Time Dependent Convection Electric Fields and Plasma Injection

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Large-scale electric fields associated with storms or substorms are responsible for inward convection and energization of plasma sheet plasma. Calculations based on steady state convection theory show that the response to such electric fields qualitatively accounts for many features of the injected particle distribution, but quantitative agreement with the theory has not yet been obtained. It is known that the predictions can be improved by introducing the concept of convection in response to a time dependent electric field. On the other hand, time dependent calculations are sensitive to the choice of initial conditions, and most models have failed to incorporate these conditions in a realistic and self-consistent manner. In this paper we present a more complete model consisting of realistic initial conditions and time dependent convection to explain a typical substorm-associated electron injection event. We find very good agreement between the observed electron flux changes and those predicted by our model.

INTRODUCTION

Transport of collisionless plasma sheet plasma to the inner magnetosphere results from large-scale electric fields associated with storms or substorms. As was shown by previous workers [Chen, 1970; Kivelson and Southwood, 1975; Stern, 1975; Cowley and Ashour-Abdalla, 1976a, b], the maximum earthward penetration of the adiabatically convecting tail plasma is delimited in the steady state by spatial boundaries determined by the magnitude of the convection electric field and the constants of the particle motion. For magnetospheric plasmas the relevant constants are the two adiabatic invariants μ and J , where μ is the magnetic moment and J the bounce integral. The convection boundaries, often called 'Alfvén layers' [Wolf, 1970], separate spatial regions in which qualitatively different particle drift orbits are present. For electrons of prescribed μ and J , an Alfvén layer separates drift orbits which are closed about the earth from those which are open to the magnetopause and tail and thus separates regions of enhanced plasma sheet electron fluxes (antiarthward of the Alfvén layer) from regions of reduced plasma sheet electron fluxes (earthward of the Alfvén layer). For protons the nature of drift paths in different spatial regions separated by Alfvén layers can be more complex [Chen, 1970].

Because satellite particle detectors measure particle energy and pitch angle, not μ and J , it is convenient to identify a type of boundary similar to an Alfvén layer but defined in terms of energy and pitch angle instead of μ and J [Kivelson and Southwood, 1975]. We introduce the nomenclature 'steady state demarcation boundary' (SSDB) for this related boundary which is determined by the convection electric field magnitude and specified for a particle energy W and pitch angle α . The SSDB is the same as the boundary called $L_A(\phi)$ by Kivelson and Southwood [1975] and called the critical boundary by Kivelson et al. [1979]. Now, the SSDB is not a particle drift orbit, since energy is not a conserved quantity, but it does delimit the spatial region inside of which an electron whose local energy and pitch angle are the prescribed W and α is on a closed orbit. Outside the SSDB an identical electron will be on an open orbit and thus may have a source in the plasma sheet. Later in this paper we extend the definition of the demarcation boundary to include time dependent effects.

Steady state convection theory has been used with differing degrees of success by many authors to explain substorm-asso-

ciated particle phenomena. One data set which has been analyzed by several authors is that of Williams et al. [1974], who observed electron flux enhancements near $L = 5$ in the dusk to midnight sector on Explorer 45 shortly after substorm onset. They tried to explain their observations by invoking plasma convection in the electric and magnetic field model developed by McIlwain [1972] to describe particle injection at geostationary orbit. Williams et al. found that the magnitude of McIlwain's steady state electric field was insufficient to inject electrons to the low L values traversed by Explorer 45. The Williams et al. paper stressed that the dispersion of arrival times differed for high- and low-energy electrons. Following substorm onset the delay of the initial flux increase at the satellite increased with energy for low-energy electrons (<8 keV) and decreased with energy for high-energy electrons. To account for this behavior, the authors postulated an ad hoc radial electric field in addition to the McIlwain field in the evening sector.

Subsequently, Kivelson and Southwood [1975] assumed a dipole magnetic field, corotation electric field, and uniform cross-magnetospheric electric field and invoked steady state adiabatic convection theory to explain the energy and time dependent electron flux increases in the same data. Assuming that the flux increase in a given low-energy channel occurred when the outbound satellite passed through the SSDB and entered a region accessible to plasma sheet electrons of that energy, they calculated the points of intersection of the satellite trajectory with the SSDBs for a uniform convection electric field. Although they were successful in explaining the energy dependence of arrival times, they obtained only qualitative agreement with the timing of the observed flux increases. They made no attempt to interpret the subsequent flux decreases.

Using the same convection model as Kivelson and Southwood [1975], Cowley and Ashour-Abdalla [1976a] interpreted the Williams et al. data in a slightly different way. For each energy channel of the detector they calculated the electric field necessary to place the corresponding SSDB at the satellite position of the observed flux increase in that energy channel. They found that different convection electric fields were required to account for increases in different energy channels. They concluded that the convection electric field had either a strong spatial or a strong time dependence.

Further applications of convection theory were presented in a companion paper by Cowley and Ashour-Abdalla [1976b], who turned their attention to the storm time proton nose event

observed near dusk by *Smith and Hoffman* [1974] on Explorer 45. They argued that the observed distribution was a natural steady state spatial feature produced by adiabatic convection, but their model predicted a nose penetration much further earthward than was observed. *Cowley* [1976] later brought the predicted and observed nose positions into agreement by invoking proton loss at the strong diffusion rate over the entire trajectory.

The above interpretations of particle injection all relied on use of steady state convection theory. Implicit in the steady state model is the assumption that the plasma particles will have sufficient time to reach their steady state configuration, an assumption which cannot be valid considering the time scale of a substorm. At substorm onset there is a major change in the magnitude and configuration of the earth's electromagnetic fields. The tail field strength typically increases by ~ 1 kV/ R_E , and the SSDB moves $\geq 1 R_E$ earthward [*Kivelson et al.*, 1979]. Particles near $L = 5$ ($B = 150 \gamma$) need ~ 100 min to adjust to the changed conditions. On the other hand, particle phenomena such as those observed by *Williams et al.* [1974] are sometimes seen ~ 10 min after substorm onset, and on such a short time scale the use of steady state theory cannot be justified.

A time dependent model of plasma sheet response to the convection electric field was presented by *M. Ejiri, R. A. Hoffman, and P. H. Smith* (unpublished manuscript, 1978), who attempted to explain the observed position ($L \sim 4.5$) and energy ($W \sim 15$ keV) of the protons injected inside the plasma-pause (proton nose event) during a magnetic storm. They assumed that all their particles started from a single boundary which was azimuthally symmetric in the range $\pm 60^\circ$ about midnight at $L = 10$. Substorm onset was simulated by a stepwise enhancement of the convection electric field. With their ad hoc particle initial condition they predicted that the nose should penetrate to $L = 4.5$ with a time delay from onset much greater than that normally observed.

MODEL OF SUBSTORM CONVECTION

In this paper we describe an adiabatic convection model in which we incorporate not only a time dependent convection electric field but also physically realistic energy dependent initial conditions consistent with quiet time observations. We assume that plasma sheet electrons are present outside of the initial electron boundaries, which are themselves determined from steady state convection theory. The initial boundaries have an energy dependence appropriate for a small convection field representative of a steady, quiet magnetosphere [*Kivelson et al.*, 1979]. We simulate substorm onset by an instantaneous enhancement of the cross-magnetospheric potential drop; evidence for enhanced convection and convection electric fields at substorm onset may be found in the work of *Pytte et al.* [1978] and *Mozer* [1971], respectively. We then calculate the position (radial distance versus azimuth as a function of time) of the inner edge of the region in which plasma sheet electrons of given W and α can be found for the assumed initial conditions and enhanced field. The calculation provides new demarcation boundaries which are not steady state but rather convecting. Therefore we supplement our definition of the SSDB with an additional definition of the 'time dependent demarcation boundary' (TDDB), a boundary which follows the inner edge of convecting plasma sheet electrons of a designated energy. A satellite will encounter freshly injected plasma sheet electrons of energy W when it moves out through the TDDB for that energy.

To illustrate the use of our model, we reinterpret the substorm electron observations of *Williams et al.* [1974], the same data set analyzed by *Kivelson and Southwood* [1975] and *Cowley and Ashour-Abdalla* [1976a]. For reasonable initial and final convection electric fields, whose amplitudes are the only free parameters, we can model the times and positions of the plasmopause and of flux enhancements and decreases in the four different energy channels of the Explorer 45 particle detector in terms of crossings of the TDDB. For a simple convection field model we obtain very good agreement with observations.

We find that in our time dependent picture of substorm-associated convection, the initial particle boundaries are as crucial to the calculation as the incorporation of the time dependence itself, since the position of a particle within the first tens of minutes after onset depends greatly on its location at onset. We suggest that any time dependent convection model must incorporate realistic initial conditions consistent with quiet time observations to give good quantitative results.

ANALYSIS OF DATA

The particle observations reexamined in this study are the low-energy (0 to 7.1 keV) electron flux increases and decreases seen aboard Explorer 45 shortly following a substorm onset on December 12, 1971 [*Williams et al.*, 1974]. As the Explorer 45 particle detectors measure particles with energies down to only 1.5 keV, the presence or absence of the thermal (~ 0 keV) component was inferred by the measurement from the on-board dc electric field probe [*Maynard and Cauffman*, 1973]. The substorm began at ~ 2200 UT, at which time the satellite was outbound at $L = 5.3$ and near 2040 LT. As time progressed, the satellite passed outward through the plasma-pause, as determined by saturation of the on-board dc electric field probe, and then 90° pitch angle electron flux increases were successively observed in the energy channels 1.5–2.1, 2.3–3.2, 3.5–4.8, and 5.3–7.1 keV. For this range of energies the observed energy dispersion was opposite to that expected if injection occurred near midnight, and gradient curvature drift then brought the electrons to the dusk-midnight sector. Later, on the inbound portion of the orbit, the low-energy electron fluxes were seen to decrease, first in the 5.3- to 7.1-keV channel and subsequently in lower energy channels. No change (increase or decrease) of electron flux in the 8.0- to 10.8-keV energy channel was seen at any point along the relevant portion of the orbit. In Table 1 we list the times of the electron flux increases and decreases along with the satellite positions at these times. The inverse energy dispersion of the flux increases and the opposite dispersion of the flux decreases are clearly seen.

As was done in the previous analyses of *Kivelson and Southwood* [1975] and *Cowley and Ashour-Abdalla* [1976a], we assume that the observed flux changes occurred when the satellite passed through demarcation boundaries, but we use the TDDBs described above.

We assume that the total magnetospheric electric field is derivable from a time dependent generalization of the potential in the equatorial plane given by *Volland* [1973]:

$$\Phi(\text{kV}) = C_1(t)L^2 \sin \phi + C_2/L \quad (1)$$

where $C_1(t)$ is proportional to the total potential drop across the magnetosphere in kilovolts and is time dependent, ϕ is azimuth measured eastwards from noon, and $C_2 = 91$ kV is the corotation potential. We adopt the L^2 dependence of the convection potential based on observational and theoretical evi-

dence that the convection electric field is partially shielded from the inner magnetosphere [Heppner, 1972; Volland, 1973; Jaggi and Wolf, 1973; Southwood, 1977]. Later we will show that the use of a shielded potential is not critical to our conclusions and that a uniform electric field in which the first term on the right-hand side of (1) is proportional to L can be used with equal success. For our calculation we assume that the earth's magnetic field is dipolar and that the field lines are equipotentials.

We use the potential of (1) to model both quiet presubstorm conditions and substorm conditions, imagining the total potential drop [$C_1(t)$] to be small and time independent prior to substorm onset. For such a steady state the SSDBs are taken to define the inner edge of the plasma sheet electron distribution, and this provides us with the needed initial conditions; i.e., we suppose the plasma sheet electrons of all energies are present at all tail locations beyond the SSDBs. For an initial convection potential drop of $\Phi = 60$ kV we have plotted in Figure 1 the SSDBs for 90° pitch angle electrons of 0-, 2.3-, and 5.3-keV energies. Kp before the substorm at 2200 UT was 3-, and the 60-kV potential drop is an upper limit estimate to what might be expected for such Kp [Kivelson, 1976]. It is seen that the boundaries are elongated so that they are farthest from the earth at dusk and closest to the earth at dawn and that higher energy boundaries lie progressively farther from the earth. The region inside the zero-energy SSDB, or plasmopause, is filled with a high-density cold plasma on closed orbits about the earth, and the cold plasma density drops sharply as the satellite moves out across the plasmopause [Chappell et al., 1970]. Because the fluxes of electrons which have convected in from the tail are excluded from regions earthward of the relevant SSDB, it is clear that finite energy electrons whose source is in the magnetotail are excluded from regions inside the plasmopause. This exclusion does not apply to positively charged particles. For example, protons with nonzero but ≤ 20 -keV energies can penetrate earthward of the plasmopause to form the proton nose structure [Smith and Hoffman, 1974].

Having selected the initial potential drop $\Phi_0 = 60$ kV, we then determined the SSDBs for 90° pitch angle electrons with initial energies from 0 to 10 keV in steps of 1 keV. Particles were placed along their respective SSDBs every 3° in azimuth from 120° to 240° (60° either side of midnight) to model a magnetotail source. To simulate substorm onset at time t_0 , we instantaneously increased the cross-magnetospheric potential to $\Phi_r = 140$ kV and tracked individual particles, calculating their position and energy every 1 min. From these computed trajectories we were able to construct the innermost locations at which plasma sheet particles of any given energy would be found as a function of time. These locations provided a time dependent demarcation boundary (TDDB) for each energy channel. For the particular data being studied, we chose boundary energies $W_0 = 0$ and W_i ($i = 1-4$) equal to 1.5, 2.3, 3.2, and 5.3 keV, corresponding to the minimum energy in each of the four lowest energy channels of the electron detector of Williams et al.

Turning to the Williams et al. [1974] observations (Table 1), we note that the data give an azimuth and radial distance at which flux changes in the electrons of energy W_i are observed at a time $t_i - t_0$ following substorm onset. Here we take t_0 to be the substorm onset time at 2200 UT as determined by Williams et al. from ground magnetograms. To compare our calculations with the observations, we predict the radial position of the TDDB for electrons of energy W_i at the satellite azimuth ϕ_i and at a time $t_i - t_0$ following the increase of the cross-

TABLE 1. Explorer 45 Electron Flux Observations, December 12-13, 1971, Orbit 86

Energy, keV	Time, UT	L	Time, LT	ϕ , deg
<i>Outbound (Flux Increase)</i>				
0	2221	5.5	2048	132
1.5-2.1	2230	5.5	2052	133
2.3-3.2	2253	5.5	2116	139
3.5-4.8	2314	5.5	2128	142
5.3-7.1	2339	5.3	2152	148
<i>Inbound (Flux Decrease)</i>				
5.3-7.1	0025	4.7	2240	160
3.5-4.8	0029	4.6	2248	162
2.3-3.2	0032	4.5	2256	164
1.5-2.1	0039	4.3	2304	166
0	0039	4.3	2304	166

Explorer 45 electron flux observations taken from Williams et al. [1974]. The universal time (UT) of the flux increases and decreases is given along with the position of Explorer 45 at the time of the observed flux change. The angle ϕ is measured eastward from noon. Substorm onset was at 2200 UT.

magnetospheric potential. For a good fit the predicted radial position should be close to the radial position of Explorer 45 at the azimuth ϕ_i . The results of our calculation are presented in Table 2. In this table we have again included the Explorer 45 radial distance and azimuth at the time of the observed flux increases and decreases in the energy channel labeled by W_i . In addition, we give the delay of the increase or decrease following substorm onset ($t_i - t_0$) and the radial positions, L_i^{pred} , of the TDDBs at $t_i - t_0$ and ϕ_i for $\alpha = 90^\circ$ electrons of energy W_i . The difference between predicted and observed values of L is entered as ΔL .

It is clear from Table 2 that there is very good agreement between the radial positions of Explorer 45 at the times of the observed flux changes and those of the TDDBs as predicted by our calculation. The maximum difference between observation and prediction occurs for the 5.3-keV electron flux decrease, but even in this worst case the TDDB lies only $\sim 0.2 R_E$ outside the Explorer 45 orbit. The average absolute difference between observation and prediction for this case is $\langle |\Delta L| \rangle = 0.1$. Figure 2 illustrates the relation of observed and predicted posi-

Fig. 1. Steady state demarcation boundaries (SSDB) for electrons ($\alpha_0 = 90^\circ$) are displayed for electron energies of 0, 2.3, and 5.3 keV for a cross-magnetospheric potential drop of $\Phi = 60$ kV. As is described in the text, these boundaries separate the spatial region in which the electrons of the specified energy are on closed orbits (earthward of the boundary) from that in which they are on open orbits (anti-earthward of the boundary).

TABLE 2. Comparison Between Explorer 45 Observations and Predictions of the Present Model

W_i , keV	L_i^{obs}	ϕ_i , deg	$t_i - t_0$, min	L_i^{pred}	ΔL
<i>Flux Increase</i>					
0	5.5	132	21	5.3	-0.2
1.5	5.5	133	30	5.4	-0.1
2.3	5.5	139	53	5.4	-0.1
3.2	5.5	142	74	5.4	-0.1
5.3	5.3	148	99	5.4	0.1
<i>Flux Decrease</i>					
5.3	4.7	160	145	4.9	0.2
3.2	4.6	162	149	4.7	0.1
2.3	4.5	164	152	4.5	0.0
1.5	4.3	166	159	4.4	0.1
0	4.3	166	159	4.2	-0.1

The Explorer 45 observations are given in the first four columns, where $t_i - t_0$ is the time after substorm onset at which the flux increases and decreases were observed. The predicted radial positions of the TDDBs at time $t_i - t_0$ at the satellite azimuth are listed in the fifth column. The difference between observed and predicted radial distances, ΔL , is given in the last column.

tions of the plasmopause and of flux increases and decreases in the four designated energy channels relative to the Explorer 45 trajectory.

DISCUSSION OF RESULTS

By use of a simple approximation to the earth's electric and magnetic fields, substorm-associated electron injection and energy dispersion can be quantitatively interpreted in terms of a convection model in which the potential drop across the magnetosphere increases discontinuously at substorm onset. The two free parameters of the calculation are the potential drop before and after the start of the substorm, and they are highly constrained. The initial potential drop gives consistent initial conditions for the calculation, and its magnitude is fixed by the time delay between substorm onset and the times of the initial

energy dependent plasma sheet boundary penetration. The final potential drop in the application of this model to the electron data of Williams *et al.* [1974] was fixed by the times of the return of the satellite to the region earthward of the plasma sheet. Other observations which can be explained in a similar fashion have recently appeared in the literature [Maeda *et al.*, 1978; Kivelson *et al.*, 1979].

The critical difference between our calculation and the previous work of Kivelson and Southwood [1975] and Cowley and Ashour-Abdalla [1976a, b] is the use of a time dependent model. Our calculation differs from that of M. Ejiri, R. A. Hoffman, and P. H. Smith (unpublished manuscript, 1978) because we introduce initial conditions consistent with the assumed model. Two different forms of the convection potential have been used in the papers cited. The present work and the work of M. Ejiri, R. A. Hoffman, and P. H. Smith (unpublished manuscript, 1978) use a convection potential (first term of (1)) which varies as L^2 , while Kivelson and Southwood [1975] and Cowley and Ashour-Abdalla [1976a, b] use a uniform convection field with a potential that varies as L . However, we find that the models are insensitive to the choice of L dependence because of the limited spatial range over which injection is observed.

We tested the sensitivity of calculations to the form of the potential in two different ways. First, we asked whether a steady state convection theory could successfully predict the locations of flux increases and decreases for either form of convection potential. We found the potential drop which minimized $\langle |\Delta L| \rangle$ for each of the assumed forms of potential. Using these values, 45 kV for the uniform field and 90 kV for the partially shielded field, we calculated the SSDBs for the two different potential forms; the results are displayed in Figure 3. (We note that the 45 kV calculated for the uniform field is less than that calculated by Kivelson and Southwood [1975]. They determined a potential drop of 55 kV by requiring that the Explorer 45 trajectory did not intersect the 8.0-keV SSDB.) The inaccuracy of the prediction is similar for the two cases, with $\langle |\Delta L| \rangle = 0.3$ for both the uniform field and the partially shielded field. Recognizing that for any given potential form a change in the magnitude of the potential drop would cause the entire SSDB pattern to shift to higher or lower L , we see from Figure 3 that no single choice of a potential could reduce the scatter illustrated or consistently account for both the flux increases and the flux decreases.

Having decided that the steady state calculation was not significantly improved by use of an L^2 dependent potential, we asked whether the time dependent calculation could be carried out with equal success for either form of the potential. Accordingly, we reran our time dependent calculation using a uniform field and, although we do not present our results here, obtained results as good as those shown in Figure 2 using values $\Phi_0 = 35$ kV and $\Phi_f = 75$ kV. Consequently, we believe that it is the time dependence and the self-consistent initial conditions which are the crucial features of our calculation, and not the form of the potential.

Although we have presented the results of only one computer run for the shielded potential, a number of such computations have been performed in which either Φ_0 or Φ_f or both were varied, and we did find the results to be sensitive to changes in these potentials. We varied Φ_0 from 20 to 60 kV and Φ_f from 100 to 160 kV; these values are typical polar cap potential drops observed by Heppner [1972] for quiet and disturbed times, respectively. We found 'good agreement' ($\langle |\Delta L| \rangle \leq 0.2$) for all combinations of Φ_0 and Φ_f such that

Fig. 2. Explorer 45 trajectory with positions of flux increases in energy channels W_1 to W_4 and flux decreases in channels W_4 to W_1 shown as vertical tick marks. Plasmopause entry and exit are marked as solid dots. The calculated radial distances to the TDDB for the i th energy channel at the time of an observed flux increase or decrease are shown by the circled numbers, and the position of the $W = 0$ TDDB at times of plasmopause encounter is shown by the circled P.

45 kV $\leq \Phi_0 \leq 60$ kV and 130 kV $\leq \Phi_f \leq 160$ kV. The results of the different runs also showed that the predicted flux increases were far more sensitive to changes in Φ_0 than to changes in Φ_f , whereas the predicted flux decreases were more sensitive to changes in Φ_f . These dependences may be easily understood by noting that the flux increases occurred shortly after substorm onset when the initial distribution determined by Φ_0 is of great importance. The flux decreases were seen after the distribution had experienced the final convection field for about 2 hours and therefore were more directly dependent on the value selected for Φ_f . These dependences also point out why a single choice of potential could not consistently account for both the flux increases and the flux decreases.

For our study we did not take t_0 , the substorm onset time, to be a free parameter, although it was pointed out by Williams *et al.* [1974] that there could have been a timing error of ± 10 min in the substorm onset time. We found that this uncertainty had negligible effect on our calculation, causing the range of Φ_0 values yielding good agreement to shift by a few kilovolts.

Throughout this paper we have stressed our use of initial conditions which are representative of a steady state unperturbed convecting plasma in the magnetosphere. To illustrate the initial conditions actually used in our calculation, we took particles with energies characteristic of each energy channel and traced them backwards in time (conserving μ and J) from their positions at $t = t_i$ to their initial positions at $t = t_0$, as shown in Figure 4. Note that the positions of these particles at $t = t_i$ were chosen to be the predicted positions of the corresponding TDDBs associated with the flux increases. The positions of the observed plasmopause crossing and subsequent flux increases given in Table 1 are indicated by the solid dots along the satellite trajectory. Two important features emerge from the calculation. First, as might be expected from the generally eastward drift at low L of electrons, the particles start at azimuths west of their final positions. They do not start close to midnight. Second, the particles start at radial distances which are within $2 R_E$ of their radial position at $t = t_i$. Note that all of the particles start inside of $L = 7$ and that the initial radial distance is strongly energy dependent.

M. Ejiri, R. A. Hoffman, and P. H. Smith (unpublished manuscript, 1978) have recently completed some calculations of time dependent proton convection in which they started particles of all energies on a single boundary which was azimuthally symmetric about midnight at $L = 10$. Our calculations indicate that such initial conditions seem inappropriate and that energy dependent boundaries at small radial distances should be assumed. In fact, a study of ATS 5 data just completed [Kivelson *et al.*, 1979] supports the view that low-energy (<15 keV) electrons and protons at synchronous orbit exhibit a steady state Alfvén boundary structure during quiet times. In the Kivelson *et al.* [1979] study, the energy dispersion seen in the electrons and protons in UCSD energy spectrograms for quiet times is quantitatively shown to be consistent with the motion of the satellite through the SSDBs for a weak convection electric field of the sort that we assume for the $t < t_0$ time period. Because we believe that SSDBs exist in the vicinity of synchronous orbit and that details of plasma injection are very sensitive to initial conditions, we are convinced that time dependent calculations which assume distant sources of injected particles and which neglect to incorporate the energy dispersion of initial conditions, such as those performed by M. Ejiri, R. A. Hoffman, and P. H. Smith (unpublished manuscript, 1978), can be misleading. In fact, we found it impossible to find quantitative 'good agreement' with the Williams *et al.*

Fig. 3. Same format as that of Figure 2; however, here the best fit SSDBs are shown for a uniform electric field (dashed circles) and for a partially shielded electric field (solid circles).

[1974] observations for any Φ_f using the Ejiri *et al.* initial condition. We propose that use of initial conditions similar to those we have adopted could reduce the unacceptably long time required for nose penetration in the Ejiri *et al.* calculation.

Finally, at this point in the analysis we must pose a philosophical question as to whether it is worthwhile to manipulate further the form of the convection potential or its time variation to improve the fit of our predictions to the observed flux changes. Although additional free parameters would yield im-

Fig. 4. Explorer 45 trajectory in the equatorial plane with trajectories of electrons with energies characteristic of each energy channel. The electrons were traced backwards in time from their positions at $t = t_i$ to their initial positions at $t = t_0$. The positions of the particles at $t = t_i$ were chosen to be the predicted positions of the corresponding TDDBs associated with the flux increases. The solid dots indicate the positions of the observed plasmopause crossing and subsequent flux increases as given in Table 1.

proved agreement with observations, such improved agreement would not necessarily improve our understanding of the injection process in this particular event or in qualitatively similar events unless we could independently determine the added parameters. We do not think that improved fits would add to the evidence presented to support the view that the inclusion of time dependent effects and self-consistent energy dependent initial boundaries in a simple model of convection is useful in understanding particle observations, similar to those presented in this study, shortly after substorm onset. The applicability of our approach over a wider range of substorm event types may be tested in the future through simultaneous measurements at two points in space; such measurements are now available from the Isee 1 and 2 spacecraft.

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