + \alpha)$$

$$S = w/(1 - w^2)^{1/2}$$

where the integrals are to be carried out along the Landau contour [Kroll and Trivelpiece, 1973]. After a lengthy procedure, it can be shown if $|\xi| \ll 1$, then

$$X_{11} = \frac{2\omega_{pe}^2}{k_r^2\nu_{iz}^2} \left[ 1 + i\omega^2/2(k_r(1 + \alpha)/|k_r| - \alpha Q_{31} + \alpha Q_{32}) \right]$$

$$+ i\omega^2/2 \alpha Q_{30} + \alpha Q_{31} + (Q_{33} - Q_{22}) \alpha \xi / (1 + \alpha)w$$  \hspace{1cm} (29)

where

$$Q_{21} = \text{erf} \left( \frac{w}{(1 - w^2)^{1/2}} \right)$$

$$Q_{22} = P \frac{1}{\pi^{1/2}} \int_{-\infty}^{+\infty} \text{erf} [S(\lambda + A')] \exp (-\lambda^2) \, d\lambda$$

$$Q_{30} = S \exp (-S^2A'^2)/2$$

$$Q_{31} = \frac{S}{2} P \frac{1}{\pi^{1/2}} \int_{-\infty}^{+\infty} \exp \left( -\left( S^2(\lambda + A')^2 - \lambda^2 \right) \right) / |\lambda| \, d\lambda$$

$$Q_{32} = -2S^2Q_{30}A'$$

$$Q_{33} = -S^2 \exp (-S^2A'^2) \left\{ 1 + wS\alpha P \frac{1}{\pi^{1/2}} \int_{-\infty}^{+\infty} \exp \left[ -\left( t + wS\alpha^2 \right)^2 \right] \, dt \right\}$$

$P$ means the principal part of the integral (see appendix); therefore, assuming $|k_r| = k_r$, the dispersion equation becomes

$$D(k_r, \omega) = 1 + \frac{2\omega_{pe}^2}{k_r^2\nu_{iz}^2(1 + \alpha)w} \left[ \frac{i\omega^2(\omega + i\gamma)}{k_r\nu_e} + \frac{\omega(k_r(1 + \alpha))}{k_r\nu_e} \right]$$

$$+ \alpha(Q_{32} - Q_{22}) \left( \frac{\omega + i\gamma}{k_r\nu_e} \right) + \frac{T_{ex}k_r\nu_e(1 + \alpha)w}{T_{ex}(\omega - k_r\nu_e + i\gamma)}$$

$$G = 1 + \frac{\omega_{pe}^2}{\Omega_e^2} + \frac{2\omega_{pe}^2}{k_r^2\nu_{iz}^2(1 + \alpha)Q_{31}}$$

$$a = \frac{k_r(1 + \alpha)}{|k_r|} - \alpha Q_{31} + \alpha Q_{32}$$

$$b = \alpha Q_{30}$$

$$\omega(k_r) = \omega(k_r) + i\gamma(k_r)$$

where $\omega(k_r)$ is the real frequency of the perturbation as a function of wave number $k_r$, and $\gamma(k_r)$ is the corresponding growth rate.

It is easy to see that in the limits of $A' \to \infty$ or $\alpha \to 0$ or $w \to 0$, $D(k_r, \omega)$ becomes

$$D(k_r, \omega) = 1 + \frac{2\omega_{pe}^2}{k_r^2\nu_{iz}^2} \left[ \frac{i\omega^2(\omega + i\gamma)}{k_r\nu_e} + \frac{T_{ex}k_r\nu_e}{T_{ex}(\omega - k_r\nu_e + i\gamma)} \right]$$

which is the expression obtained for a Maxwellian distribution.

Now we have deduced the local dispersion equation from which the lower hybrid-drift instability can be studied by determining $\omega$ and $\gamma$.

Usually $(Q_{33} - Q_{22})$ is of the order of 1, (for example, for $\alpha = 4$, $w = 0.6$, $|\alpha(Q_{33} - Q_{22})| = 5.6$ or $0.57$ for $A' = 0.55$ or 1.0, respectively). Consequently, from (31), we have, in limit $|\xi| \ll 1$
\[ \omega_i = \frac{2 \omega_{i0}^2}{k_y^2 v_{iL}^2} \left( \frac{T_{iL}}{T_{i0}} \right) \frac{k_y \rho_i}{G} \]  
where  
\[ G = \frac{\omega_{i0}^2}{k_y^2 v_{iL}^2} \]  
Equation (41) has the same form as for a Maxwellian distribution.

Assuming \( \gamma/\omega_i \ll 1 \) and dividing (36) into real and imaginary parts, as

\[ D_r = G + \frac{2 \omega_{i0}^2}{k_y^2 v_{iL}^2} \left( 1 + \alpha \right) u \frac{T_{iL}}{T_{i0}} \frac{k_y \rho_i}{\left( \omega_i - k_y \rho_i \right)} \]
\[ D_i = \frac{2 \omega_{i0}^2 \gamma^{1/2}}{k_y^2 v_{iL}^2} \left( 1 + \alpha \right) u \left( \frac{\omega_i}{k_y \rho_i} + b \right) \]
we find [Krall and Trivelpiece, 1973].

\[ \gamma = \pi^{1/2} \frac{T_{iL}}{T_{i0}} \frac{\left( \omega_i - k_y \rho_i \right)^2}{\left( 1 + \alpha \right) u k_y \rho_i} \left( \frac{\omega_i}{k_y \rho_i} + b \right) \]  

Although it has been used that \( |\omega/k_y v_{iL}| \ll 1 \), numerical computations show that these expressions are in fact accurate up to \( |\omega/k_y v_{iL}| \sim 1 \). After some straightforward algebra, it can be seen that the maximum growth rate \( \gamma \) occurs at wave number

\[ \frac{\omega_i^2}{\Omega_{iH}^2} = \frac{10}{3} \frac{T_{iL}}{T_{i0}} \frac{k_y^2 v_{iL}^2}{v_{iL}} \]  

The real frequency of maximum growth is

\[ \tilde{\omega} = \left[ \frac{10}{3} \right]^{1/2} \left( \frac{T_{iL}}{T_{i0}} \right)^{1/2} \frac{T_{eL}}{T_{e0}} \frac{v_{iL}}{v_{eL}} \left( \frac{\xi}{5} - \frac{3}{5} b a \right) \frac{\Omega_{iH}}{c} \]  

where

\[ \Omega_{iH} = \frac{eB}{(m_m v_{iL})^{1/2}} \]  

is the lower hybrid frequency; \( v_{eL} \) is the effective drift velocity

\[ v_{eL} = v_e + \frac{b}{a} v_{iL} \]

and

\[ \frac{\partial F_r}{\partial v_r} \bigg|_{v_r = \omega_r} = 0 \]  

Obviously, if \( \alpha \to 0 \) or \( \omega_i \to 0 \), or \( \alpha' \to \infty \), then \( b \to 0, a \to 1 \). At that time \( \gamma, \tilde{\gamma}, \tilde{\omega} \) and \( \tilde{\omega} \) all tend to values corresponding to those in a Maxwellian distribution.

Figure 4 shows the computations of \( \gamma \) and \( \tilde{\omega} \) in the neutral sheet. Note that \( \gamma > 0.1 \Omega_{iH} \) through the entire region \( 0.1 \ll \alpha' \ll 2 \) which was previously identified as the region of applicability of our approximations. When \( \alpha' \) goes to smaller or larger values, \( \gamma \) decreases, indicating that the instability tends to vanish. Figure 5 shows the ratio of \( \gamma/\gamma_m \) and \( \tilde{\gamma}/\tilde{\omega}_m \) where \( \gamma_m \) and \( \tilde{\omega}_m \) are the maximum growth rate and the corresponding real frequency in the 'Maxwellian sheet' where \( \alpha = 0 \) or \( \omega_i = 0 \). It is clear that in the unstable region \( \tilde{\omega}_m \sim 1 \) while \( \gamma_m \) is larger than 1, implying that the non-Maxwellian distribution may amplify the growth rate of the lower-hybrid-drift instability. For the non-Maxwellian distribution, the frequency spectrum is not much changed, but the growth rate is 3 times larger than that in the Maxwellian neutral sheet.

Why can the growth rate be increased by a non-Maxwellian distribution? To understand the reason, it should be noticed that if, instead of the frame of reference used above, we take one in which \( v_e = 0 \), then the average velocity of the ions be-
comes $-v_d + \delta v_d$, which is just the ion diamagnetic velocity $v_d$ (see section 3). As pointed out by Davidson [1978] and Huba et al. [1978], the lower-hybrid-drift instability is excited by a resonance between the ion motion and a drift wave, and $\gamma \approx -(\partial F_d/\partial v_d)_\omega_\omega$. From (36) the resonantly unstable region of ion velocity space covers the velocity range $0 < v_y < v_y$. In this region, we can see from Figure 3 that the slope of the non-Maxwellian distribution is larger than that of a Maxwellian distribution, so the growth rate in a non-Maxwellian sheet can be amplified. In that case, there is another part of ion motion associated with the non-Maxwellian distribution; therefore, the resonance is enhanced and so is the growth rate.

5. SATURATION LEVEL AND THE ANOMALOUS RESISTIVITY

The anomalous transport equations in the quasilinear approximation are [Davidson and Glad, 1975]

$$\frac{\partial}{\partial t} (n m \rho_d) = \int 2k \rho_d I_m (X_i) dk_y$$ (50)

$$\frac{\partial}{\partial t} (n m \rho_e) = -\int 2k \rho_e I_m (X_i) dk_y$$ (51)

$$Q_{an}^t = -2 \int e \gamma I_m \left[ (\omega_e - k_\omega e + i \gamma) X_i \right] dk_y$$ (52)

$$Q_{an}^t = -2 \int e \gamma I_m \left[ (\omega_e - k_\omega e + i \gamma) X_i \right] dk_y$$ (53)

and

$$Q_{an}^t + Q_{an}^t + \int \frac{\partial}{\partial t} (n m \rho_d^2/2) + \frac{\partial}{\partial t} (n m \rho_e^2/2) = 0$$ (54)

where it is assumed that $\langle v_d \rangle = \bar{v}_d$, $\langle v_e \rangle = \bar{v}_e$, where the angle brackets denote ensemble average and $e_e$ is the energy density of the electric field fluctuations at $k_y$.

$$e_e = |\delta E_k|^2/8\pi$$ (55)

Equations (50) and (51) describe momentum exchange of ions and electrons with the electrostatic perturbation field; (52) and (53) describe heating of ions and electrons; (54) implies local conservation of energy. Obviously, from (50) and (51) the total momentum in the system is conservative.

In studying any instability problem, an important aspect is to obtain the saturation level. An upper bound to the saturation level can likewise be determined by estimating the total amount of free energy available in the system [Fowler, 1968]. For a non-Maxwellian, unstable state, the fluctuation energy cannot exceed that which would be given up in a transition to a neighboring Maxwellian state. Thus an upper bound to the energy gain, $\bar{e}_e$, can be calculated. It is found that in most regions of the neutral sheet model described in section 3, $\bar{e}_e$ is about $n T_e/10$ which is 3 orders of magnitude larger than that in the Maxwellian sheet. This is, however, an implausible estimate. Although more free energy is contained in the non-Maxwellian distribution (1), only a part of the excess can be tapped by LHD instability. Actually, the fluctuations of LHD waves are driven by the relative drift between electrons and ions. This, in turn, constitutes only a small fraction of the total energy associated with the distribution (1). Thus, it seems reasonable to expect that once the energy is exhausted, the mode will saturate ($\gamma \rightarrow 0$). This will occur at a level much smaller than predicted by Fowler's thermodynamic method, so another instability will be necessary to relax the ions completely to a Maxwellian state.

Several mechanisms which may be responsible for the quasi-linear or nonlinear development of the LHD instability have been suggested such as ion trapping, plateau formation, current relaxation, and electron resonance broadening [Winske and Liewey, 1978; Davidson, 1978, Huba and Papadopoulos, 1978, Gary, 1980]. Computer simulations indicate that ion trapping is the dominant nonlinear effect in the large drift velocity regime [Winske and Liewey, 1978]. In the low drift velocity regime, it has been proven that electron resonance broadening is an effective stabilization mechanism when $T_e \approx 0.5 T_i$, and $v_e(t = 0) \approx 1.5 v_i$ [Huba and Papadopoulos, 1978]. Quasi-linear analysis shows that saturation can occur by either plateau formation or current relaxation in the case of $T_e \ll T_i$ and $v_e(t = 0) < v_e/4$. It has been found [Huba et al., 1978] that if the drift velocity is low enough, say $v_e(t = 0) < 0.18 v_i (1 + T_e/T_i)$

$$\frac{v_e(t = 0)}{v_e} < \frac{0.18}{(1 + T_e/T_i)}$$

plateau formation will be completed before $v_e$ relaxes to zero. On the other hand, if $v_e(t = 0)$ is a bit larger, say $v_e(t = 0) > 0.18 v_i (1 + T_e/T_i)$

$$\frac{v_e(t = 0)}{v_e} > \frac{0.18}{(1 + T_e/T_i)}$$

it is energetically favorable for stabilization to occur through current relaxation with $v_e \rightarrow 0$. In calculating the growth rate in section 4, we found that $\gamma$ decreases drastically with decreasing $v_e(t = 0)$ if $v_e(t = 0) < v_e/4$. So, under the conditions of the earth's magnetotail with $T_e \ll T_i$ and $v_e(t = 0) < v_e/4$, current relaxation is probably the most effective stabilizing process for unstable LHD waves.

In the case of the non-Maxwellian distribution of ions in this paper, the current relaxation comes from the decrease in the relative motion between electrons and ions. We notice from (40) that $\gamma \rightarrow 0$ when $v_e \rightarrow 0$. The saturation level can thus be estimated by assuming a narrow band in $k_y$ space over which the growth rate maximizes. Approximating $Q_{an}^t$ and $Q_{an}^t$ as

$$Q_{an}^t = \int \frac{(\omega_e - k_\omega e + i \gamma) X_i}{\gamma} \frac{\partial e}{\partial t} \int 2 e_{ei} \beta d k_y$$

$$Q_{an}^t = \int \frac{(\omega_e - k_\omega e + i \gamma) X_i}{\gamma} \frac{\partial e}{\partial t} \int 2 e_{ei} \beta d k_y$$

where

$$\frac{\partial e}{\partial t} = \int 2 e_{ei} \beta d k_y$$ (55)

and using (44), (45), (46), and (54) we find
Fig. 6. Variations of $\varepsilon_F/(\varepsilon_F)_m$ with $A'$ calculated by using two different sets of parameters. $\varepsilon_F$ and $(\varepsilon_F)_m$ the saturation levels of electric energy in the non-Maxwellian and the Maxwellian neutral sheet, respectively.

$$f_{\text{sat}}^e = -\frac{\partial [nm_e (\varepsilon_{pe} - \varepsilon_{pe}^0)]/\partial t}{nm_e (\varepsilon_{pe} - \varepsilon_{pe}^0)}$$

and

$$\eta_m = \frac{m_{\text{eff}}^e}{ne^2}$$

respectively. From (49), (50), (55), and (56), it can be obtained that

$$\eta_m = \frac{\int 2k_e \varepsilon_m \varepsilon_{im} \, dk_y}{\varepsilon_{im} \varepsilon_{im}}$$

Within the context of maximum growth estimates of (68), we calculate the anomalous resistivity in the non-Maxwellian sheet based upon the calculated values of $\varepsilon_F$ in Figure 6. The variations of $\eta/\eta_m$ versus $A'$ for $\zeta = 1$ ($v_{pe}/v_{pe} = 1/2$) and $\zeta = 1/2$ ($v_{pe}/v_{pe} = 1/4$) are plotted in Figure 7, where $\eta_m$ is the anomalous resistivity in the Maxwellian sheet. It is clear that $\eta$ is 3–4 times larger than $\eta_m$. Notice that $(\eta/\eta_m)$ is very close to $\xi/\xi_m$.

6. DISCUSSION

To summarize the results of this paper, we review our conclusions:

1. We have demonstrated that in the thin neutral sheet with a non-Maxwellian distribution of ions the lower-hybrid-drift instability can be excited. The unstable region exists throughout the whole sheet except very close to the null line.

2. In the most unstable region, the frequency spectrum of the 'non-Maxwellian lower-hybrid-drift' waves is nearly the
The non-Maxwellian distribution (1) may amplify the growth rate of the instability because the resonance between the ion's motion and the drift wave is enhanced.

4. In the neutral sheet with a non-Maxwellian distribution (1) the fluctuation level at saturation, $\epsilon_f$, may be 2–3 times larger than that in the Maxwellian neutral sheet. The available free energy comes from the enhanced cross-field current and the enhanced inhomogeneities in both plasma and magnetic field.

5. The anomalous resistivity associated with the lower-hybrid-drift instability in the non-Maxwellian neutral sheet with distribution (1) can be 3–4 times larger than that in the Maxwellian sheet.

To understand the role of the lower-hybrid-drift instability in the reconnection process in the non-Maxwellian neutral sheet, it will be necessary in the future to consider the instability as electromagnetic, to consider the nonlinear development, and to investigate what happens in the null region.

**APPENDIX**

For the electrostatic approximation, we have

$$X_4 = \frac{4\pi e^2}{mk^2} \int k \delta F_0 / \delta v_x \, d^3 v$$

$$= \omega^2 / k^2 \int d^3 v \omega / k \mu^2 + \omega^2 / k^2 \int d^3 v \omega / k \mu^2$$

$$= \frac{2\omega^2}{k^2 \mu^2} \left[ 1 + \xi Z_2 + Z_3 \right]$$

(A1)

where

$$Z_2 = \frac{1}{\omega} \int_0^\infty \left[ 1 - R \text{erf} \left[ S \left( \lambda + A' \right) \right] \lambda - \xi \right] \, d\lambda$$

$$= \left( Z - RZ_2 \right)$$

(A2)

$$Z = \frac{1}{\omega} \int_0^\infty \exp \left( -\lambda^2 \right) \lambda - \xi \, d\lambda$$

(A3)

is the plasma function and

$$Z_1 = \int_0^\infty \text{erf} \left[ S \left( \lambda + A' \right) \right] \exp \left( -\lambda^2 \right) \lambda - \xi \, d\lambda$$

(A4)

$$Z_3 = \frac{RS}{2\pi} \int_0^\infty \exp \left[ -S^2 \left( \lambda + A' \right)^2 - \lambda^2 \right] \lambda - \xi \, d\lambda$$

(A5)

$$R = \frac{\alpha}{1 + \alpha}$$

$$S = \frac{\omega}{1 + \omega}$$

where the integrals are to be carried out along the Landau contour [Krall and Trivelpiece, 1973].

In limit of $|\xi| \ll 1$, we can expand

$$Z_4(\xi) = \sum_{n=0} Z_n(0) \xi^n$$

where

$$Z_4(0) = \frac{1}{\sqrt{\pi}} \int_0^\infty \text{erf} \left[ S \left( \lambda + A' \right) \right] \exp \left( -\lambda^2 \right) \lambda \, d\lambda$$

$$= -i \sqrt{\pi} Q_{21} + Q_{22}$$

(A8)

$$Q_{21} = \text{erf} \left( SA' \right)$$

(A9)

$$Q_{22} = P \frac{1}{\sqrt{\pi}} \int_0^\infty \text{erf} \left[ S \left( \lambda + A' \right) \right] \exp \left( -\lambda^2 \right) \lambda \, d\lambda$$

(A10)

$$Z_n(0) = \lim_{\xi \to \frac{\pi}{\sqrt{2}}} \int_0^\infty \frac{\text{erf} \left[ S \left( \lambda + A' \right) \right] \exp \left( -\lambda^2 \right) \lambda}{\lambda - \xi} \, d\lambda$$

$$= -\lim_{\xi \to \frac{\pi}{\sqrt{2}}} \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{\text{erf} \left[ S \left( \lambda + A' \right) \right] \exp \left( -\lambda^2 \right) \lambda}{\lambda - \xi} \, d\lambda$$

(A11)

$$R_{21} = -S \exp \left( -S^2 A'^2 \right)$$

(A12)

$$R_{22} = \frac{2}{\sqrt{\pi}} \int_0^\infty \text{erf} \left[ S \left( \lambda + A' \right) \right] \exp \left( -\lambda^2 \right) \lambda \, d\lambda$$

$$= -SP \frac{1}{\sqrt{\pi}} \int_0^\infty \exp \left[ -S^2 \left( \lambda + A' \right)^2 - \lambda^2 \right] \lambda \, d\lambda$$

(A13)

$$Z_n(0) = \frac{RS}{2\pi} \int_0^\infty \exp \left[ -S^2 \left( \lambda + A' \right)^2 - \lambda^2 \right] \lambda \, d\lambda$$

$$= R \left( m^{1/2} Q_{10} + Q_{11} \right)$$

(A14)

$$Q_{30} = \frac{S}{2} \exp \left( -S^2 A'^2 \right)$$

(A15)

$$Q_{31} = \frac{S}{2} \frac{1}{\sqrt{\pi}} \int_0^\infty \exp \left[ -S^2 \left( \lambda + A' \right)^2 - \lambda^2 \right] \lambda \, d\lambda$$

(A16)

$$Z_n(0) = \frac{RS}{2\pi} \frac{1}{\sqrt{\pi}} \lim_{\xi \to 0} \int_0^\infty \exp \left[ -S^2 \left( \lambda + A' \right)^2 - \lambda^2 \right] \lambda \, d\lambda$$

$$= -\frac{RS}{2\pi} \frac{1}{\sqrt{\pi}} \lim_{\xi \to 0} \int_0^\infty \exp \left[ -S^2 \left( \lambda + A' \right)^2 - \lambda^2 \right] \lambda \, d\lambda$$

$$= R \left( m^{1/2} Q_{32} + Q_{33} \right)$$

(A17)

$$Q_{32} = -2S^2 Q_{30} A'$$

(A18)
\[ Q_{33} = -S^2 \exp\left(-\frac{\omega^2 A'}{2}\right) \left[ 1 + \frac{wSA'}{\sqrt{2}} \right] \int_{-\infty}^{\infty} \exp\left[-(t + wSA')^2/2t\right] dt \] (A19)

It is well known that in the limit \(|\xi| \ll 1\)

\[ Z(\xi) = \frac{i\omega^{1/2} k_v}{|k_v|} \left[ 1 + \frac{\omega^{1/2} A(1 + \alpha) - \alpha Q_{31} + \alpha Q_{33}}{1 + \alpha Q_{30} + \alpha (Q_{33} - Q_{32})} \right] \] (A20)

thus, from (A6), (A7), and (A1), we have

\[ X' = \frac{2\omega_{pe}^2}{k^2 v_s^2} \frac{1}{1 + \alpha u} \left\{ 1 + i\omega^{1/2} A(1 + \alpha) - \alpha Q_{31} + \alpha Q_{33} \right\} \]

\[ + i\omega^{1/2} A\phi + \alpha (Q_{33} - Q_{32}) \] (A21)

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