Abstract. An empirical analysis of solar wind-magnetosphere energy coupling functions is reported. Using the technique of linear prediction filtering with 2.5 minute data, we examine the relationship of auroral zone geomagnetic activity to solar wind power input functions which depend on the solar wind quantities \( V_B \), \( V_B^2 \), or \( V_B^2 B \). In this analysis a least squares prediction filter or impulse response function which relates a solar wind power function to an auroral zone geomagnetic index is designed directly from the data. We find that the computed impulse response functions have the characteristics of a low pass filter with a time delay which may be dependent on the strength of the energy input. While the AL index is reasonably well related to the solar wind energy functions, the AU index shows a substantially poorer relationship. In addition, high frequency variations of the auroral indices and some substorm expansions are not predictable with solar wind information alone, suggesting that internal magnetospheric processes partially control the AL index. We also find that the \( \epsilon \) parameter which depends on \( V_B^2 \) in the solar wind has a poorer relationship to auroral zone geomagnetic activity than a power parameter having a \( V_B \) solar wind dependence.

Introduction

The relationships between indices of geomagnetic activity and a variety of interplanetary quantities have been studied extensively using empirical techniques. These studies have established the importance of such interplanetary parameters as the north-south magnetic field component (\( B_n \)), the solar wind speed (\( V \)), the interplanetary magnetic field (IMF), magnitude (\( B \)), and various combinations of these parameters (see reviews by Burch, 1974; Akasofu, 1977; and Russell, 1979). In general, the best correlations have been obtained using the quantities \( V_B \) and \( V_B^2 \) where,

\[
B_0 = 0, B_n > 0 \\
B_0 = -B_n, B_n < 0
\]

(Aubry and McPherron, 1971; Rostoker et al., 1972; Murayama and Hakameda, 1975; Crooker et al., 1977; Maezawa, 1979). Because of this, Burton et al. (1975) proposed that the magnetosphere acts as a half wave rectifier of the interplanetary electric field. This notion was supported by deriving a functional relationship between \( V_B \) and the ring current index \( D_n \) and showing that this relation was a good predictor of \( D_n \).

Recently, strong criticism of the half wave rectifier model has appeared in the literature (Perreault and Akasofu, 1978; Akasofu, 1979). These authors claim that \( E \) can be identified semi-quantitatively as the dynamo power delivered from the solar wind to the open magnetosphere. To make this argument, however, several assumptions are required. These assumptions include a particular theory of merging and merging geometry, assumptions pertaining to the distribution of plasma and fields within the magnetosheath, and assumptions about the dissipation of energy within the magnetosphere. Different assumptions have led others to far different conclusions (Siscoe and Crooker, 1974; Atkinson, 1978).

Following the theoretical development of Siscoe and Crooker (1974) we have related the quantities \( V_B \) and \( V_B^2 \) to power input to the magnetosphere. The approach assumes that energy is transferred from the solar wind by means of a tangential stress on the geomagnetic tail. This approach is equivalent, however, to integrating the Poynting flux into the tail. As merged field lines are carried to the geomagnetic tail they are stretched, exerting a tangential force which acts against the solar wind flow. Energy from the flow is thus transferred to the stretched field. Details of the derivation of the power input to the magnetosphere can be found in Siscoe and Crooker (1974). We simply state their result here in CGS units:

\[
P = \frac{1}{4\pi} \frac{L_T}{L_m} B_T [B_T^2 V]
\]

where

\[
P = \text{power into the magnetosphere} \\
L_T = \text{length of the geomagnetic tail} \\
L_m = \text{length of the merging line} \\
B_T = \text{tangential field at the tail boundary} \\
|B_T^2| = \text{merging component of the magnetic field} \\
V = \text{upstream solar wind speed}
\]

If we assume that \( |B_T| = B_n \), this result can be rewritten as

\[
P_{V_B} = (\frac{\epsilon}{2\pi}) [B_n V_B^2]
\]

where \( \epsilon = \frac{L_T}{L_m} \). To obtain a \( V_B \) dependence we multiply (3) by the dimensionless quantity \( V/V^* \) here \( V^* \) is a characteristic solar wind speed.

\[
P_{V_B} = \frac{\epsilon B_n}{2\pi} V_B^2
\]

We take the quantities in parentheses in equations (3) and (4) to be constant. In this report the input power parameters given by equations (3) and (4) are examined critically and compared to the \( \epsilon \) parameter defined in equation (1)

\[
P = V_B^2 \sin^2(\theta/2)
\]
Two time intervals are examined, February 3-12, 1967 and February 17-28, 1967. The first interval is characterized by a period of very strong geomagnetic activity, while the second interval is characterized by more moderate activity. During both time intervals information about the solar wind was obtained from the Explorer 33 spacecraft located directly upstream of the magnetosphere between 20 and 60 Rg.

We examine three suggested power input parameters and their relation to auroral zone magnetic activity measured by AU, AL, and AE. The technique which we are using computes the most general linear relationship between the solar wind input time series and the geomagnetic activity output time series. Since only correlated portions of the input and output time series affect the determination of the impulse response function, the problems created by noisy data are reduced substantially. Finally, by calculating a prediction efficiency parameter, we are able to perform a critical comparison of the three power functions as predictors of energy release measured by auroral zone geomagnetic indices.

Analysis and Results

The computer algorithms used in this work are adapted from Robinson (1967). For the initial application of this analysis to the time periods used in this study, we created hourly averages of the data and duplicated the results of Iyemori and Maeda (1980). Our results are essentially identical to their figures 1 and 2 even though we obtained them from substantially shorter time series.

In figure 2 we show the prediction filters which relate the three power input parameters defined above to the AL index. The filters obtained from the moderately disturbed period are shown in the top panels of the figure, and those from the more strongly disturbed period are shown in the bottom panels. The horizontal axis indicates time lag. The filter values are actually the quantity \( h(r) \), from equation (6) shown in units of \( \sqrt{150 \text{ erg}} \). These units result from using 150 sec digital data and by dividing the solar energy input by \( 10^7 \text{ erg/sec} \). A smooth curve obtained by low pass filtering is drawn through each of the impulse response functions. This curve is shown as an aid for the reader. The dominant feature of all of the filters is the delay of the solar wind. The filters have the characteristic shape of a low pass filter with a time delay. The extremely spiky, high-frequency component of the filter functions is primarily due to the uncertainties which result from correlations based on the small number of active periods during the short (10 or 12 day) time intervals used in this analysis. A less noisy filter will result from analysis on longer time intervals which have many repeated correlated variations in the input and output time series.

![Figure 1](image1.png)

**Figure 1.** A comparison of three proposed predictors of solar wind power input to the magnetosphere during moderate and severe magnetic disturbance. The power parameters are calculated from 2.5 minute averages of solar wind data.

For purposes of comparison, we have chosen \( \epsilon \) to provide values of \( P_{\text{VH}} \) and \( P_{\text{VMB}} \), comparable to \( P \), under typical conditions which are taken to be:

- \( <V> = 390 \text{ km/sec} \)
- \( <B> = -2.5 \gamma \)
- \( <E> = 6.4 \gamma \)
- \( P = 2 \times 10^{16} \text{ erg/sec} \)
- \( V = 390 \text{ km/sec} \)

These values give \( \epsilon = 10 \text{ Rg} \).

A comparison of the three measures of solar wind power input functions \( P_{\text{VH}} \), \( P_{\text{VMB}} \), and \( P_{\text{VMB}} \) defined above is shown in Figure 1 for two five day intervals in 1967. The top panel shows the three power functions during a period of moderate geomagnetic activity and the bottom panel shows the functions during a period including very strong geomagnetic activity. The data are plotted at 2.5 minute time resolution. It is clear that the three functions are very similar. Indeed, the time variations are nearly identical, while the magnitudes of the functions display occasional differences.

A powerful empirical technique suitable to the study of input-output relationships such as energy coupling was recently utilized by Iyemori et al. (1979) and Iyemori and Maeda (1980). This technique is linear prediction filtering. In the application of this technique, the relation between the solar wind and geomagnetic activity is described in terms of an input (the solar wind), a filter (the magnetosphere), and an output (geomagnetic activity index). This is described mathematically by the model equation

\[
O(t) = \int h(r)I(t - r)dr
\]

where \( I(t) \) is the input data, \( h(r) \) is the prediction filter or impulse response function, \( O(t) \) is the output data and \( t \) denotes time. In this application \( h(r) \) characterizes properties of the magnetosphere. The function \( h(r) \) can be calculated from \( O(t) \) and \( I(t) \) using a least squares technique developed by Weiner (1949) and applied to discrete time series by Levinson (1949).

While Iyemori and Maeda (1980) were able to achieve considerable success in predicting \( D_{\text{st}} \), AL and AU from \( V_{\text{B}} \), their study was limited by use of hourly average data. Our report presents preliminary results of a study which extends the work of Iyemori and Maeda (1980) using high time resolution data (2.5 minute).

Two time intervals are examined, February 3-12, 1967 and February 17-28, 1967. The first interval is characterized by a period of very strong geomagnetic activity, while the second interval is characterized by more moderate activity. During both time intervals information about the solar wind was obtained from the Explorer 33 spacecraft located directly upstream of the magnetosphere between 20 and 60 Rg.

We examine three suggested power input parameters and their relation to auroral zone magnetic activity measured by AU, AL, and AE. The technique which we are using computes the most general linear relationship between the solar wind input time series and the geomagnetic activity output time series. Since only correlated portions of the input and output time series affect the determination of the impulse response function, the problems created by noisy data are reduced substantially. Finally, by calculating a prediction efficiency parameter, we are able to perform a critical comparison of the three power functions as predictors of energy release measured by auroral zone geomagnetic indices.

**Figure 2.** Linear prediction filters relating various proposed solar wind power input parameters to AL index during two intervals of moderate and severe geomagnetic activity. Time lag is plotted along the horizontal axis and the filter value is plotted along the vertical axis. The inset shows the vertical scale in units of \( \sqrt{150 \text{ erg}} \). Note decrease in magnetospheric time delay for strong activity. Also note non-linear response of \( P \), filter to changes in level of activity.
It is interesting to consider the differences which exist between the moderate activity and strong activity impulse response functions. These differences may suggest important nonlinearities in the energy coupling processes or important deficiencies in the input power parameters. For example, we note that there is a shorter lag time for all three of the impulse response functions during the strong activity period. The negative peak of the impulse response functions occurs at about 60 minutes lag for the moderate activity period and at about 30 minutes lag for the strong activity period. This strongly suggests more efficient magnetospheric convection during periods of high energy input. On the other hand, during the period of strong geomagnetic activity, the impulse response function obtained using \( P \) shows a greatly reduced amplitude as compared to the impulse response function obtained using \( P_{V2} \), which shows almost no amplitude reduction. This suggests immediately that \( P \) is not linearly related to the AL index. Of course, many more intervals of comparable activity must be examined to confirm these observations and properly interpret their significance.

In figure 3 we show plots of AL in the heavy trace at the top of the figure and repeated in a thinner line on the lower axes. The dark curves over the lighter AL traces are the predictions of AL generated by convolving the respective unfiltered impulse response function shown in figure 2 with the appropriate input power parameter. Since the filter is created from correlated variations in the input and output time series, when the impulse response function is convolved with the input time series, the correlated output response is obtained and the uncorrelated portion is missing. Several points are evident from examination of this figure. First, the high-frequency fluctuations of the solar wind input parameters (see figure 1) are not present in the predictions, indicating that the magnetosphere acts as a low pass filter of the input data. Second, we note occasional systematic variations in the geomagnetic index which are not predicted by solar wind information alone. Indeed, several substorms are missed entirely in the predictions. This strongly suggests that there are internal processes within the magnetosphere which are uncorrelated with the solar wind but which release energy into the ionosphere. Finally, it appears that each of the parameters does about equally well at predicting AL at high time resolution during moderate geomagnetic activity.

To quantify the prediction success we have computed a prediction efficiency parameter defined as:

\[
\text{Eff} = 1 - \frac{\sigma^2_{\text{res}}}{\sigma^2_{\text{ori}}}
\]  

where \( \sigma^2_{\text{res}} \) is the variance of the residuals generated by subtracting the predicted index from the observed index, and \( \sigma^2_{\text{ori}} \) is the variance of the observed index. This prediction efficiency parameter was computed by taking running 6-hour averages. In figure 4 we have tallied the occurrence of the various values of Eff and plotted histograms to determine the efficiency of \( P \) and \( P_{V2} \) as predictors of both the AL and AU indices. The plots are separated according to geomagnetic index and activity level. The efficiency of prediction for AL is shown in the top panels and the efficiency of prediction for AU is shown in the bottom panels. Solid lines indicate the efficiency for \( P \) and the dotted lines indicate the efficiency for \( P_{V2} \). The vertical axes give the number of occurrences of a given efficiency, while the horizontal axes show the prediction efficiency over the range -1 to +1. A value of +1 indicates a perfect prediction, 0 implies that the prediction is no better than fitting the data with its average value, and negative values imply that the wrong fluctuations were predicted. For the period of moderate activity it is clear that \( P_{V2} \) was a superior predictor of AL and during the period of strong activity \( P_{V2} \) was comparable or slightly superior to \( P \). Similar conclusions about the relative superiority of \( V2 \) to \( e \) as a predictor of auroral activity have been obtained by Baker et al. (1981) using cross correlation analysis with high time resolution data. By examining the bottom panels of the figure, it is clear that neither \( P_{V2} \) or \( P \) are satisfactory predictors of AU. These poor results may in part be due to the fact that during strong activity the auroral oval expands and the eastward electrojet moves to latitudes below the AE station chain. But even during the period of moderate activity the prediction efficiency is, in general, substantially poorer for AU than for AL. The results obtained using \( P_{V2} \) were roughly intermediate between those obtained using \( P_{V2} \) and \( P \), and we will not discuss them in detail.
Summary and Conclusions

In summary we find that there is a close similarity between the three power input functions examined. This similarity makes the study of the relative efficacy of the functions difficult. However, we find that linear prediction filtering provides a tool whereby new insight into the relationship between the three proposed energy coupling functions and geomagnetic activity can be obtained.

Using 2.5 minute data, we showed that the three input power functions, $P_{\text{ion}}$, $P_{\text{pel}}$, and $P_{\text{sw}}$ produce impulse response functions which have the characteristics of a low pass filter with a time delay. The lag time of the peak of the impulse response function is about 60 minutes during the moderate activity period and about 30 minutes during the period of strong activity. We also find occasional substorm expansions observed in the AL index which are not correlated with the solar wind input function. This suggests that the processes within the magnetosphere can control the release of energy into the ionosphere. In addition, we find that the AU index is not predicted as well as AL. This fact is perplexing but it suggests to us that the use of the AE index (which is composed of both AL and AU) for studies such as this may not be appropriate. Finally, our results indicate that the $e_\text{E}$ parameter shows a poorer relationship with auroral geomagnetic activity than a power parameter having a VB, solar wind dependence.

Acknowledgments. This work was supported at UCLA by the Office of Naval Research under Contract ONR N00014-76-C-0207, by the National Aeronautics and Space Administration under Grant NGL 05-007-004.

This work was supported in part at Stanford by the Office of Naval Research under Contract N00014-76-C-0207, by the National Aeronautics and Space Administration under Grant NGR 05-020-559, and Contract NAS5-24420, by the Division of Atmospheric Sciences, Solar-Terrestrial Research Program of the National Aeronautics and Space Administration, and by the Max C. Fleischmann Foundation.

Digital ground magnetic data used in this work were provided by the World Data Center A for Solar-Terrestrial Physics in Boulder, CO. Spacecraft data were obtained from the National Space Science Data Center Greenbelt, MD.

References


(Received May 1, 1981; accepted May 28, 1981.)