Approximations for the Study of Drift Boundaries in the Magnetosphere

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INTRODUCTION

The establishment or enhancement of an electric field oriented from dawn to dusk across the magnetosphere is widely recognized as a significant feature of geomagnetically active periods [Axford, 1969; Mozer, 1971; McIlwain, 1972; Heppner, 1973; Gurnett and Frank, 1973; Gurnett and Akasofu, 1974], and the resulting inward convection and adiabatic acceleration of tail plasma [Axford and Hines, 1961; Brice, 1967; DeForest and McIlwain, 1971] contribute to the substorm-associated particle flux enhancements observed in the quasi-dipolar regions of the magnetosphere [Arnoldy and Chan, 1969; Lezniak and Winckler, 1970; Konradi et al., 1973; Williams et al., 1974]. However, it is not possible for tail plasma to be convected into all portions of the near magnetosphere (L < 10) because there are 'forbidden zones' into which particles cannot penetrate by electric convection [Alfvén and Fälthammar, 1963; Schield et al., 1969]. The boundary of the forbidden zone is referred to as the 'Alfvén layer,' and its geometry is a sensitive function of the adiabatic invariants of the particles whose motion it characterizes [Chen, 1970, 1974] and of the magnitude of the convection electric field. Solutions have been obtained for a number of representative cases for 90° pitch angle particles in a dipole magnetic field [Chen, 1970, 1974], but the quantitative results have been published only for the midnight meridian. In this paper we derive some approximate solutions which can be used to calculate properties of Alfvén layers in the high- and low-energy limits and which apply to particles of arbitrary equatorial pitch angle. As we shall see, the high- and low-energy limits provide a very good guide to the behavior of electrons of arbitrary energy. The results are formulated as answers to the question, What approximate solutions can be used to interpret the substorm-associated electron fluxes reported recently by Williams et al. [1974] from Explorer 45 measurements.

DEFINING EQUATIONS FOR THE ALFVÉN LAYER

An Alfvén layer is defined for a particle of magnetic moment μ as a flow line on which there is a stagnation point where electric field drifts and magnetic drifts exactly balance. These boundaries can be very complex for low-energy protons, whose magnetic and corotation drifts can produce resonances, allowing them to penetrate into the inner magnetosphere [Chen, 1970]. Figure 1, adapted from Chen [1970], shows the possible complexity of these proton orbits. We will merely remark on the existence of multiple stagnation points for such protons and develop approximations to describe cases for which the Alfvén layer is geometrically simple (cf. Figure 3).

Convective flow conserves the total energy including the potential energy of the assumed uniform dawn-dusk electric field and of the corotation electric field. In the earth's dipole magnetic field the corotation potential is ΩBzRE:L - in terms of Bz, the equatorial field at 1 RE, and Ω = 2π/day. Magnetic field lines are assumed to be equipotentials in the region traversed by the particles being studied. Thus for convective flow,

\[ W + qER_{\phi}L \sin \phi - \left( q/|q| \right) CL^{-1} = \text{const} \]  \hspace{1cm} (1)

where for the earth, \( C = |q|ΩBzR_{E}^2 \approx 90 \text{ keV} \).

The azimuthal angle is measured counterclockwise from midnight. The vanishing flow velocity at the stagnation point requires

\[ \frac{\partial W}{\partial L} = -qER_{\phi} \sin \phi - \left( q/|q| \right) CL^{-3} \]  \hspace{1cm} (2)

The variation of W with L for a charged particle whose initial equatorial pitch angle is sin α_{eq} is well approximated by the expression

\[ \frac{W}{W_0} = \left( \frac{L_0}{L} \right)^\nu \quad \nu = 2.1 + 0.9 \sin \alpha_{eq} \]  \hspace{1cm} (3)

as demonstrated by Southwood and Kivelson [1975]. Cowley and Ashour-Abdalla [1974] have shown that for \( L_0/L \leq 5 \) this
energy estimate is accurate to better than 2% for $\alpha_{eq} > 5^\circ$. At $\alpha_{eq} = 0^\circ$ the energy estimate is less satisfactory, but the correct value is obtained by setting $\nu = 2$. From (3) the stagnation condition becomes

$$\nu W_s = eER_s L_s \sin \phi_s + (q/q_e)C L_s^{-1}$$

and $\cos \phi_s = 0$, as is evidently required by the symmetry of the system. The positions of the stagnation points are

$$L_s = \frac{\nu W_s \pm \sqrt{[\nu W_s]^2 - 4[q]E R_s C \sin \phi_s}}{2qER_s \sin \phi_s}$$

Thus electrons of energy $W$ have one stagnation point on the dusk meridian, while protons of energy $W$ have one stagnation point on the dusk meridian and two additional stagnation points on the dawn meridian if $\nu W_s > (4qER_s C)^{1/2}$. Conversely, for protons at fixed $\mu$ there is one stagnation point on the dawn meridian and two additional ones on the dusk meridian if $\mu < (C^2/4qER_s)$, surprisingly not an inconsistent statement.) The Alfvén layer of interest for high-energy protons is the one which passes through the larger $L_s$ at dawn, for this defines the region within which closed orbits encircle the earth and outside of which orbits are open. It is immediately clear that for the usual magnetospheric electric fields ($E \leq 3 \text{kV} R_e^{-1}$), protons of energy $>(39/\nu)$ keV on the dawn meridian are always on closed orbits, for unless $L_s$ is less than $\sim 10 R_e$, the boundary lies outside the magnetopause and if, in addition, $E \leq 3 \text{kV} R_e^{-1}$, (4) implies $W_s > (39/\nu)$ keV.

The locus of a boundary which passes through the stagnation point at $L_s$ is found by setting the total energy of (1) equal to its value at the stagnation point. Working nominally with electrons with $q = -e$ and $\phi_s = 3\pi/2$, we find

$$W - W_s = eER_s(L_s + L \sin \phi) + CL_s^{-1} - CL^{-1}$$

Eliminating $eER_s$ from (4) and (6), we obtain an equation for $y = L/L_s$

$$\nu y^{\nu+1} \sin \phi + (\nu + 1)y - 1 = (C/LW)(1 - 2y - y^2 \sin \phi)$$

Equation (7) can be solved numerically for electrons of any energy, in which case the only approximation is the use of (3). In the next section, however, we develop solutions which are useful in the high- and low-energy limits.

HIGH- AND LOW-ENERGY LIMITS

In the high-energy limit, $C/WL \ll 1$, the corotation terms on the right-hand side are small and may be neglected to lowest order. This condition is always satisfied for proton orbits corresponding to cases for which $L_s$ is the larger solution of (5) on the dawn meridian; for the condition that the root be real, $(\nu W_s)^2 > 4eER_s C$ can be combined with the inequality $L_s > \nu W_s/(eER_s)$ to give

$$\frac{CL}{LW} = \frac{C}{L_s W_s} \left( \frac{L_s}{L} \right)^{-1} \leq eER_s C \left( \frac{L_s}{L} \right)^{-1} \leq \nu \frac{(L_s/L)}^{\nu-1} \leq \frac{\nu}{4}$$

since the stagnation point is always the point at which the Alfvén layer is farthest from the earth. The zeroth-order approximation to $y$ satisfies

$$\nu \sin \phi \ y_{0}^{\nu-1} + (\nu + 1)y - 1 = 0$$

For $\nu = 3$, (8) reduces to the solution obtained in the high-energy limit by Alfvén [Alfvén and Fichtlmann, 1963]. Plots of $y_0$ as a function of $\sin \phi$ for $\nu = 2$ and $\nu = 3$ are given in Figure 2. Energetic protons are described by (8) with $\sin \phi = -\sin \phi$. The curves of Figure 2 indicate that when corotation can be neglected, the electron Alfvén layers near dawn (or proton Alfvén layers near dusk) are quite insensitive to equatorial
pitch angle but that both the asymmetry of the Alfvén layers and their dependence on equatorial pitch angle become large in the direction of the stagnation point. On this side of the earth the particle energy is lowest, and convection is naturally more important. One should note that the fractional change of electron energy on a closed orbit between midnight and dawn is of the order of 30% for $2 \leq \nu \leq 3$.

The correction of order $C/WL$ to the solution of (7) is

$$\delta y = \frac{1 - 2y_0 - y_0^2 \sin \phi}{\nu(\nu + 1)\gamma_0^{\nu+1}(\gamma_0 \sin \phi + 1)}$$

where $y = y_0 + (C/WL)\delta y$. Values of $\delta y$ are plotted in Figure 2 and are seen to be small. The equivalent result for energetic protons is found by setting $\phi = -\phi$, $W = -W$.

To zeroth order in $C/WL$, the drift orbit of a particle on the Alfvén layer is found by expressing the stagnation condition ((4) with $C = 0$) in terms of $y$ as

$$L = \nu W \gamma_0(\phi)/(|q|E_{Rf})$$

Hence a particle at longitude $\phi$ with energy $W$ has an orbit given by

$$L = \frac{\nu W (\gamma_0(\phi))' \gamma_0(\phi)}{|q|E_{Rf}}$$

if it is on an Alfvén layer.

In the low-energy limit the corotation term in (7) dominates, so an approximate solution of the form $y_0' = LW/C\delta y'$ is given by

$$\sin \phi y_0' + 2y_0' - 1 = 0$$

$$\delta y' = \frac{y_0'}{2(y_0' - 1)} \left[ y_0'' + 1 \right] \sin \phi + (\nu + 1)y_0' - 1$$

$$= \frac{1}{2(1 + y_0' \sin \phi)} \left[ y_0'' + 1 \right] \sin \phi + (\nu + 1)y_0' - 1$$

The quantities $y_0'$ and $\delta y'$ are also plotted in Figure 2. The result for low-energy protons, found by setting $W = -W$, is useful only for the portion of the Alfvén layer near the dusk stagnation point labeled $a$ in Figure 1, because of the complexities of the multiple-valued low-energy solutions. For electrons, such complications do not arise.

**Electric Field Producing an Alfvén Layer for Given $(L, W, \phi)$**

As satellite experiments typically give information on particles in a specific energy range, it is of practical value to be able to answer the following questions. For a given value of $W$ and $\alpha_{eq}$, what cross-magnetosphere field $E_a = E_a(W, \alpha_{eq}; L, \phi)$ would place the Alfvén layer at $(L, \phi)$, or alternatively, for given values of $W, \alpha_{eq}$, and cross-tail potential, what is the locus $L_a = L_a(W, \alpha_{eq}; E; \phi)$ of the boundary between closed and open orbits? Figure 3 shows schematically how the boundary $L_a$ is related to the Alfvén layers of high-energy protons, which conserve magnetic moment $\mu$ in their drift. Within the boundary $L_a$ an energetic proton observed with energy $W$ is on a closed trajectory in the dipole field, whereas at positions beyond $L_a$ the trajectories of particles of the same energy must be open.

As $L_a$ is defined by an equation of third or fourth order, depending on the pitch angle of interest, it is more sensitive to small errors in the approximate solution than $E_a$ is. Hence we first obtain an approximate solution for $E_a$ and invert it to obtain $L_a$. In the high-energy limit we find from (4), (8), and (9)

$$\frac{eE_{Rf}}{L} = \frac{W}{\nu + 1 + y_0' \sin \phi} + \frac{C}{L} y_0(1 - y_0')$$

$$= \frac{W}{L} + \frac{\beta}{L}$$

For a known cross-magnetosphere field, (12) can be solved to give $L_a$. In these expressions, $y_0$ is a function of $\phi$ defined by (8) and plotted in Figure 2. The equivalent results for high-energy protons are obtained by setting $C = -C$ and $\phi = -\phi$ in (8) and (12). The parameters $\alpha$ and $\beta$ for electrons of 0° and 90° pitch angle versus sin $\phi$ are shown in Figure 4 in units which give $eE_{Rf}$ in keV.

The dawn-dusk asymmetry of the boundary $L_a$ is much greater than the asymmetry of the Alfvén layers themselves, as can be seen when corotation is ignored by comparing the curves for $y_0(\phi)$ in Figure 2 with the corresponding curves for $\alpha(\phi)$ in Figure 4. The larger asymmetry of the boundary for fixed energy occurs because $L_a(\phi)$ crosses Alfvén layers of increasingly large $\mu$ as it approaches the side of the magnetosphere where the kinetic energy of the particle species is smallest. These Alfvén layers with large $\mu$ are found at great distances from the dipole center.

The ratio of $L_a$ for 0° particles to $L_a$ of 90° particles is significant, for it indicates that small pitch angle particles can penetrate further than 90° pitch angle particles into the dipole field from a source region in the tail. For example, near midnight the boundary for energetic 0° particles lies about 25% further in than the 90° boundary. The possibility of large energy dependent pitch angle anisotropies occurring in the region between the boundaries $L_a(\nu = 3)$ and $L_a(\nu = 2)$ has interesting consequences for the possible development of wave-particle instabilities, a possibility which is being investigated.
The corotation contributions to $E_A$ are positive for electrons because the addition of the corotation velocity effectively increases the magnetic angular velocity relative to the electric field drift velocity. In order to obtain balance at the stagnation point, the electric field must increase. By similar reasoning, if the electric field is fixed, the stagnation point must move outward to decrease magnetic drift effects relative to electric field drift effects. As $L_s$ increases, the Alfvén layer moves out at all $\phi$.

The boundary $L_A$ lies at larger radial distances for electrons, whose corotation and magnetic drifts supplement each other, than for protons, whose corotation and magnetic drifts oppose each other. In the Jovian magnetosphere, where the dipole orientation is opposite to that of the earth, the corotation drift opposes the magnetic drift for electrons, which can therefore be convected in to smaller radial distances than protons. (Swift [1971] has commented on this feature of convective flow in a reversed geomagnetic dipole field.) Correspondingly, electrons can gain more energy than protons during inward convection, and this may play a role in producing the intense energetic electron flux observed in the dipole field region on Jupiter [Opp, 1974, and references cited therein].

For low-energy electrons the values of $E_A$ and $L_A$ are obtained analogously by use of (11) in the limit $WL/R \ll 1$, which yields

$$eE_AR_E = \frac{C}{L^3} y_0 + \frac{W}{L} \left( y_0' - 1 \right)$$

where $y_0'$ is defined by (11a). Curves for $\beta'$ and $\alpha'$ are given in Figure 5. Equation (13) can be solved for $L$ to give $L_A$ for known $E$. The low-energy approximation for protons, useful near the dusk stagnation point, is obtained by setting $W = -W$.

The coefficients $\alpha$ and $\beta$ are remarkably close to the coefficients $\alpha'$ and $\beta'$, as is seen in Figure 6, where curves of $(\alpha - \alpha')/\alpha$ and $(\beta' - \beta)/\beta$ versus $\sin \phi$ are plotted for the case $\nu = 3$; the values for $\nu = 2$ are of the same order. This suggests that the linear expression is a good approximation for any electron energy (Chen's [1970] results for intermediate energy protons indicate that this could not be good for protons). We can in fact show that approximations (12) and (13) provide lower bounds to $E_A$. From (4) we obtain

$$eE_AR_E = \frac{C}{L^3} y_0 + \frac{W}{L} \left( y_0' - 1 \right)$$

with $\eta = eE_AR_EL/C$ and $\rho = WL/C$ we have

$$\eta = \rho y + y^2$$

where $y$ is an implicit function of $\rho$ by virtue of (7). Now

$$\frac{\partial \eta}{\partial \rho} = y y' + \frac{\rho y}{1 + y y'}$$

and so

$$\frac{\partial \eta}{\partial \rho} = \frac{y}{1 + y y'}$$

where $y$ is defined by (7).
slowly, that it has a high probability of being precipitated near the boundaries is a consequence of the fact that near the precipitation boundaries and has shown that although they coincide closely over a large part of the nighttime magnetosphere. He has also noted that the near coincidence of they coincide on the day side of the magnetosphere, may be well separated on the day side of the magnetosphere, electric field to calculate electron Alfvén layers and may be well separated on the day side of the magnetosphere, the alternative possibility that the boundary is a precipitation boundary must be considered. Wolf [1970] has used different models of the convection electric field to calculate electron Alfvén layers and precipitation boundaries and has shown that although they may be well separated on the day side of the magnetosphere, they coincide closely over a large part of the nighttime magnetosphere. He has also noted that the near coincidence of the boundaries is a consequence of the fact that near the stagnation point on the Alfvén layer the plasma flows so slowly that it has a high probability of being precipitated before it is convected away.

The application of an Alfvén layer analysis is probably most clearly relevant to observations of the type discussed by Kivelson and Southwood [1975], who interpreted the asymmetric distribution of particle flux at fixed energy observed in the early part of a geomagnetic storm [Frank, 1970]. For analysis of this type of experimental data we have found the approximate expressions of the preceding section to be a useful alternative to exact numerical solutions. As an example of further experimental observations which can be interpreted consistently using the arguments of this paper, we consider the complex substorm-associated increases of electron flux reported by Williams et al. [1974]. The electron flux was measured by the Explorer 45 satellite as it moved through the premidnight magnetopause near apogee at L = 5.5. (In this spatial region, Alfvén layers and precipitation boundaries are likely to be approximately coincident.) Williams et al. note that although the high-energy (35–560 keV) flux peaks arrived at the satellite with a dispersion consistent with magnetic gradient drift to the satellite from a source near midnight, the highest-energy particles arriving earliest, the low-energy (1.5–10.8 keV) fluxes arrived in the reverse order, the lowest-energy particles arriving first at the satellite. Williams et al. suggest that the source of the low-energy electrons could be the plasma sheet, which is convected in by an electric field for which they obtain a crude estimate of 0.7 kV R\(^{-1}\). They reject this interpretation, because they note that particle trajectories calculated by McIlwain [1972] for a model electric field developed to interpret particle measurements at geostationary orbit do not penetrate deeply enough to bring plasma sheet electrons to the Explorer 45 orbit. Noting, however, that the minimum penetration distance \(L_a(\phi)\) is a rather sensitive function of the cross-tail field, we can determine how large an electric field is required to allow electrons of \(W < 8 \text{ keV}\) to convect inward to the Explorer 45 orbit and then examine other aspects of the experimental observations to see whether they are reasonably consistent with the assumed electric field.

Relevant features of the Williams et al. [1974] measurements in the low-energy channels are given in Table 1. If the significant aspects of these observations are to be interpreted in terms of convection boundaries, we must postulate an electric field in which the Explorer 45 orbit intersects the cutoff boundaries given by (13) for \(W \leq 5.3 \text{ keV}\) and does not intersect the cutoff boundary for \(W = 8.0 \text{ keV}\). The requirement is met for 1.8 kV R\(^{-1}\) < \(E < 2.2 \text{ kV R}\)\(^{-1}\), and such electric fields are in the range usually associated with substorms [Mozier, 1971; Heppner, 1973; Gurnett and Frank, 1973]. In Figure 7 we

### TABLE 1. Positions of Initial Increases Observed in Low-Energy Electron Fluxes [Williams et al., 1974] and Predicted Intersections of the Explorer 45 Orbit

<table>
<thead>
<tr>
<th>Energy Range</th>
<th>Observations</th>
<th>Predictions for (E = 1.9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>([0.80, 10.8])</td>
<td>([\cdots, \cdots, \cdots])</td>
<td>([5.5, 45^\circ])</td>
</tr>
<tr>
<td>([5.3, 7.1])</td>
<td>([5.3, -32^\circ])</td>
<td>([5.4, -51^\circ])</td>
</tr>
<tr>
<td>([3.5, 4.8])</td>
<td>([5.5, -38^\circ])</td>
<td>([5.3, -55^\circ])</td>
</tr>
<tr>
<td>([2.3, 3.2])</td>
<td>([5.5, -40^\circ])</td>
<td>([5.2, -58^\circ])</td>
</tr>
<tr>
<td>([1.5, 2.1])</td>
<td>([5.5, -47^\circ])</td>
<td>([5.2, -58^\circ])</td>
</tr>
<tr>
<td>Plasmapause out</td>
<td>([5.5, -48^\circ])</td>
<td>([4.9, 63^\circ])</td>
</tr>
<tr>
<td>Plasmapause in</td>
<td>([4.5, -14^\circ])</td>
<td>([3.2, 8^\circ])</td>
</tr>
</tbody>
</table>

The cutoff boundaries were obtained from (13) for the lowest-energy electrons in each channel in a field \(E = 1.9 \text{ kV R}\)\(^{-1}\). Dots indicate that no rise was observed and that no rise was predicted.

\[
\left. \frac{\partial^2 E}{\partial \phi^2} \right|_{E=0} = \left( \frac{\partial y}{\partial \phi} \right)^2 \left( 2 + \frac{\partial y}{\partial \phi} + 1 \right) \frac{1}{1 + y \sin \phi} \quad (16)
\]
would have simultaneously produced convection and acceleration. Kivelson [1975] has introduced, and which may be thought to have developed earth, as suggested by Williams et al. The assumed electric field were somewhat skewed [e.g., Volland, 1973], the crossings of the boundaries as well as the plasmapause encounters would coincide more closely with the observed flux increases. The highly idealized model of this paper can at best apply only to average behavior, so we have not attempted to adjust parameters to obtain better agreement with the observations.

The Williams et al. [1974] observations can now be interpreted by considering how electrons in the range 1.5-560 keV would respond to the expected combination of tail magnetic field collapse [McPherron et al., 1973] and an ~2 kV Re depressive electric field associated with the substorm onset before the first flux increases were observed. During substorms, significant increases in the magnetic field are typically observed within 3 hours of local midnight [Clauer and McPherron, 1974]. As Explorer 45 was initially outside this local time region and moved in local time at a rate of less than or approximately equal to the corotation angular velocity, it did not encounter particles which had been locally accelerated by the changing magnetic field. However, high-energy electrons initially within the region of magnetic field increases near midnight and accelerated at the substorm onset encountered the satellite after drifting once around the earth, as suggested by Williams et al. The assumed electric field would have some effect on the energetic electrons but would not change the picture in any qualitative way [Walker and Kivelson, 1975].

For the low-energy electrons the electric field which has been introduced, and which may be thought to have developed 30 min to 1 hour before the substorm onset [Moser, 1971], would have simultaneously produced convection and acceleration [Kivelson and Southwood, 1975] with little azimuthal drift. Thus these electrons would have reached the satellite mainly by being convected inward. For 1 Re of inward displacement from L = 6.5 to L = 5.5, adiabatic convection leads to energy increases of the order of 60%, and this could easily account for the peaks in the low-energy channels. The interpretation of the arrival times of the low-energy particles has already been given.

Turning to the proton measurements which are also reported by Williams et al. [1974], we can merely note that the energetic protons are found within their Alfvén layers but that we have not analyzed the lower-energy protons because of the complexity of low-energy proton orbits previously noted. However, Smith and Hoffman [1974] have developed arguments with a philosophy similar to that of the arguments presented here in their interpretation of proton fluxes observed by Explorer 45 during the beginning of several magnetic storms in 1971 and 1972.

Conclusions
The convection boundaries associated with the cross-magnetosphere electric fields occurring during the early stage of magnetic storms and substorms depend strongly on particle energy and azimuthal position. The spatial and temporal variations of the particle flux may be largely decoupled across such boundaries, so that sharp gradients or very different time dependence may be observed for particles whose energies lie above or below some cutoff energy. We hope that the simple approximations given in this paper will prove useful in identifying such boundaries in experimental data. The simplicity of these results is inevitably tied to the simplicity of the model we have assumed, and we feel that their main use should be as a rule of thumb guide, most powerful when applied to electrons. Naturally, the further the magnetic field departs from a dipole (and the further the electric field departs from a uniform dawn-dusk field), the greater should be the degree of skepticism accorded to our estimates. We would like to point out the usefulness of considering the boundary L = (W, φ) as it was described here in future more sophisticated work. One should further bear in mind that the model fields are static and that, accordingly, there is an inherent time lag in the complete formation of an Alfvén layer which is comparable with the drift time about the earth.

The analysis of the Williams et al. [1974] electrons in this paper and recent studies of proton fluxes during the development phase of magnetic storms [Kivelson and Southwood, 1975; Smith and Hoffman, 1974] show that drift boundaries can be significant and must be considered in an analysis of particle behavior. During the later (main and recovery) phases of storms the simple sharp boundary produced by a time independent electric field is no longer found, both because nonadiabatic processes become increasingly important over longer time intervals and because repeated injection events implying time-varying electric fields (a type of nonadiabatic process) can move particle boundaries to new positions. For this reason the energy and L dependent enhancements of energetic proton flux measured during the main phase and recovery phase of a storm by Konradi et al. [1973] cannot be interpreted in terms of an Alfvén layer for a reasonable electric field. Analogous measurements made during the development phase of a storm should show some of the features which we have described and would be of great interest.

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